Master MOSIG – Mathematical Foundations of Machine Learning – Dec 10, 2024.

Firstname : .....
Lastname : .....

Please be **concise** and **precise**. All documents are allowed. Electronic devices are not allowed.
Duration : 30min.

We want to compute the optimal strategy of a gambler. Initially, the gambler starts with $S_0 \in \{1 \dots, 4\}$ euros. At each time step, he/she can bet up to S_t euros (bets are integer between 0 and S_t).

If the gambler bets A_t, then with probability $p \in (0, 1)$, he/she wins $2U_t$ and with probability $q = 1 - p$, the gambler looses A_t:

$$S_{t+1} = \begin{cases} S_t + 2A_t & \text{with probability } p \\ S_t - A_t & \text{with probability } q = 1 - p \end{cases}$$

1. We assume that the gambler wants to maximize the probability of reaching $S_t \geq 5$ euros within T bets. Explain why this can be modeled as a MDP and provide the state space, action space, reward and transition probabilities. Shall we use a finite-horizon or a discounted reward criterion?

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2. Let $V_t(s)$ be the probability that the gambler ends with 5 or more euros given that he/she started with s euros after t bets. We set $V_T(5) = 1$ and $V_T(s) = 0$ for $s < 5$. Write below a set of equations that link the values of $V_t(s)$, $V_{t+1}(s')$, p and q (for $t < T$):

   - $V_t(0) =$ .....
   - $V_t(1) =$ .....
   - $V_t(2) =$ .....
   - $V_t(3) =$ .....
   - $V_t(4) =$ .....
   - $V_t(5) =$ .....

3. Assume that $T = 1$. Compute the optimal strategy and its expected performance.

4. Assume that $T = 4$ and $p = 1/2$. Complete the following table of $V_t(s)$ (justify below).

V	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = T = 4$
$s = 0$	$V_0(0) = \ldots$				
$s = 1$					
$s = 2$					
$s = 3$					
$s = 4$					
$s = 5$					

5. Describe the optimal strategy (for $p = 1/2$).