Parallel Algorithms

Design
and
Implementation

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Overview

- Machine model and work-stealing
- Work and depth
- Fundamental theorem : Work-stealing theorem
- Parallel divide & conquer
- Examples
    - Accumulate
    - Monte Carlo simulations

- Part2: Work-first principle - Amortizing the overhead of parallelism
- Prefix/partial sum

    - Sorting and merging

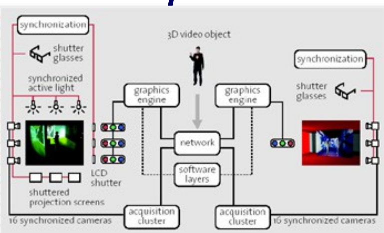
- Part3: Amortizing the overhead of synchronization and communications
- Numerical computations : FFT, marix computations; Domain decompositions

Interactive parallel computation?

Any application is "parallel":

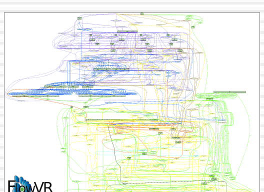
•composition of several programs / library procedures (possibly concurrent) ;

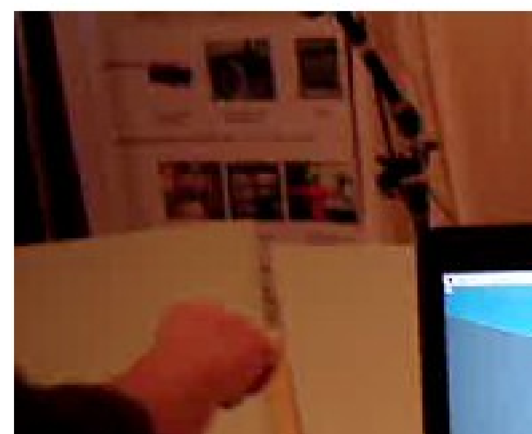
•each procedure written independently and also possibly parallel itself.



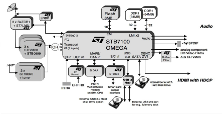
Interactive Distributed Simulation

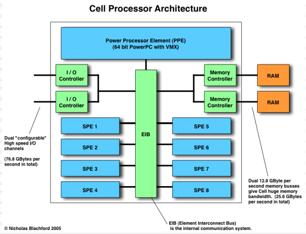
3D-reconstruction
+ simulation
+ rendering
[B Raffin &E Boyer]
- 1 monitor
- 5 cameras,
- **6** PCs





New parallel supports from small too large

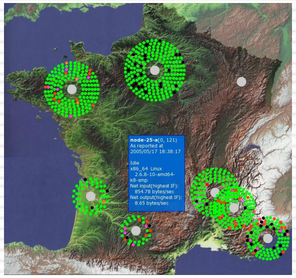




- **Parallel chips & multi-core architectures**:
  - **MPSoCs** (Multi-Processor Systems-on-Chips)
  - **GPU** : graphics processors (and programmable: Shaders; Cuda SDK)
  - MultiCore processors (Opterons, Itanium, etc.)
  - Heteregoneous multi-cores : **CPUs + GPUs + DSPs+ FPGAs** (Cell)

- **Commodity SMPs**:
  - 8 way PCs equipped with multi-core processors (AMD Hypertransport) + 2 GPUs

- **Clusters**:
  - 72% of top 500 machines
  - Trends: more processing units, faster networks (PCI- Express)
  - Heterogeneous (CPUs, GPUs, FPGAs)



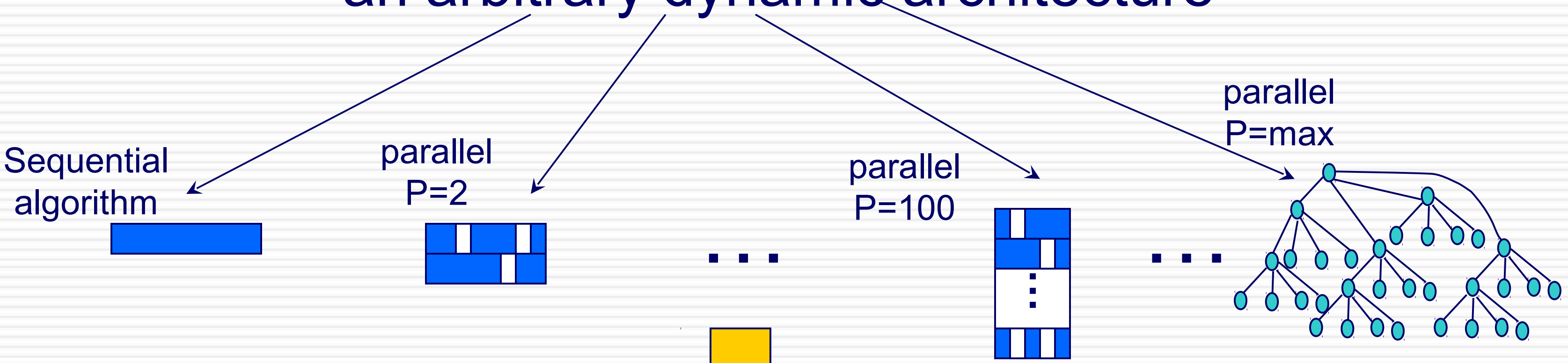
- **Grids**:
  - Heterogeneous networks
  - Heterogeneous administration policies
  - Resource Volatility

Grimage platform

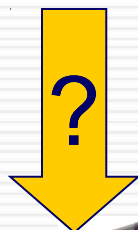
- **Dedicated platforms**: eg Virtual Reality/Visualization Clusters:
  - Scientific Visualization and Computational Steering
  - PC clusters + graphics cards + multiple I/O devices (cameras, 3D trackers, multi-projector displays)

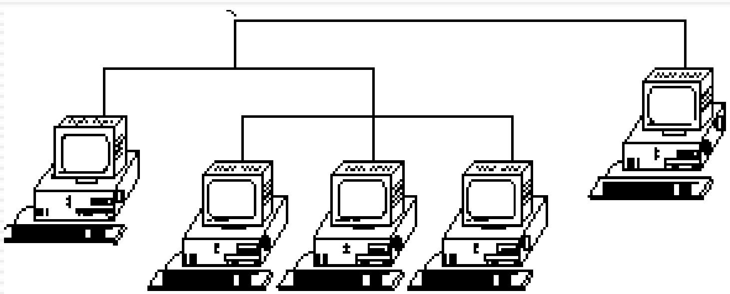
The problem

To design a single algorithm that computes efficiently prefix(a) on an arbitrary dynamic architecture



Which algorithm to choose ?





Heterogeneous network

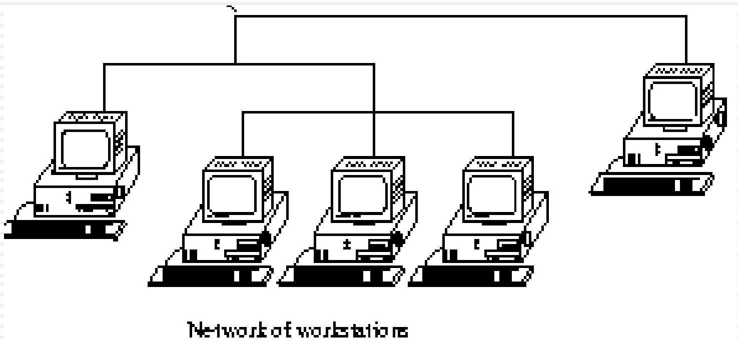
Multi-user SMP server

Grid

Dynamic architecture : non-fixed number of resources, **variable speeds**
eg: *grid*, … but not only: *SMP server in multi-users mode*

Processor-oblivious algorithms

Dynamic architecture : non-fixed number of resources, variable speeds
eg: grid, SMP server in multi-users mode,….



=> motivates the design of «**processor-oblivious**» parallel algorithm that:

+ is **independent** from the underlying architecture:
no reference to *p* nor $\Pi_i(t)$ = *speed of processor i at time t nor …*

+ on a given architecture, has **performance guarantees** :
behaves as well as an optimal (off-line, non-oblivious) one

2. Machine model and work stealing

- Heterogeneous machine model and work-depth framework
- Distributed work stealing
- Work-stealing implementation : work first principle
- Examples of implementation and programs:
  Cilk , Kaapi/Athapascan
- Application: Nqueens on an heterogeneous grid

Heterogeneous processors, work and depth

Processor speeds are assumed to change arbitrarily and adversarially:

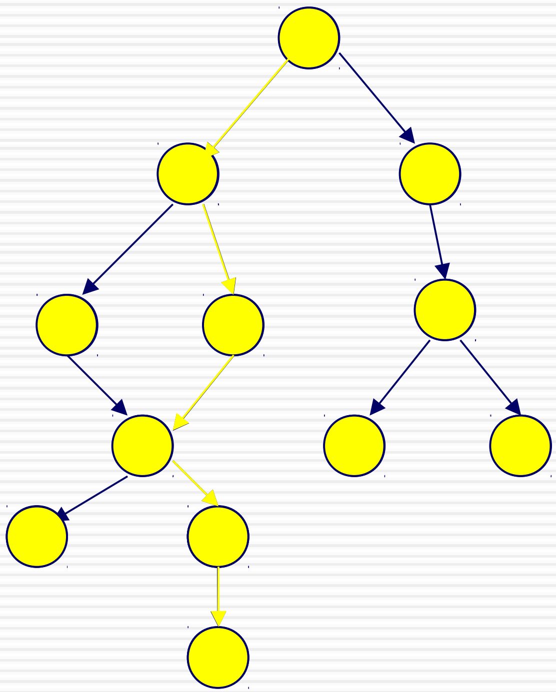
model [Bender,Rabin 02] $\Pi_i(t)$ = ***instantaneous speed*** *of processor i at time t*

