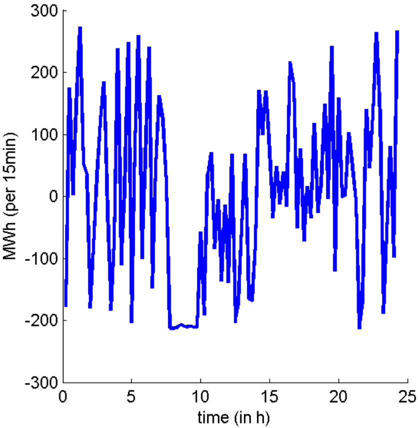
Efficiency and Prices in Real-Time Electricity Markets

Nicolas Gast

EPFL and Inria

April 12, 2014

Quiz: what is the value of energy?



Average price is 20\$/MWh.
Average production is 0.

1. 0\$.
2. $150k\$$
3. $-150k\$$.

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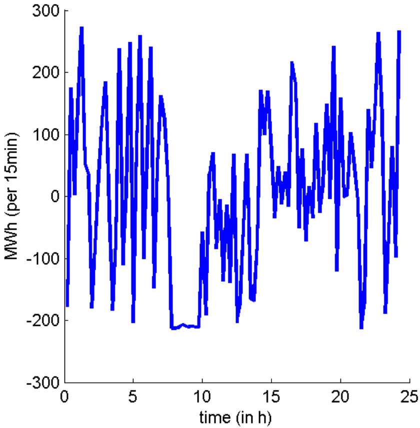

Average price is 20\$/MWh.
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1. 0\$.
   YES: If you are a private consumer.
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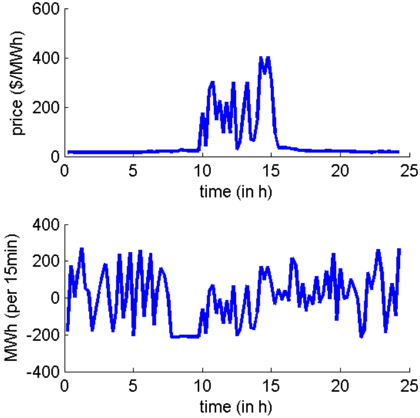

Average price is 20$/MWh.
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1. 0$.
   YES: If you are a private consumer.
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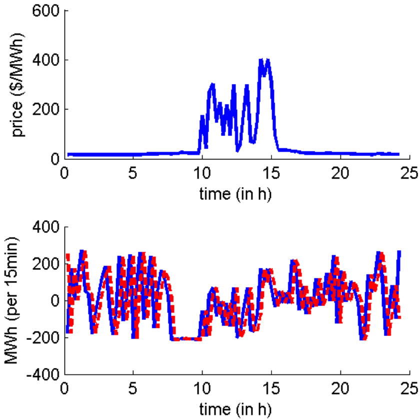
   YES: If you buy on the real-time electricity market (Texas, mar 3 2012)
3. $-150k$$.

Quiz: what is the value of energy?



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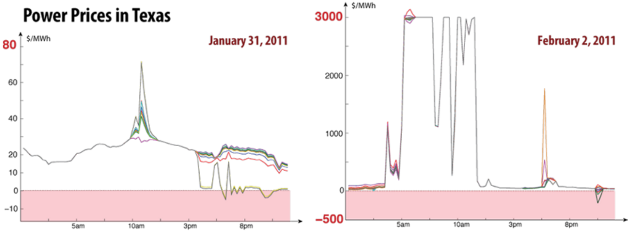
1. 0\$.
   YES: If you are a private consumer.
2. $150k\$$
   YES: If you buy on the real-time electricity market (Texas, mar 3 2012)
3. $-150k\$$.
   NO (but YES for the red curve! Texas, march 3rd 2012)

Can we explain real-time electricity markets?

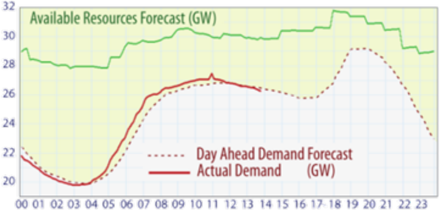


Is it price manipulation or an efficient market?

Issue 1: The electric grid is a large, complex system

It is governed by a mix of economics (efficiency) and regulation (safety).