(in #unit operations per second)

Assumption : $\mathrm{Max}_{i,t}\{ \Pi_i(t) \} < constant \, . \, \mathrm{Min}_{i,t}\{ \Pi_i(t) \}$

Def: for a computation with duration T

- **total speed:** $\Pi_{tot} = (\sum_{i=0,..,P} \sum_{t=0,..,T} \Pi_i(t)) / T$
- **average speed** per processor: $\Pi_{ave} = \Pi_{tot} / P$



"Work" W = #total number operations performed

"Depth" D = #operations on a critical path

(~parallel "time" on ∞ resources)

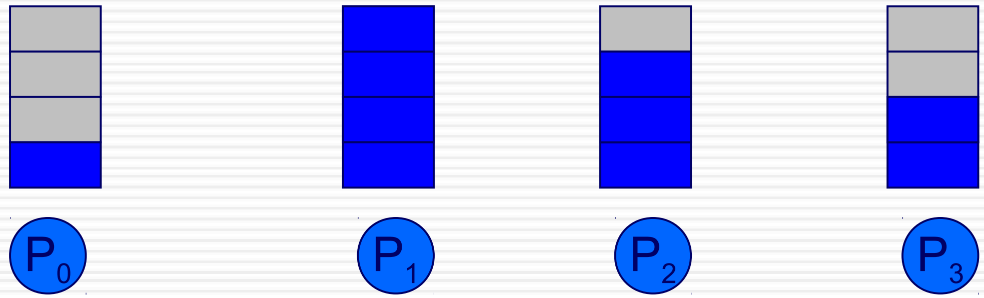
For any greedy *maximum utilization* schedule:

[Graham69, Jaffe80, Bender-Rabin02]

$$makespan \le \frac{W}{p.\Pi_{ave}} + \left(1 - \frac{1}{p}\right)\frac{D}{\Pi_{ave}}$$

The work stealing algorithm

- **A distributed and randomized algorithm that computes a greedy schedule :**
  - Each processor manages a local task (depth-first execution)



The work stealing algorithm

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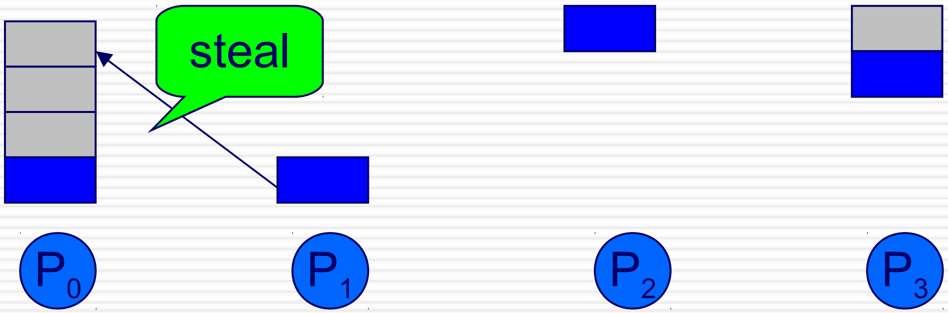


  - When idle, a processor steals the topmost task on a remote -non idle- victim processor (randomly chosen)
  - **Theorem**: With good probability, [Acar,Blelloch, Blumofe02, BenderRabin02]
    - **#steals = O(*p.D*)** **and execution time** $\leq \frac{W}{p.\Pi_{ave}} + O\left(\frac{D}{\Pi_{ave}}\right)$
  - **Interest**:
    if ***W* independent of *p*** and ***D* is small**, work stealing achieves **near-optimal** schedule

Proof

- **Any parallel execution can be represented by a binary tree:**
  - Node with 0 child = TERMINATE instruction
    - End of the current thread
  - Node with 1 son = sequential instruction
  - Node with 2 sons: parallelism = instruction that
    - Creates a new (ready) thread
      - eg fork, thread_create, spawn, …
    - Unblocks a previously blocked thread
      - eg signal, unlock, send

Proof (cont)

- Assume the local ready task queue is stored in an array: each ready task is stored according to its depth in the binary tree
- **On processor i at top t :**
  - $H_i(t)$ = the index of the oldest ready task
- **Prop 1:** When non zero, $H_i(t)$ is increasing
- **Prop 2: H(t) =** $\mathrm{Min}_{(i\ \text{active at}\ t)}\{ H_i(t) \}$ is increasing
- **Prop 3: Each steal request on i makes H_i strictly increase** (i.e. $H_i(t+1) \geq H_i(t) + 1$).
- **Prop 4:** For all i and t: $H_i(t) \leq$ Height(Tree)
- **Corollary:** if at each steal, the victim is a processor i **with minimum $H_i(t)$** then

  #steals ≤ (p-1).Height(tree) ≤ (p-1).D

Proof (randomized, general case)

- **Group the steal operations in blocks of consecutive steals:** [Coupon collector problem]
  - Consider p.log p consecutive steals requests after top t, Then with probability > ½, **any** active processor at t have been victim of a steal request.
    - Then $Min_i H_i$ has increased of at least 1
- **In average, after (2.p.log p.M) consecutive steals requests, $\mathbf{Min_i\ H_i \geq M}$**
  - Thus, in average, after (2.p.log p.D) steal requests, the execution is completed !
- [Chernoff bounds] **With high probability (w.h.p.),**
  - **#steal requests = O(p.log p.D)**

Proof (randomized, additional hyp.)

- **With additional hypothesis:**
  - Initially, only one active processor
  - When several steal requests are performed on a same victim processor at the same top,
    only the first one is considered (others fail)
  - [Balls&Bins] Then **#steal requests = O(p.D)** w.h.p.

- **Remarks:**
  - This proof can be extended to
    - asynchronous machines (synchronization = steal)
    - Other steal policies then steal the “topmost=oldest” ready tasks, but with impact on the bounds on the steals

Steal requests and execution time

- **At each top, a processor j is**
  - Either active: performs a “work” operation
    - Let wj be the number of unit work operations by j
  - Either idle: performs a steal requests
    - Let sj be the number of unit steal operations by j

- **Summing on all p processors :**

$$\text{Execution time} \leq \frac{W}{p.\Pi_{ave}} + O\left(\frac{D}{\Pi_{ave}}\right)$$

Work stealing implementation

Scheduling

efficient policy (close to optimal) ← → control of the policy (realisation)

Difficult in general (coarse grain)

But easy if *D* is small [Work-stealing]

Execution time $\leq \frac{W}{p.\Pi_{ave}} + O\left(\frac{D}{\Pi_{ave}}\right)$

(fine grain)

Expensive in general (fine grain)

But small overhead if a small number of tasks

(coarse grain)

If D is small, a work stealing algorithm performs a ***small number of steals***

=> ***Work-first principle****:* "scheduling overheads should be borne by the critical path of the computation" [Frigo 98]

Implementation: since all tasks but a few are executed in the local stack, overhead of task creation should be as close as possible as sequential function call

At any time on any non-idle processor,
efficient local *degeneration* of the *parallel* program in a *sequential execution*

Work-stealing implementations following the work-first principle : Cilk

- **Cilk-5** http://supertech.csail.mit.edu/cilk/ **: C extension**
  - **Spawn** f (a) ; **sync** (serie-parallel programs)
  - Requires a shared-memory machine
  - Depth-first execution with synchronization (on sync) with the end of a task :
    - Spawned tasks are pushed in double-ended queue
  - "Two-clone" compilation strategy [Frigo-Leiserson-Randall98] :
    - on a successfull steal, a thief executes the continuation on the topmost ready task ;
    - When the continuation hasn't been stolen, "sync" = nop ; else synchronization with its thief

```
cilk int fib (int n)
{
     if (n < 2) return n;
     else
     {
        int x, y;

        x = spawn fib (n-1);
        y = spawn fib (n-2);

        sync;

        return (x+y);
     }
}
```

```
int fib (int n)
{
    fib_frame *f;                   frame pointer
    f = alloc(sizeof(*f));          allocate frame
    f->sig = fib_sig;               initialize frame
    if (n<2) {
        free(f, sizeof(*f));        free frame
        return n;
    }
    else {
        int x, y;
        f->entry = 1;               save PC
        f->n = n;                   save live vars
        *T = f;                     store frame pointer
        push();                     push frame
        x = fib (n-1);              do C call
        if (pop(x) == FAILURE)      pop frame
            return 0;               frame stolen
        ...                         second spawn
        ;                           sync is free!
        free(f, sizeof(*f));        free frame
        return (x+y);
    }
}
```

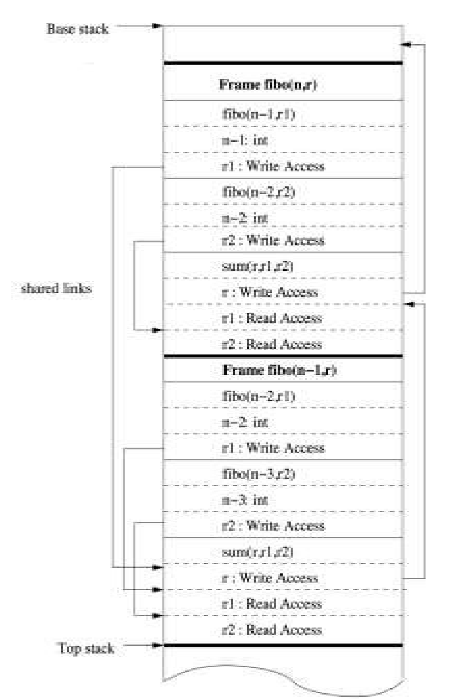
- *won the 2006 award "Best Combination of Elegance and Performance" at HPC Challenge Class 2, SC'06, Tampa, Nov 14 2006 [Kuszmaul] on SGI ALTIX 3700 with 128 bi-Ithanium]*