Issue 2: Mix of forecast (day-ahead) and real-time control



Mean error: 1–2%

Mean error: 20%

Main message

- Real-time prices can be used for control
  - Decentralized control
- But:
  - Price fluctuation
  - Under-investment, observability

Outline

1. Real-Time Market Model and Competitive Equilibria
2. Numerical Computation of an Equilibrium and Distributed Optimization
3. Consequences of the efficiency of the pricing scheme
4. Summary and Conclusion

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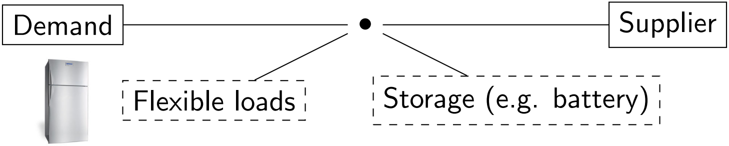
1. Real-Time Market Model and Competitive Equilibria

2. Numerical Computation of an Equilibrium and Distributed Optimization

3. Consequences of the efficiency of the pricing scheme

4. Summary and Conclusion

We consider the simplest model that takes the dynamical constraints into account (extension of Cho-Meyn 2006)



- Generator constraints

- Uncertainty of renewable and consumption.

- Storage :

- Demand-response:

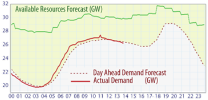
We consider the simplest model that takes the dynamical constraints into account (extension of Cho-Meyn 2006)

Demand — • — Supplier

Flexible loads

Storage (e.g. battery)

- Generator constraints : $\zeta^- \le \dfrac{G(t) - G(s)}{t - s} \le \zeta^+$
- Uncertainty of renewable and consumption.



- Storage : Finite power and energy capacity. Efficiency $\eta \le 1$.
- Demand-response:
  - Flexible consumption (temperature dead-band). For example:

We assume perfect competition between 2, 3 or 4 players

Players={supplier, demand, storage operator, flexible demand aggregator}

Player i maximizes:

$$\arg\max_{E_i \in \text{internal constraints of } i} \mathbb{E}\left[\int_0^\infty \underbrace{W_i(t)}_{\text{internal utility}} - \underbrace{P(t)}_{\text{spot price}} \cdot \underbrace{E_i(t)}_{\text{bough/sold energy}} dt\right]$$

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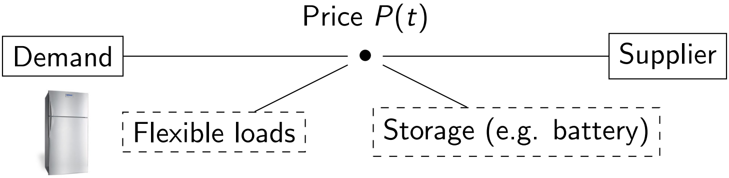
Players are assumed price-takers: they cannot influence $P(t)$.



Each player has internal utility/constraints and exchange energy

	Internal utility	Sold energy	Intern. constraints
Supplier	Generation cost	Generated energy	ramping
Demand	-disutiliy of b/o	Consumed energy	
Storage operator	Aging	(dis)charged	power/efficiency
Flexible load	Undesirable states	Consumed energy	temperature dead-band

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Special case (cho-meyn 2006): linear cost functions.

- **Linear cost of generation:** $cG(t)$
- **Demand:** $v\underbrace{\min(D(t),E(t))}_{\text{satisfied demand}}-c^{bo}\underbrace{\max(D(t)-E(t),0)}_{\text{frustrated demand}}$.
- **Storage :** $0 \le B_0 + \sum_{s=1}^{t} E_s(t) \le B_{\max}$ **and** $-D_{\max} \le E_S \le C_{\max}$.