Work-stealing implementations following the work-first principle : KAAPI

- **Kaapi / Athapascan** http://kaapi.gforge.inria.fr **:** C++ library
  - **Fork**<f>()(a, …) with **access mode** to parameters (value;read;write;r/w;cw) **specified in f prototype** (macro dataflow programs)
  - Supports distributed and shared memory machines; heterogeneous processors
  - Depth-first (*reference order*) execution with synchronization on data access :
    - Double-end queue (mutual exclusion with compare-and-swap)
    - on a successful steal, one-way data communication (write&signal)

```
struct sum {
   void operator()(Shared_r < int > a,
                   Shared_r < int > b,
                   Shared_w < int > r )
   { r.write(a.read() + b.read()); }
 } ;

 struct fib {
  void operator()(int n, Shared_w<int> r)
  { if (n <2) r.write( n );
    else
    { int r1, r2;
      Fork< fib >() ( n-1, r1 ) ;
      Fork< fib >() ( n-2, r2 ) ;
      Fork< sum >() ( r1, r2, r ) ;
    }
  }
} ;
```

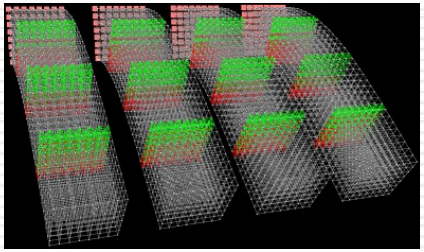


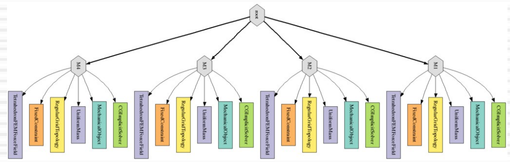
(b) Shared links

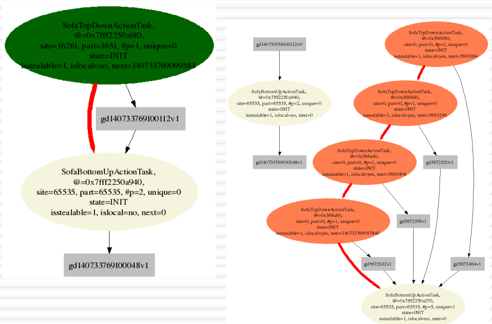
- *Kaapi won the 2006 award “Prix special du Jury” for the best performance at NQueens contest, Plugtests-*

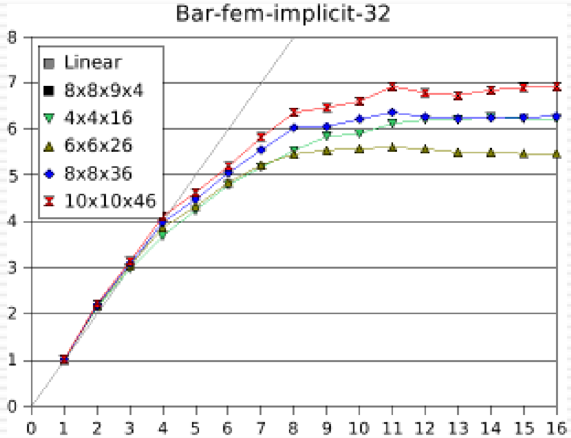
Experimental results on SOFA [CIMIT-ETZH-INRIA]

[Allard 06]

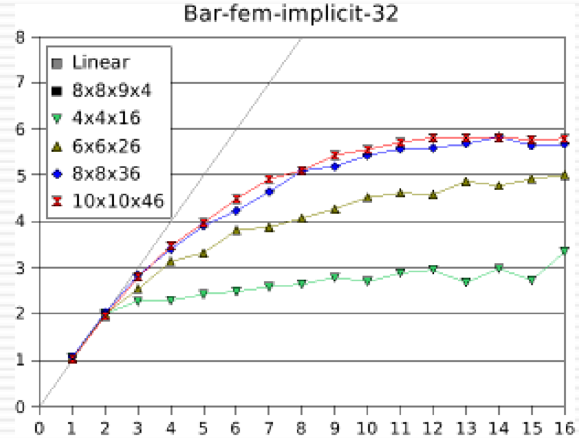








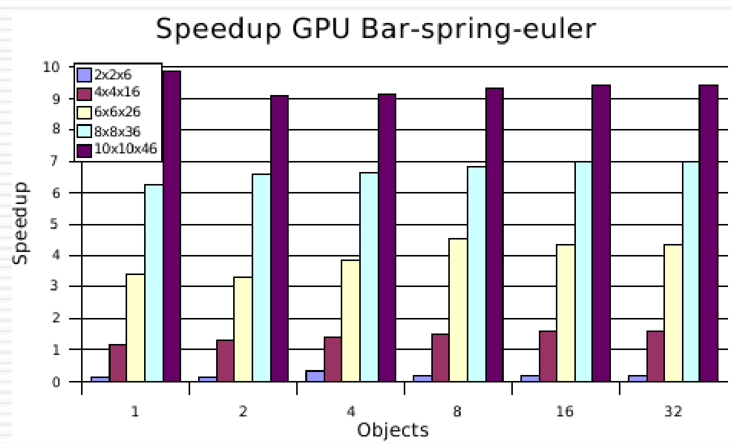
Kaapi (C++, ~500 lines)



Cilk (C, ~240 lines)

Preliminary results on GPU NVIDIA 8800 GTX

- speed-up ~9 on Bar 10x10x46 to Athlon64 2.4GHz
  - 128 “cores” in 16 groups
  - CUDA SDK : “BSP”-like, 16 X [16 .. 512] threads
  - Supports most operations available on CPU
  - ~2000 lines CPU-side + 1000 GPU-side



Algorithm design

$$\textbf{Execution time} \leq \frac{W}{p.\Pi_{ave}} + O\left(\frac{D}{\Pi_{ave}}\right)$$

- **From work-stealing theorem, optimizing the execution time by building a parallel algorithm with both**
  - **$W = T_{seq}$**

  and

  - small depth D

- Double criteria
  - **Minimum work W** (ideally T_{seq})
  - **Small depth D**: ideally polylog in the work: $= \log^{O(1)} W$

Examples

- **Accumulate**

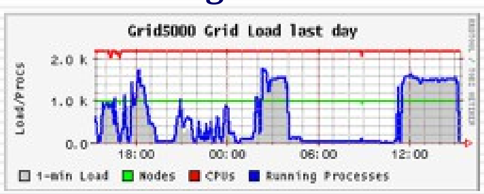
- **=> Monte Carlo computations**

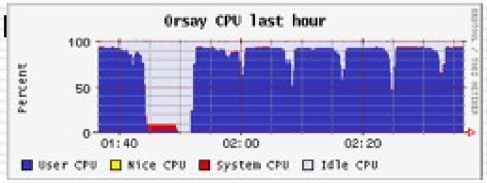
Example: Recursive and Monte Carlo computations

- *X Besseron, T. Gautier, E Gobet, &G Huard won the nov. 2008 Plugtest-Grid&Work'08 contest – Financial mathematics application (Options pricing)*

- In 2007, the team won the Nqueens contest; Some facts [on *on Grid'5000*, a grid of processors of heterogeneous speeds]
  - NQueens(21) in 78 s on about 1000 processors
  - Nqueens (22) in 502.9s on 1458 processors
  - Nqueens(23) in 4435s on 1422 processors [~24.10^{33} solutions]
  - 0.625% idle time per processor
  - < 20s to deploy up to 1000 processes on 1000 machines [Taktuk, Huard]
  - 15% of improvement of the sequential due to C++ (te

Grid'5000 utilization during contest

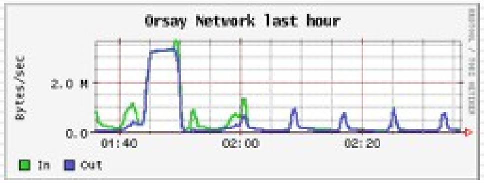




CPU

6 instances Nqueens(22)

Network



Algorithm design

- **Cascading divide & Conquer**

  - $W(n) \le a.W(n/K) + f(n)$ with $a>1$
    - If $f(n) << n^{\log_K a}$ => $W(n) = O(n^{\log_K a})$
    - If $f(n) >> n^{\log_K a}$ => $W(n) = O(f(n))$
    - If $f(n) = \Theta(n^{\log_K a}$ => $W(n) = O(f(n) \log n)$

  - $D(n) = D(n/K) + f(n)$
    - If $f(n) = O(\log^i n)$ => $D(n) = O(\log^{i+1} n)$

  - $D(n) = D(\mathrm{sqrt}(n)) + f(n)$
    - If $f(n) = O(1)$ => $D(n) = O(\log\log n)$
    - If $f(n) = O(\log n)$ => $D(n) = O(\log n)$!!

Examples

- Accumulate

- Monte Carlo computations

- **Maximum on CRCW**
- Matrix-vector product – Matrix multiplication -- Triangular matrix inversion

- **Exercise: parallel merge and sort**
- Next lecture: Find, Partial sum, adaptive parallelism, communications

Algorithm design

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Lecture 2 – Processor oblivious algorithms

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Lecture 2

- **Remind: Work W and depth D :**
  - With work-stealing schedule:
    - #steals = O(pD)
    - Execution time on p procs = W/p + O(D) w.h.p.
    - Similar bound achieved with processors with changing speed or multiprogrammed systems.
- **How to parallelize ?**
  - 1/ There exists a fine-grain parallel algorithm that is optimal in sequential
    - Work-stealing and Communications
  - 2/ Extra work induced by parallel can be amortized
  - 3/ Work and Depth are related
    - Adaptive parallel algorithms

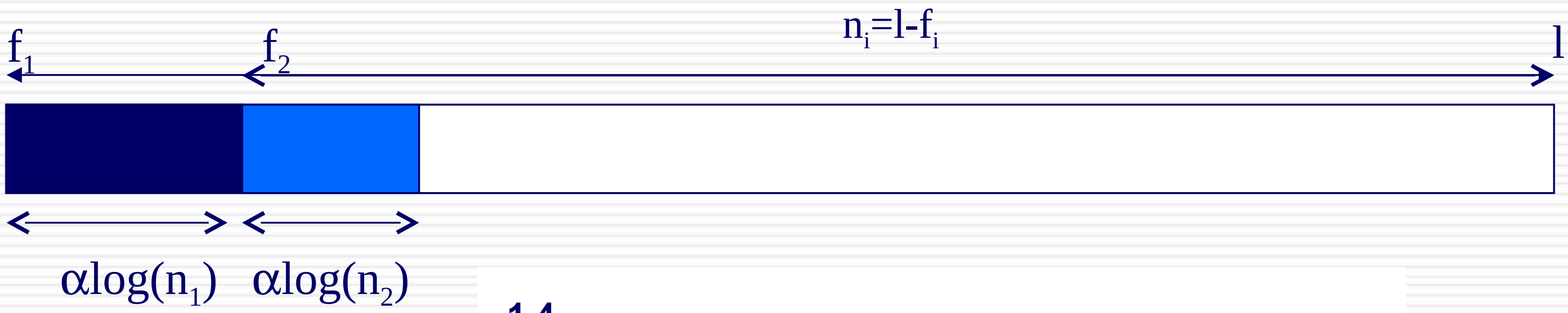
First examples

- **Put overhead on the steals :**
  - Example Accumulate

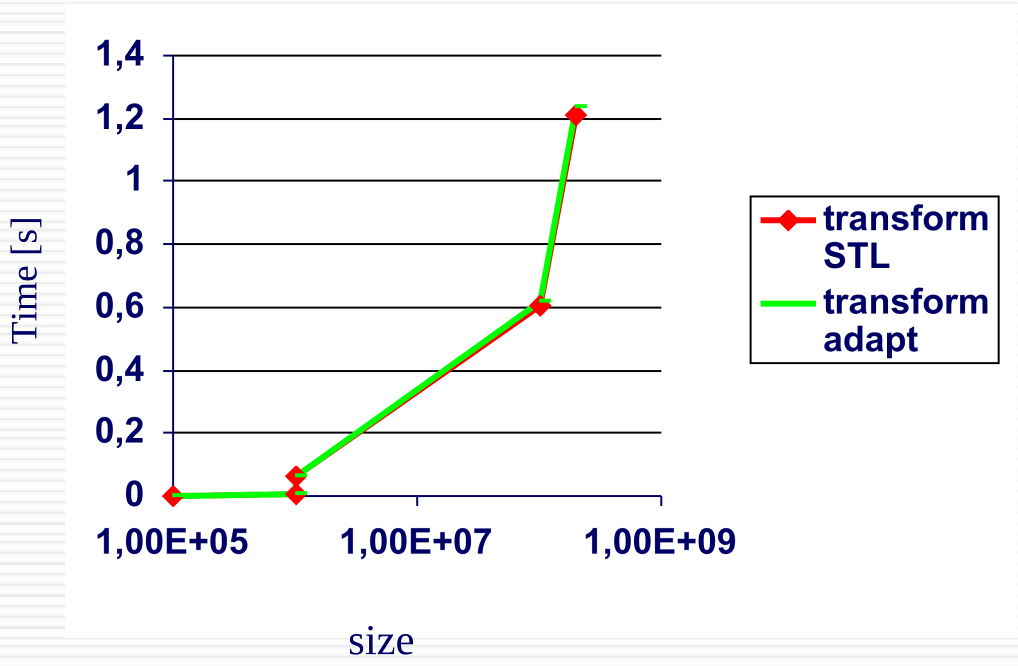
- **Follow an optimal sequential algorithm:**
  - Example: Find_if

Adaptive coupling: Amortizing synchronizations (parallel work extraction)

Example : STL **transform** STL : loop with n independent computations

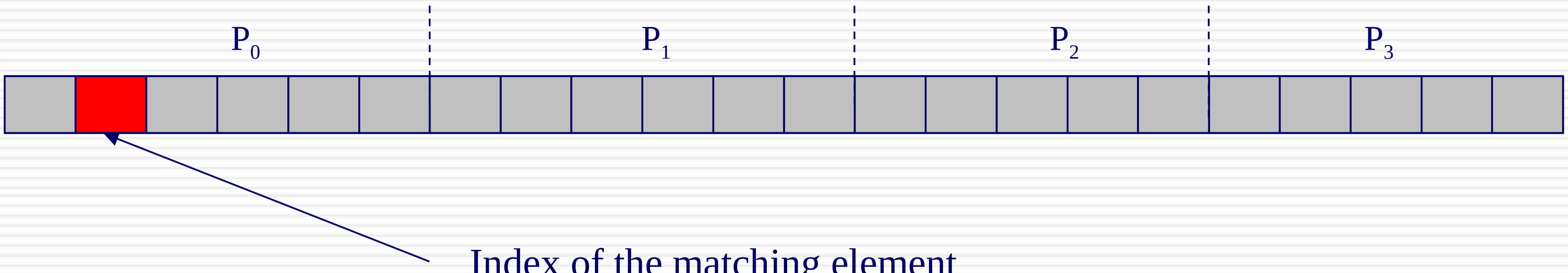


Machine :
AMD Opteron Opteron 875
2,2 Ghz,
Compiler gcc, option –O2



Amortizing Parallel Arithmetic overhead: example: find_if

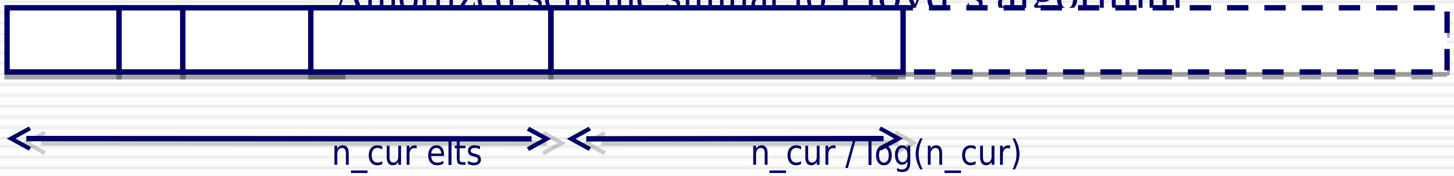
- **For some algorithms:**
  - W_{seq} unknown prior to execution
  - Worst case work W is not precise enough: we may have $W >> W_{seq}$
- **Example: find_if : returns the index of the first element that verifies a predicate.**



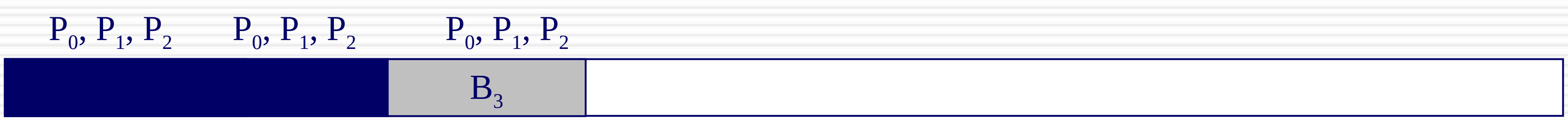
- Sequential time is $T_{seq} = 2$
- Parallel time= time of the last processor to complete: here, on 4 processors: $T_4 = 6$

Amortizing Parallel Arithmetic overhead: example: find_if

- To adapt with provable performances ($W_{par} \sim W_{seq}$) : compute in parallel no more work thant the work performed by the sequential algorithm (**Macro-loop** [Danjean, Gillard, Guelton, Roch, Roche, PASCO'07]), Amortized scheme similar to Floyd's algorithm