Energy balance and the social planner's problem

$(E_1^e, \dots, E_j^e)$ is socially optimal if it maximizes $\mathbb{E}\left[\int_0^\infty \underbrace{\sum_{i \in \text{ players}} W_i(t)\, dt}_{\text{social utility}}\right]$,

subject to

- For any player i, E_i^e satisfies the constraints of player i.
- The energy balance condition: for all t:

$$\sum_{i \in \text{players}} E_i^e(t) = 0.$$

Definition: competitive equilibrium

$(P^e, E_1^e, \dots, E_j^e)$ is a competitive equilibrium if:

- For any player i, E_i^e is a selfish best response to P:

$$\arg\max_{E_i \in \text{internal constraints of } i} \mathbb{E}\left[\int_0^\infty \underbrace{W_i(t)}_{\text{internal utility}} - \underbrace{P(t)E_i(t)}_{\text{bough/sold energy}} dt\right]$$

- For all t: $\sum_{i \in \text{players}} E_i^e(t) = 0$

The market is efficient (first welfare theorem)

Theorem

Any competitive equilibrium is socially optimal.

Very general result (Cho-Meyn 2006, Wang et al. 2012, Gast et al. 2013, 2014).

Proof. The first welfare theorem is a Lagrangian decomposition

For any price process P:

social planner's problem

$$\boxed{\max_{\substack{E_i \text{ satisfies constraints } i \\ \forall t : \sum_i E_i(t) = 0}} \mathbb{E}\left[\sum_{i \in \text{players}} \int W_i(t)dt\right]}$$

selfish response to prices

$$\leq \sum_{i \in \text{players}} \boxed{\max_{E_i \text{ satisfies constraints } i} \mathbb{E}\left[\int (W_i(t) + P(t)E_i(t))dt\right]}$$

If the selfish responses are such that $\sum_i E_i(t) = 0$, the inequality is an equality.

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selfish response to prices

$$= \sum_{i \in \text{players}} \boxed{\max_{E_i \text{ satisfies constraints } i} \mathbb{E}\left[\int (W_i(t) + P(t)E_i(t))dt\right]}$$

If the selfish responses are such that $\sum_i E_i(t) = 0$, the inequality is an equality.

What is the price equilibrium? Is it smooth?

What is the price equilibrium? Is it smooth?

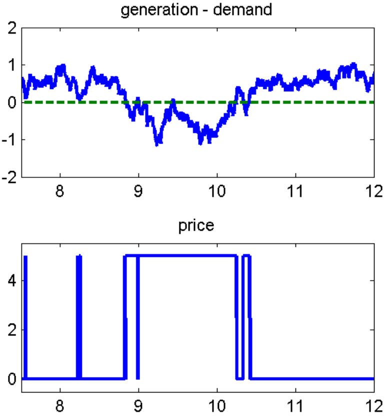
Reminder

Price is such that: for any player i, E_i^e is a selfish best response:

$$\underset{E_i \in \text{internal constraints of } i}{\arg\max} \mathbb{E}\left[\int_0^\infty \underbrace{W_i(t)}_{\text{internal utility}} - \underbrace{P(t)E_i(t)}_{\text{bough/sold energy}} dt\right]$$

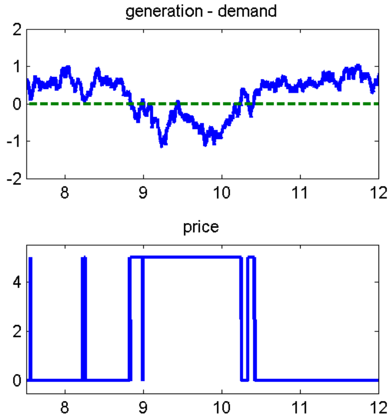
- Production has ramping constraints,
- Demand does not.

Fact 1. Without storage, prices are never equal to the marginal production cost.

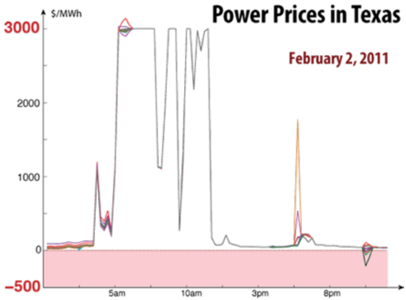


No storage

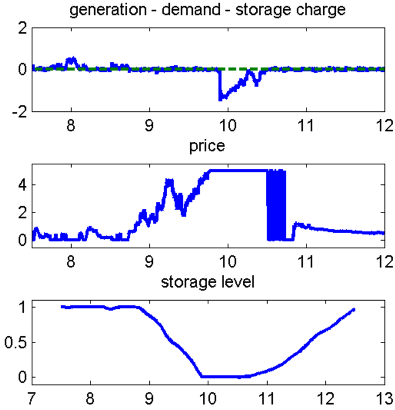
Fact 1. Without storage, prices are never equal to the marginal production cost.