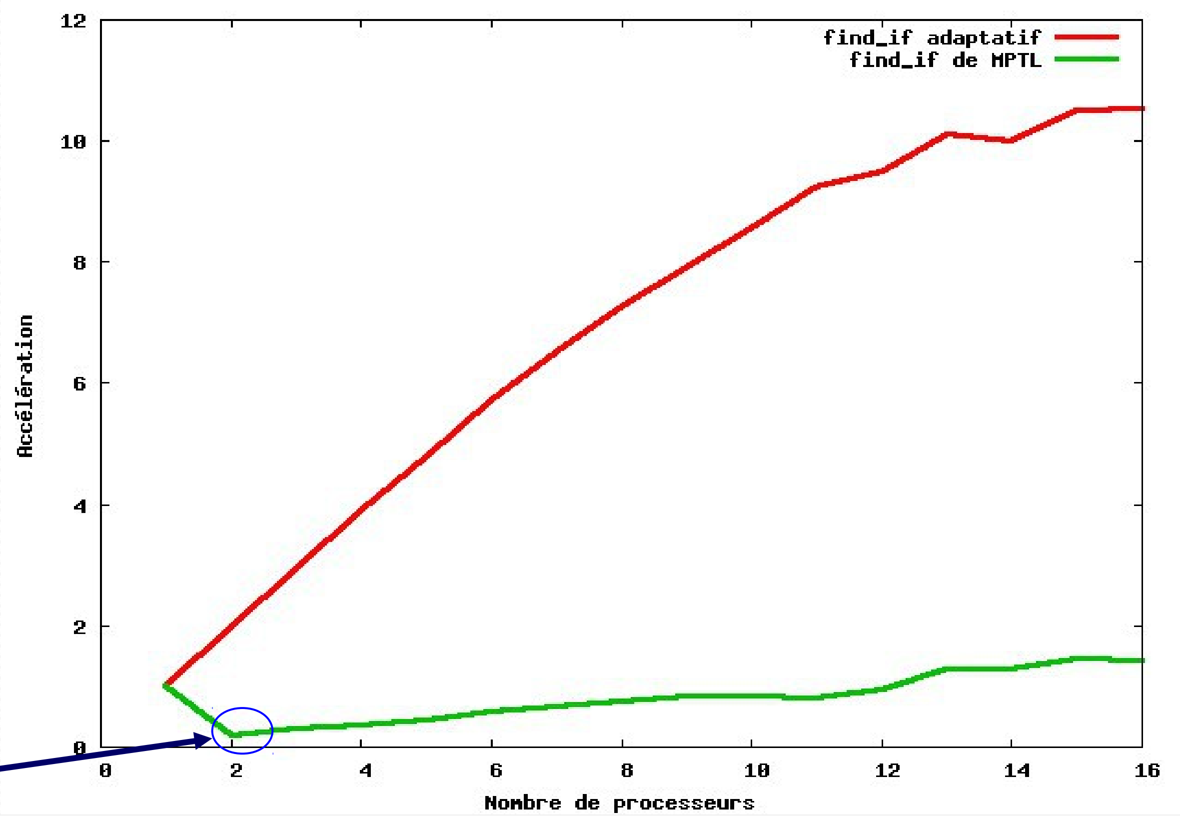
- Example : **find_if**



Amortizing Parallel Arithmetic overhead: example: find_if [Daouda Traore 2009]

- **Example : find_if STL**
  - Comparison with find_if parallel MPTL [Baertschiger 06]

Machine :
AMD Opteron (16 cœurs);
Data: doubles;
Array size: 10^6;
Position element: 10^5;
TimeSTL : 3,60 s;
Predicate time ≈ 36µ

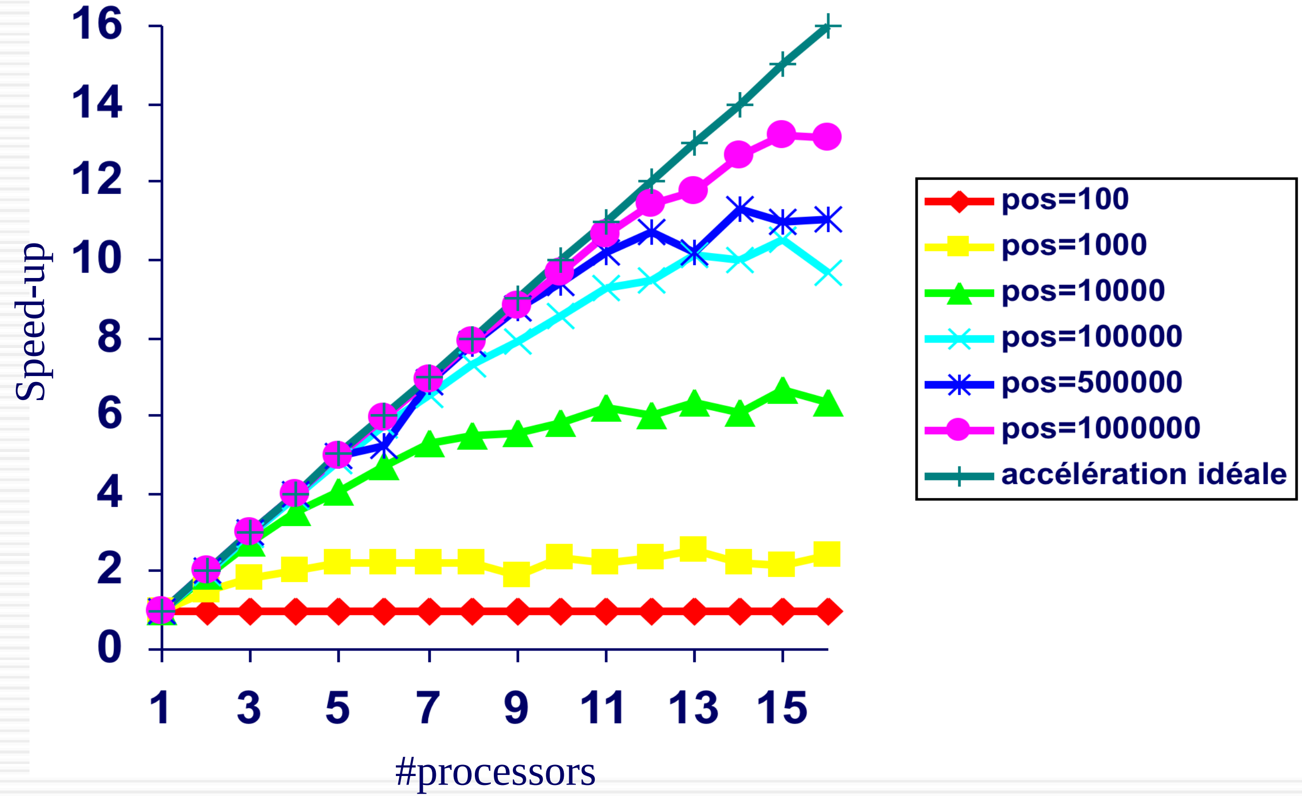


Speed-down (speed-up < 1)

Amortizing Parallel Arithmetic overhead: example: find_if [Daouda Traore 2009]

- **Example : find_if STL**
  - Speed-up w.r.t. STL sequential tim and the position of the matching element.

Machine :
AMD Opteron (16 cœurs);
Data: doubles;
Size Array: 10^6;
Predicate time≈ 36µ



Overview

- Introduction : interactive computation, parallelism and processor oblivious
  - Overhead of parallelism : parallel prefix
- Machine model and work-stealing
- **Scheme 1: Extended work-stealing : concurently sequential and parallel**

3. Work-first principle and adaptability

- **Work-first principle**: -implicit- dynamic choice between two executions :
  - a **sequential *"depth-first"*** execution of the parallel algorithm (local, default) ;
  - a **parallel "*breadth-first*"** one.
- Choice is performed at runtime, depending on resource idleness:
  rare event if Depth is small to Work
- **WS adapts parallelism to processors with practical provable performances**
  - Processors with changing speeds / load (data, user processes, system, users,
  - Addition of resources (fault-tolerance [Cilk/Porch, Kaapi, …])
- **The choice is justified only when the sequential execution of the parallel algorithm is an efficient sequential algorithm:**
  - Parallel Divide&Conquer computations
  - …

-> But, this may not be general in practice

How to get both optimal work W_1 and D=W_∞ small?

- General approach**: to** mix **both**
  - a **sequential** algorithm with optimal work $\boldsymbol{W_1}$
  - and a fine grain **parallel** algorithm with minimal depth ***D*** = critical time $\boldsymbol{W_\infty}$

- Folk technique **:** *parallel, than sequential*
  - Parallel algorithm until a certain « grain »; then use the sequential one
  - Drawback : $\boldsymbol{W_\infty}$ increases ;o) …and, also, the number of steals

- *Work-preserving speed-up technique* **[Bini-Pan94]** *sequential, then parallel* **Cascading [Jaja92] : Careful interplay of both algorithms to build one with both**

  W_∞ **small** and W_1 **= O(** W_{seq} **)**

  - Use the work-optimal sequential algorithm to reduce the size
  - Then use the time-optimal parallel algorithm to decrease the time
  - Drawback : sequential at coarse grain and parallel at fine grain ;o(

Extended work-stealing: concurrently sequential and parallel

Based on the work-stealing and the ***Work-first*** *principle* :

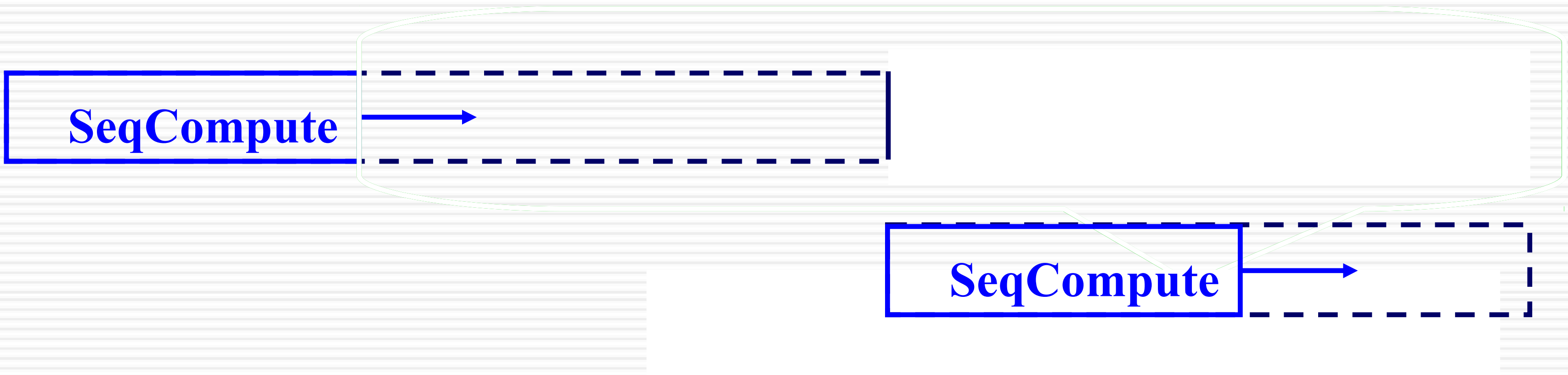
Instead of optimizing the **sequential execution** of the **best parallel** algorithm, let optimize the **parallel execution** of the ***best sequential*** algorithm

Execute always a sequential algorithm to reduce parallelism overhead

⇒ parallel algorithm is used only if a processor becomes **idle** (ie *workstealing*) [Roch&al2005,…] to ***extract parallelism*** from the remaining work a sequential computation

Assumption : two concurrent algorithms that are complementary:

- - one sequential : ***SeqCompute*** (always performed, the priority)
  - the other parallel, fine grain : ***LastPartComputation*** (often not performed)



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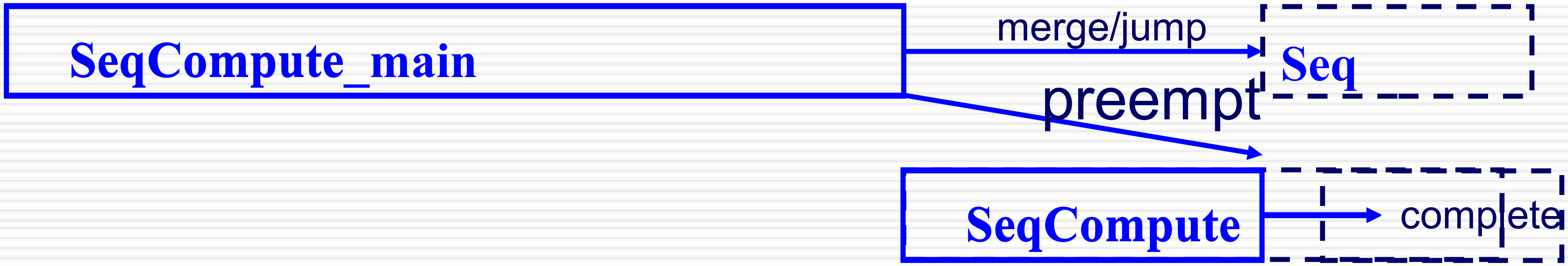

Note:

- **merge and jump** operations to ensure non-idleness of the victim
- Once *SeqCompute_main* completes, it becomes a work-stealer

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- **Scheme 2: Amortizing the overhead of synchronization (Nano-loop)**

Extended work-stealing and granularity

- **Scheme of the sequential process : nanoloop**

```
While (not completed(Wrem) ) and (next_operation hasn't been stolen)
{
    atomic { extract_next k operations ; Wrem -= k  ; }
    process the k operations extracted ;
}
```

- ***Processor-oblivious* algorithm**
  - Whatever *p* is, it performs O(*p.D*) preemption operations (« continuation faults »)
    -> ***D*** should be as small as possible to maximize both speed-up and locality
  - If no steal occurs during a (sequential) computation, then its *arithmetic work* is optimal to the one W_{opt} of the sequential algorithm (no spawn/fork/copy)
    -> ***W*** should be as close as possible to W_{opt}
- Choosing **k = Depth(W_{rem})** does not increase the depth of the parallel algorithm while ensuring ***O(W / D)*** *atomic operations* **:**
  since $D > \log_2 W_{rem}$, **then if** $p = 1$**:** $W \sim W_{opt}$
- **Implementation :** atomicity in nano-loop based without lock
  - Efficient mutual exclusion between sequential process and parallel work-stealer
- ***Self-adaptive granularity***

Interactive application with time constraint

Anytime Algorithm:

- Can be stopped at any time (with a result)
- Result quality improves as more time is allocated

In Computer graphics, anytime algorithms are common:

Level of Detail algorithms (time budget, triangle budget, etc…)

Example: Progressive texture loading, triangle decimation (Google Earth)

Anytime processor-oblivious algorithm:

On ***p*** processors with average speed Π_{ave}, it outputs in a fixed time ***T***

a result with the same quality than

a sequential processor with speed Π_{ave} in time $p.\Pi_{ave}$.

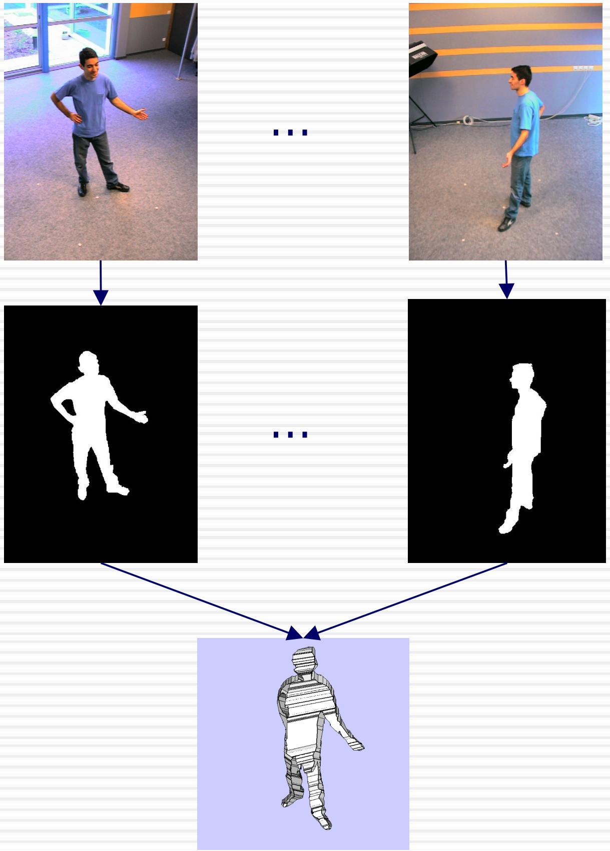
Example: Parallel Octree computation for 3D Modeling

Parallel 3D Modeling

3D Modeling :

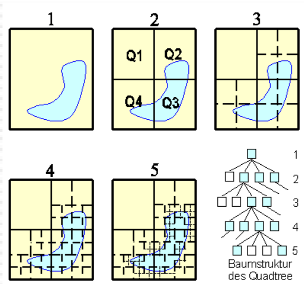
build a 3D model of a scene from a set of calibrated images

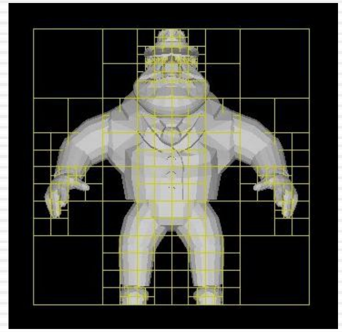
On-line 3D modeling for interactions: 3D modeling from multiple video streams (30 fps)

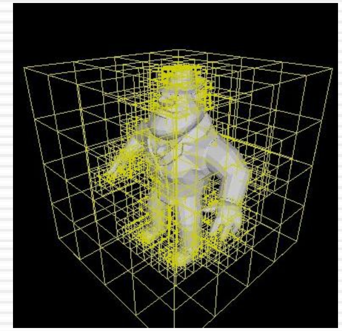


Octree Carving [L. Soares 06]

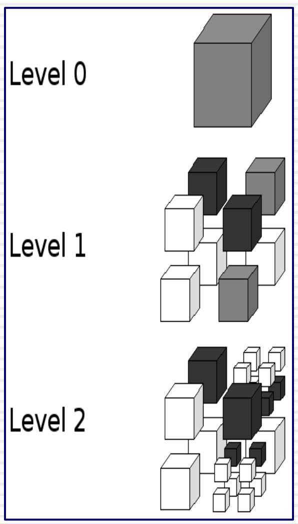
A classical recursive anytime 3D modeling algorithm.





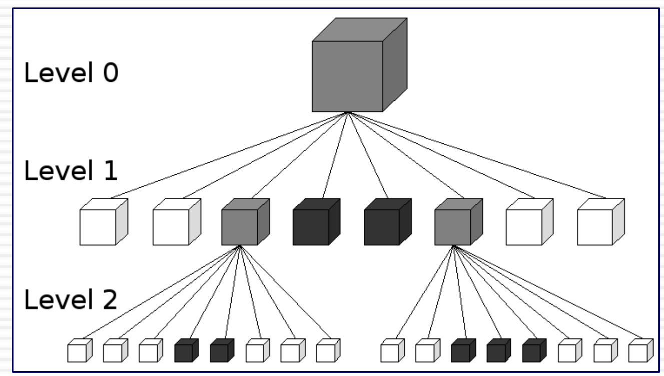


Standard algorithms with time control:



Depth first
+ iterative deepening

State of a cube:
- Grey: mixed => split
- Black: full : stop
- White: empty : stop



Width first

At termination: quick test to decide all grey cubes time control

Width first parallel octree carving

Well suited to work-stealing

-Small critical path, while huge amount of work (eg. D = 8, W = 164 000)

- non-predictable work, non predictable grain :

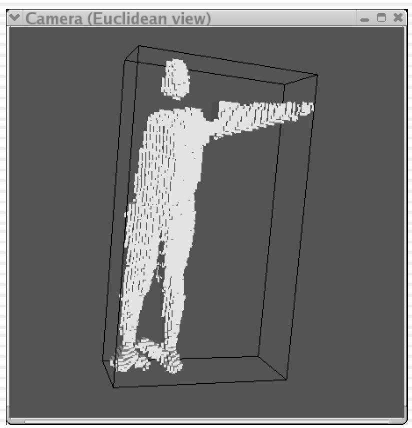
For cache locality, each level is processed by a self-adaptive grain :

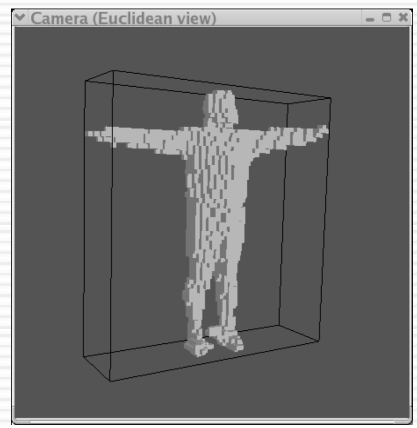
“sequential iterative” / ”parallel recursive split-half”

Octree needs to be “balanced” when stopping:

- Serially computes each level *(with small overlap)*
- Time deadline (30 ms) managed by signal protocol

Unbalanced





Balanced

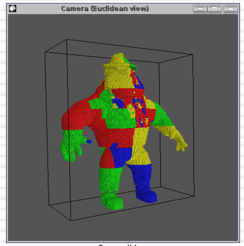
Theorem: W.r.t the adaptive in time T on p procs., the sequential algorithm:

- goes at most one level deeper : $\mathbf{| d_s - d_p | \leq 1}$;

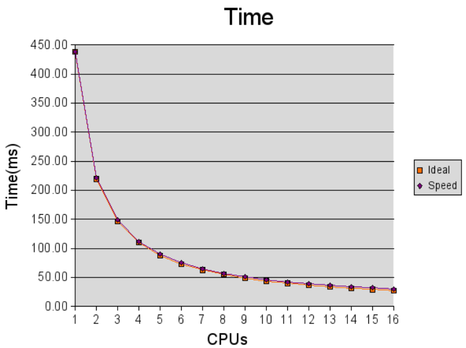
- computes at most : $\mathbf{n_s \leq n_p + O(\log n_s)}$.