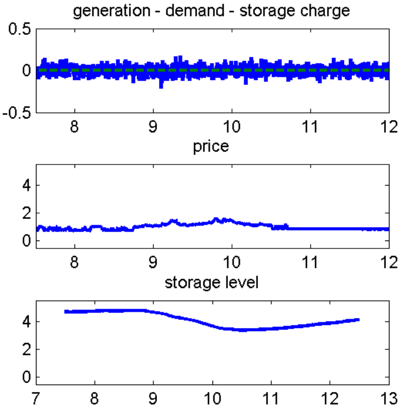
No storage



Fact 2. Storage leads to a price concentration

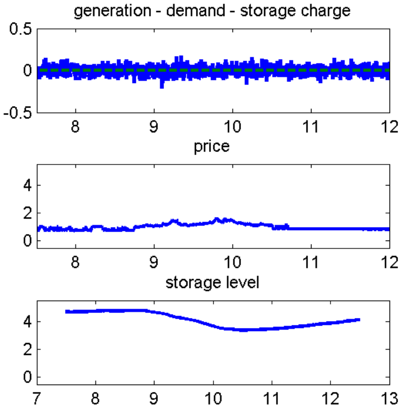


Small storage

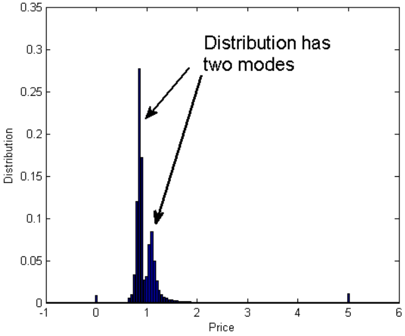


Large storage

Fact 3. Because of (in)efficiency, the price oscillates, even for large storage



Large storage



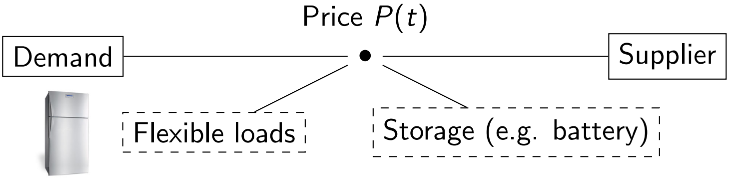
Two modes in $\sqrt{\eta}$ and $1/\sqrt{\eta}$

Outline

Reminder

If there exists a price such that selfish decisions leads to energy balance.

- These decisions are optimal.



- There exists such a price.
- We can compute it.

We design a decentralized optimization algorithm based on an iterative scheme

Iterate

Price $P(t)$

Generator

Demand

$\vdots$

Fridges

Difficulties

1. Forecast errors
2. Stochastic behavior of appliances
3. Convergence guarantee

We design a decentralized optimization algorithm based on an iterative scheme

Iterate

Price $P(t) \xrightarrow{\textbf{1.}\ \text{forecasted price}\ P(1),\dots,P(T)}$

Generator

Demand

$\vdots$

Fridges

Difficulties

1. Forecast errors
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We design a decentralized optimization algorithm based on an iterative scheme

Iterate

Price $P(t)$ $\underset{\textbf{2. forecasted consumption}}{\overset{\textbf{1. forecasted price } P(1),\dots,P(T)}{\rightleftarrows}}$ Generator / Demand / $\vdots$ / Fridges

1. forecasted price $P(1), \dots, P(T)$
2. forecasted consumption

Generator

Demand

$\vdots$

Fridges

Difficulties

1. Forecast errors
2. Stochastic behavior of appliances
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We design a decentralized optimization algorithm based on an iterative scheme

Iterate

Generator

Price $P(t)$ $\overset{\textbf{1. } \text{forecasted price } P(1),\dots,P(T)}{\underset{\textbf{2. } \text{forecasted consumption}}{\rightleftarrows}}$ Demand