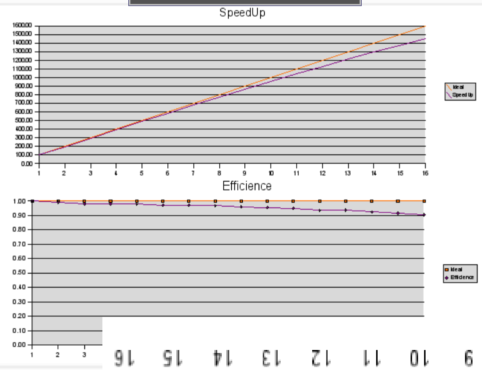
Results

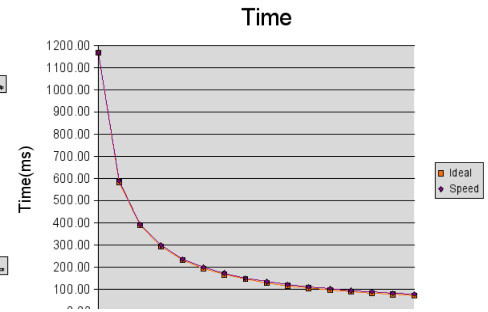
[L. Soares 06]

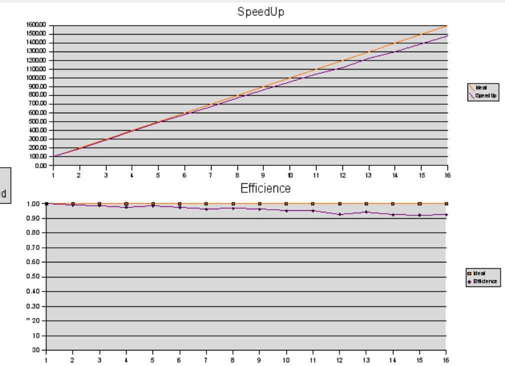


- 16 core Opteron machine, 64 images
- Sequential: 269 ms, 16 Cores: 24 ms
- 8 cores: about 100 steals (167 000 grey cells)

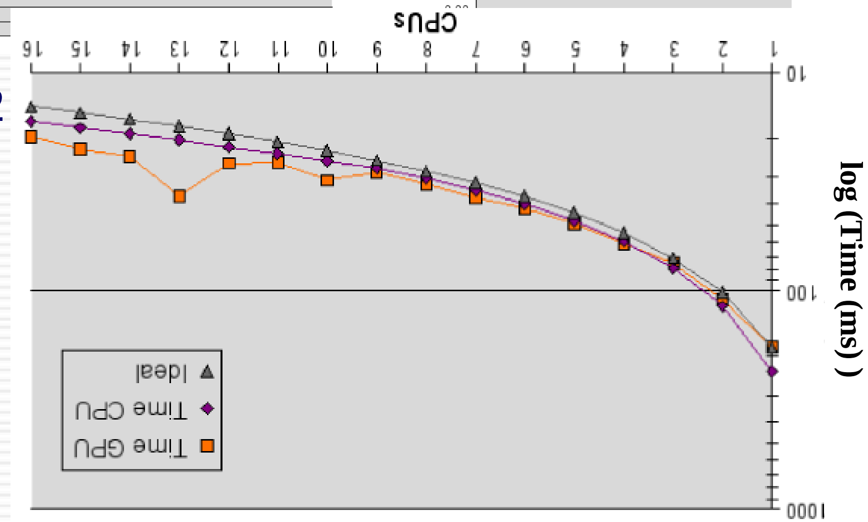








8 cameras, levels 2



els 2 to 7

s+GPU

enGL

but

Overview

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- **Scheme 3: Amortizing the overhead of parallelism (Macro-loop)**

4. Amortizing the arithmetic overhead of parallelism

Adaptive scheme : extract_seq/nanoloop // extract_par

- ensures an optimal number of operation on 1 processor
- but no guarantee on the work performed on p processors

Eg (C++ STL): find_if (first, last, predicate)

locates the first element in [First, Last) verifying the predicate

This may be a drawback (unneeded processor usage) :

- undesirable for a library code that may be used in a complex application, with many components
- (or not fair with other users)
- increases the time of the application :
  - *any parallelism that may increase the execution time should be avoided*

Motivates the building of **work-optimal** parallel adaptive algorithm (**processor oblivious**)

4. Amortizing the arithmetic overhead of parallelism (cont'd)

Similar to nano-loop for the sequential process :
- that balances the -atomic- local work by the depth of the remaindering one

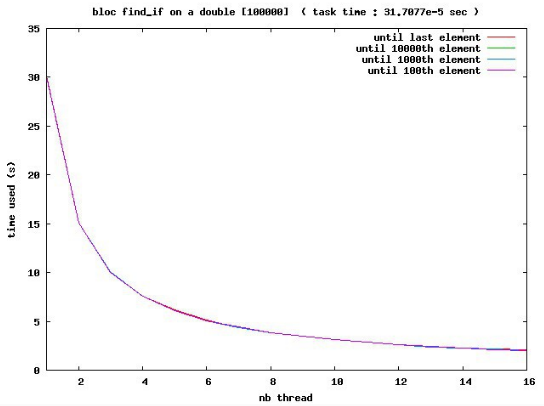
Here, by **amortizing** the work induced by the extract_par operation, ensuring this **work to be *small*** enough :
- Either w.r.t the -useful- **work already performed**
- Or with respect to the - useful - **work yet to performed** (if known)
- or both.

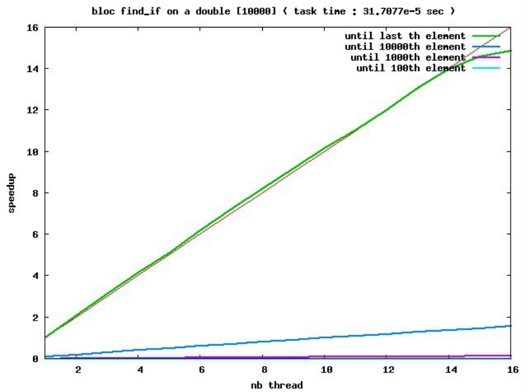
Eg : find_if (first, last, predicate) :
- only the work already performed is known (on-line)
- then prevent to assign more than $\alpha(W_{done})$ operations to work-stealers
- Choices for α(n) **:**
    - **n/2 : similar to Floyd's iteration** (approximation ratio = 2)
    - **n/log* n :** to ensure optimal usage of the work-stealers

Results on find_if

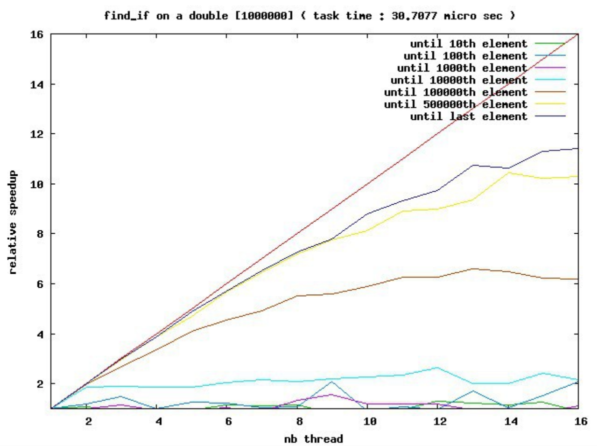
[S. Guelton]

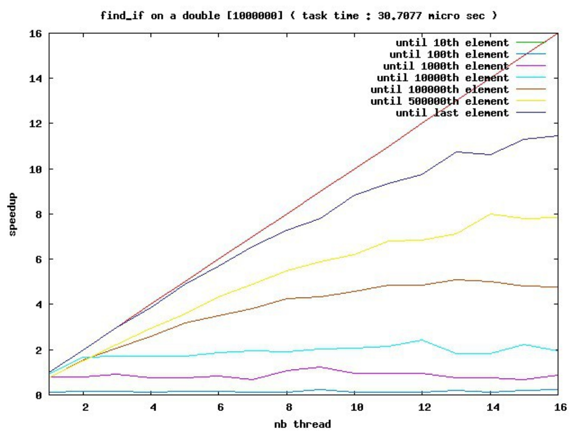
N doubles : time predicate ~ 0.31 ms





With no amortization macroloop





With amortization macroloop

5. Putting things together
processor-oblivious prefix computation

Parallel algorithm based on :

- **- compute-seq / extract-par scheme**

- **-** nano-loop for compute-seq

- \- macro-loop for extract-par

Parallelism induces overhead : e.g. Parallel prefix on fixed architecture

- **Prefix problem** :
  - input : $a_0, a_1, \ldots, a_n$
  - output : $\pi_1, \ldots, \pi_n$ with