$\vdots$

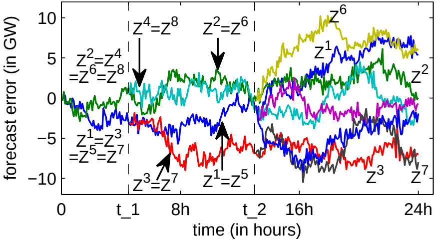
3. Update price

Fridges

Difficulties

1. Forecast errors
2. Stochastic behavior of appliances
3. Convergence guarantee

Solution 1: We represent forecast errors by multiple discrete-time trajectories



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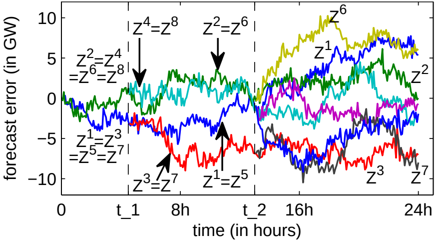
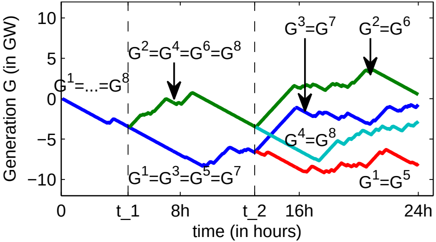
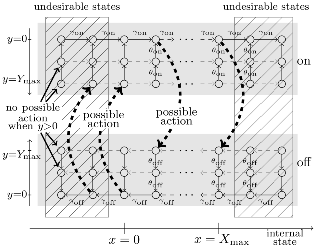




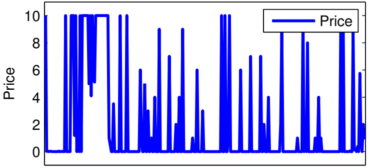
Finite number of observation point.

Solution 2: Each flexible appliance computes its best-response to price

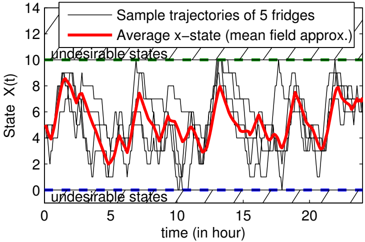


$=$

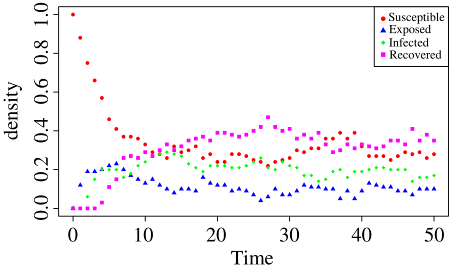
Object $=$ Markov chain



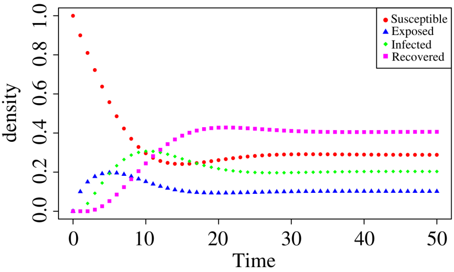
$\Downarrow$ best response



The global behavior of the flexible appliances can be approximated by a mean-field approx.



Original system



Mean-field approximation
(limit as number of appliances is large)

The problem is convex.

$$\begin{array}{ll} \text{minimize} & \displaystyle\sum_{i \in \text{players}} \mathbb{E}\left[W_i(E_i)\right] \\ \text{subject to} & E_i \text{ satisfies constraints of player } i \\ & \text{For all } t\text{: } \sum E_i(t) = 0 \end{array}$$

Constraints = observability + generator / demand / storage / flexible load.

Dual ascent method is decentralized but not robust

Lagrangian:

$$L_0(E,P) := \sum_{i\in\text{players}} W_i(E_i) + \sum_t P(t)\left(\sum_i E_i(t)\right)$$

Dual ascent method:

$$\begin{aligned} E^{k+1} &\in \arg\max_E L_0(E,P^k) \\ P^{k+1} &:= P^k - \alpha^k\left(\sum_i E_i^{k+1}\right) \end{aligned}$$

Good: distributed.

Bad: converges... under some conditions.

Method of multiplier is robust but not distributed

Augmented Lagrangian:

$$L_{\rho}(E,P) := \sum_{i\in\text{players}} W_i(E_i) + \sum_t P(t)\left(\sum_i E_i(t)\right) - \frac{\rho}{2}\left(\sum_t \sum_i E_i(t)\right)^2$$

Method of multipliers:

$$\begin{aligned} E^{k+1} &\in \arg\max_E L_{\rho}(E,P^k) \\ P^{k+1} &:= P^k - \rho(\sum_i E_i^{k+1}) \end{aligned}$$

Good: (almost) always converge.
Bad: not distributed

Solution 3: add extra variables and use ADMM

Augmented Lagrangian:

$$L_\rho(E,P) := \sum_{i \in \text{players}} W_i(E_i) + \sum_t P(t)\left(\sum_i E_i(t)\right) - \frac{\rho}{2}\sum_{t,i}\left(E_i(t) - \bar{E}_i(t)\right)^2$$

ADMM (alternating direction method of multipliers):

$$\begin{aligned}
E^{k+1} &\in \arg\max_E L_\rho(E, \bar{E}^k, P^k) \\
\bar{E}^{k+1} &\in \arg\max_{\bar{E} \text{ s.t. } \sum_i \bar{E}_i = 0} L_\rho(E^{k+1}, \bar{E}, , P^k) \\
P^{k+1} &:= P^k - \rho(\sum_i E_i^{k+1})
\end{aligned}$$

Good: distributed, always converge if convex.

Outline

Reminder

There exists a price such that:

- Selfish decision leads to a social optimum.

We know how to compute the price.

Price $P(t)$

Demand — Supplier

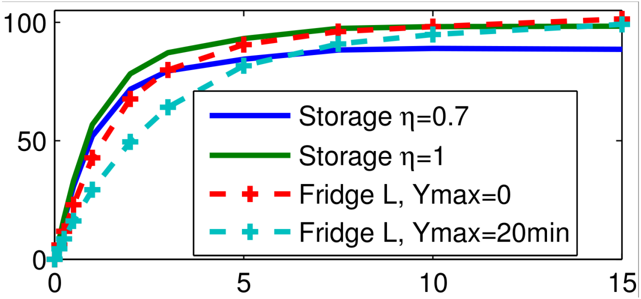
Flexible loads

Storage (e.g. battery)

We can evaluate the effect of more flexible load / more storage.

In a perfect world, the benefit of demand-response is similar to perfect storage

Social Welfare

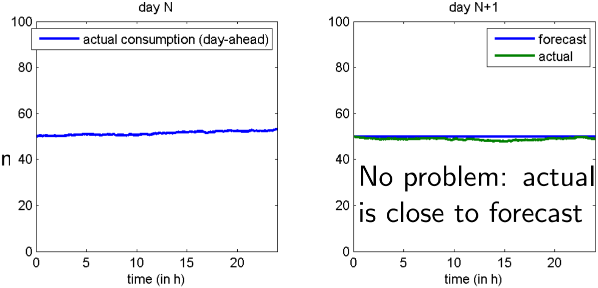


Installed flexible power (in GW for UK)

Problem 1: synchronization leads to observability problem

No demand-response

Total consumption

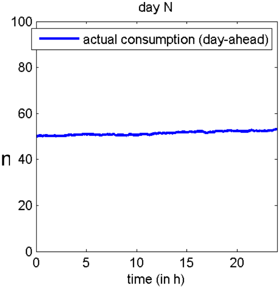


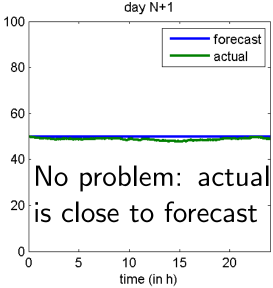
No problem: actual consumption is close to forecast

Problem 1: synchronization leads to observability problem

No demand-response

Total consumption

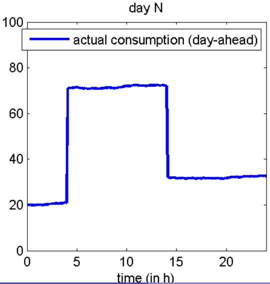


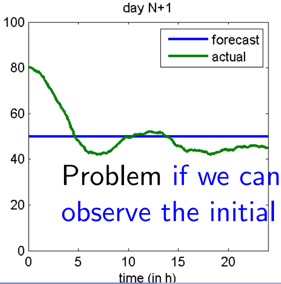


No problem: actual consumption is close to forecast

With demand-response

Total consumption





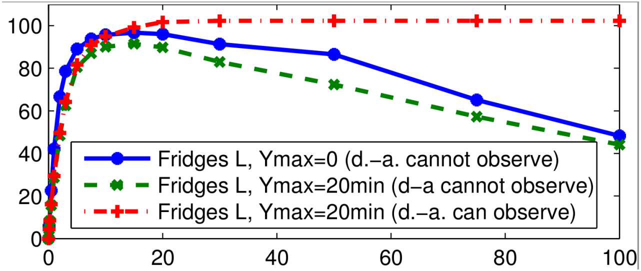
Problem if we cannot observe the initial state

Problem 1. Observablity is detrimental if the penetration is large

We assume that:

- The demand-response operator knows the state of its fridges
- The day-ahead forecast does not.

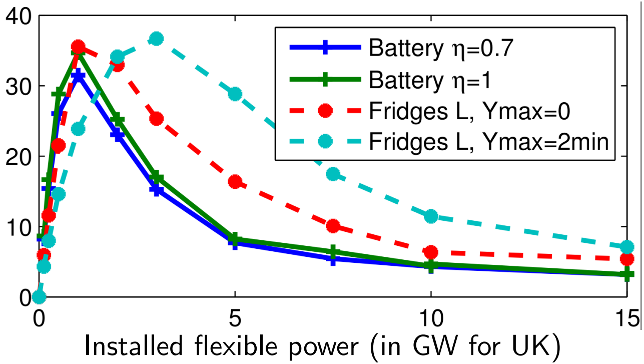
Social Welfare



Installed flexible power (in GW for UK)

Problem 2. The market structure might lead to under-investment

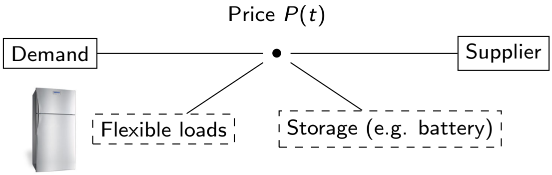
Welfare for storage owner



Outline

Summary

1. Real-time market model (generation dynamics, flexible loads, storage)



2. A price such that selfish decisions are feasible leads to a social optimum.

3. We know how to compute the price.

- Trajectorial forecast and ADMM

4. Benefit of demand-response: flexibility, efficiency
Drawbacks: non-observability, under-investment

Conclusion and perspective

- Methodology:
  - Distributed Lagrangian (ADMM) is powerful
  - Use of trajectorial forecast makes it computable
  - Can be used for learning
- Real-time Market
  - Efficient but not robust
    - Efficiency disregards safety, security, investment,...
    - Who wants real-time prices at home?
  - Interesting applications: electric cars, voltage control

I belong to the Quanticol project

A Quantitative Approach to Management and Design of Collective and Adaptive Behaviors.

- FET project, cousin of CASSTING.

Objectives:

- Build a modelization tool
  - Stochastic models, fluid approximation, optimization, verification
- Applications: smart-cities
  - Buses
  - Bike-sharing systems
  - Smart-grids

Nicolas Gast — http://mescal.imag.fr/membres/nicolas.gast/

Model

- Dynamic competitive equilibria in electricity markets, G. Wang, M. Negrete-Pincetic, A. Kowli, E. Shafieepoorfard, S. Meyn and U. Shanbhag, *Control and Optimization Methods for Electric Smart Grids*, 35–62 2012,
- A Control Theorist's Perspective on Dynamic Competitive Equilibria in Electricity Markets. G. Wang, A. Kowli, M. Negrete-Pincetic, E. Shafieepoorfard, S. Meyn and U. Shanbhag.

Storage and Demand-response

- Impact of storage on the efficiency and prices in real-time electricity markets. N Gast, JY Le Boudec, A Proutière, DC Tomozei, e-Energy 2013
- Impact of Demand-Response on the Efficiency and Prices in Real-Time Electricity Markets. N Gast, JY Le Boudec, DC Tomozei. e-Energy 2014

ADMM

- Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein. Foundations and Trends in Machine Learning, 3(1):1-122, 2011.

Supported by quanticol — http://www.quanticol.eu