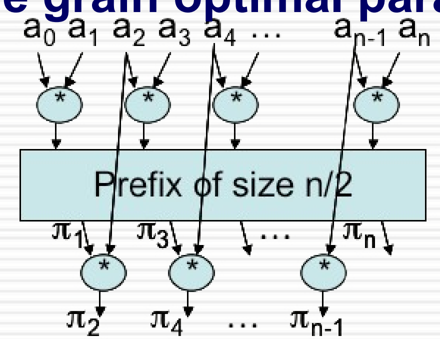
$$\pi_i = \prod_{k=0}^{i} a_k$$

- **Sequential** algorithm :
  - for ($\pi[0] = a[0]$, i = 1 ; i <= n; i++) $\pi[i] = \pi[i-1] * a[i]$;

performs only **n** *operations*

- **Fine grain optimal parallel** algorithm :

[Ladner-Fisher-81]



Critical time = 2. *log* n
but performs **2.n ops**

Parallel requires ***twice*** *more operations than sequential !!*

- Tight lower bound on **p identical processors**:

[Nicolau&al. 1996]

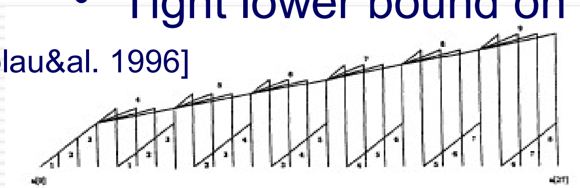


Figure 7: The Pipelined Schedule for p = 7.

Optimal time T_p = 2n / (p+1)
but performs **2.n.p/(p+1) ops**

Lower bound(s) for the prefix

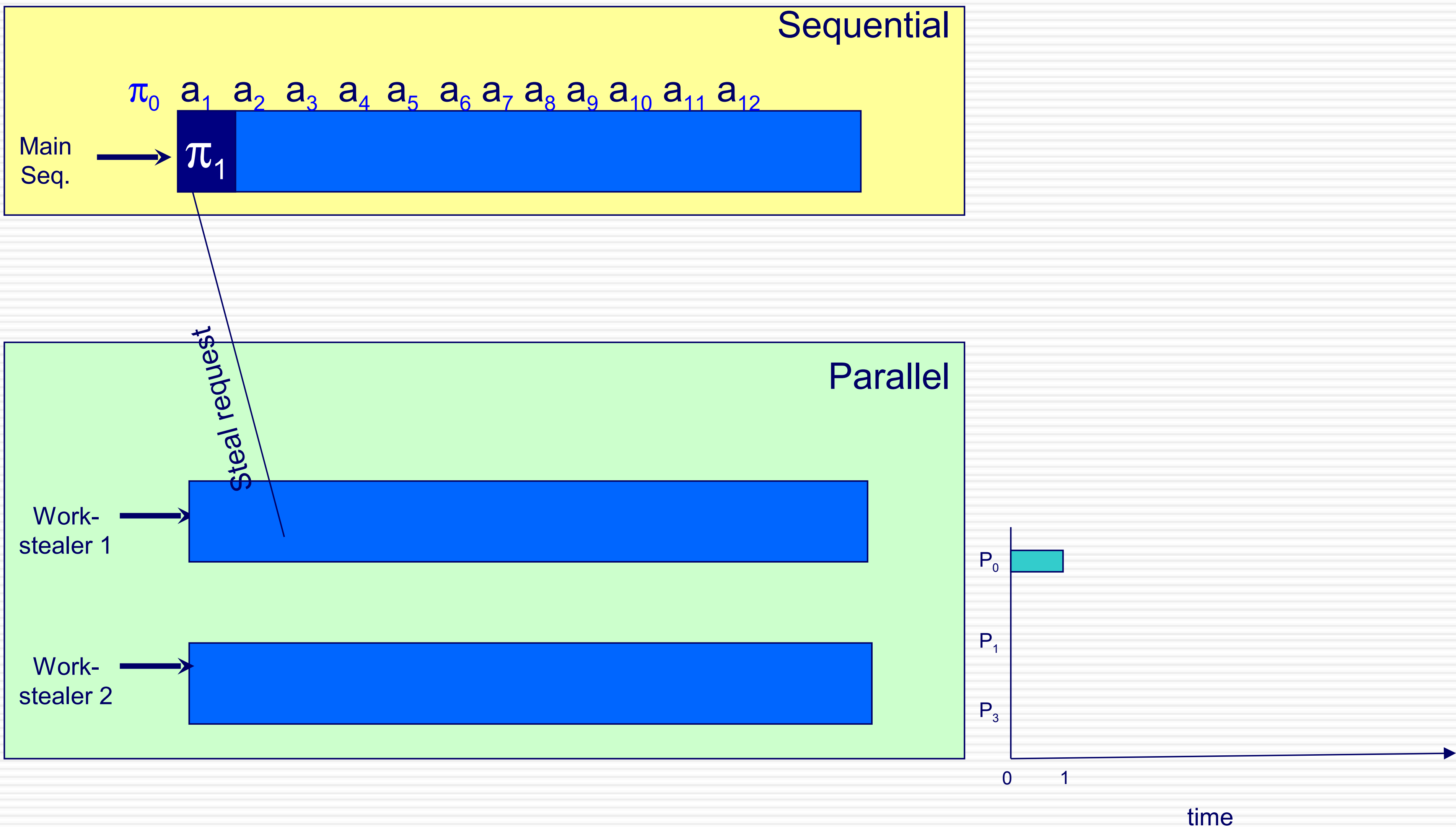
Prefix circuit of depth d

$\Downarrow$ [Fitch80]

#operations > 2*n* - *d*

$$\text{parallel time} \geq \frac{2n}{(p+1).\Pi_{ave}}$$

P-Oblivious Prefix on 3 proc.



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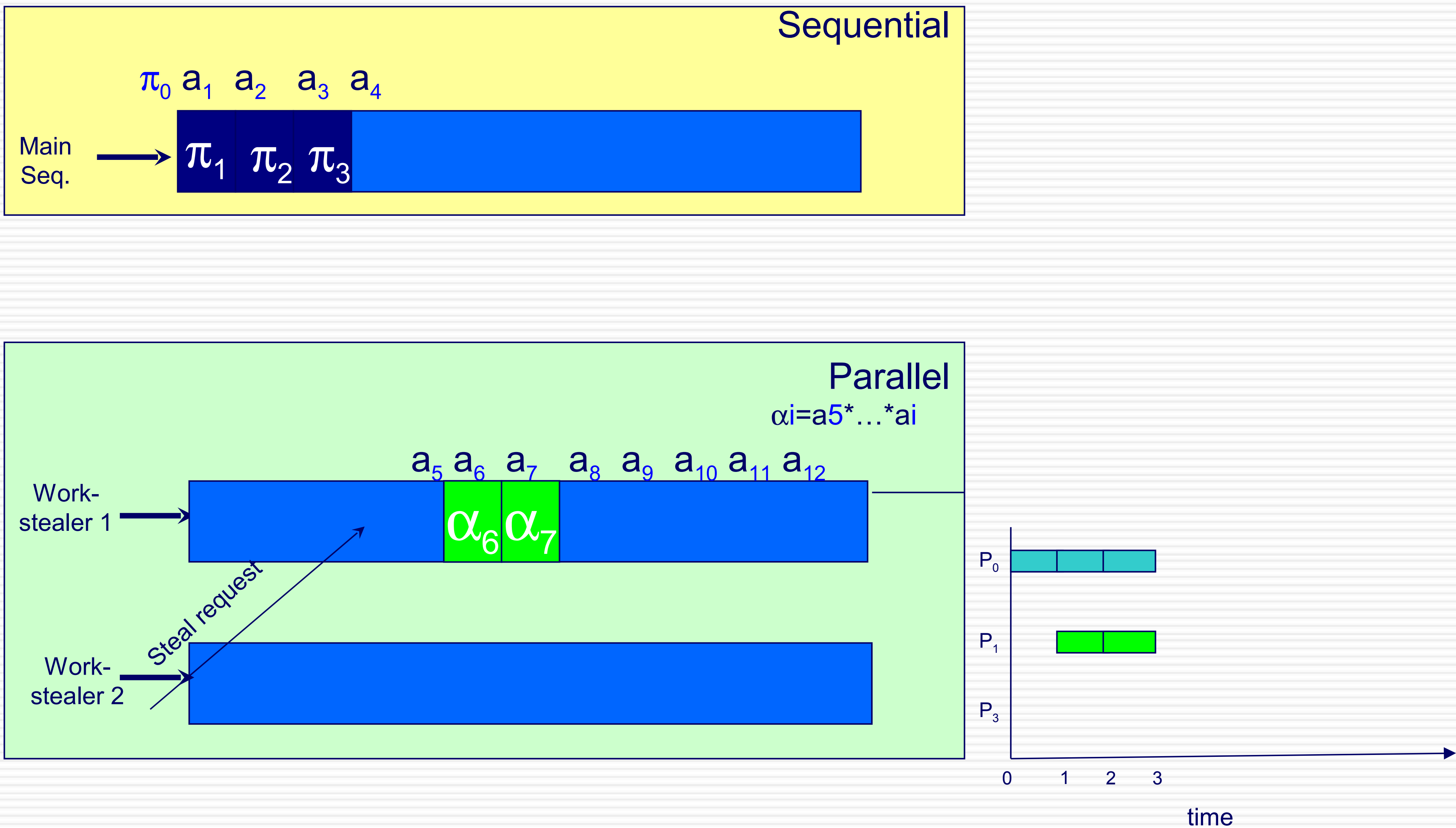

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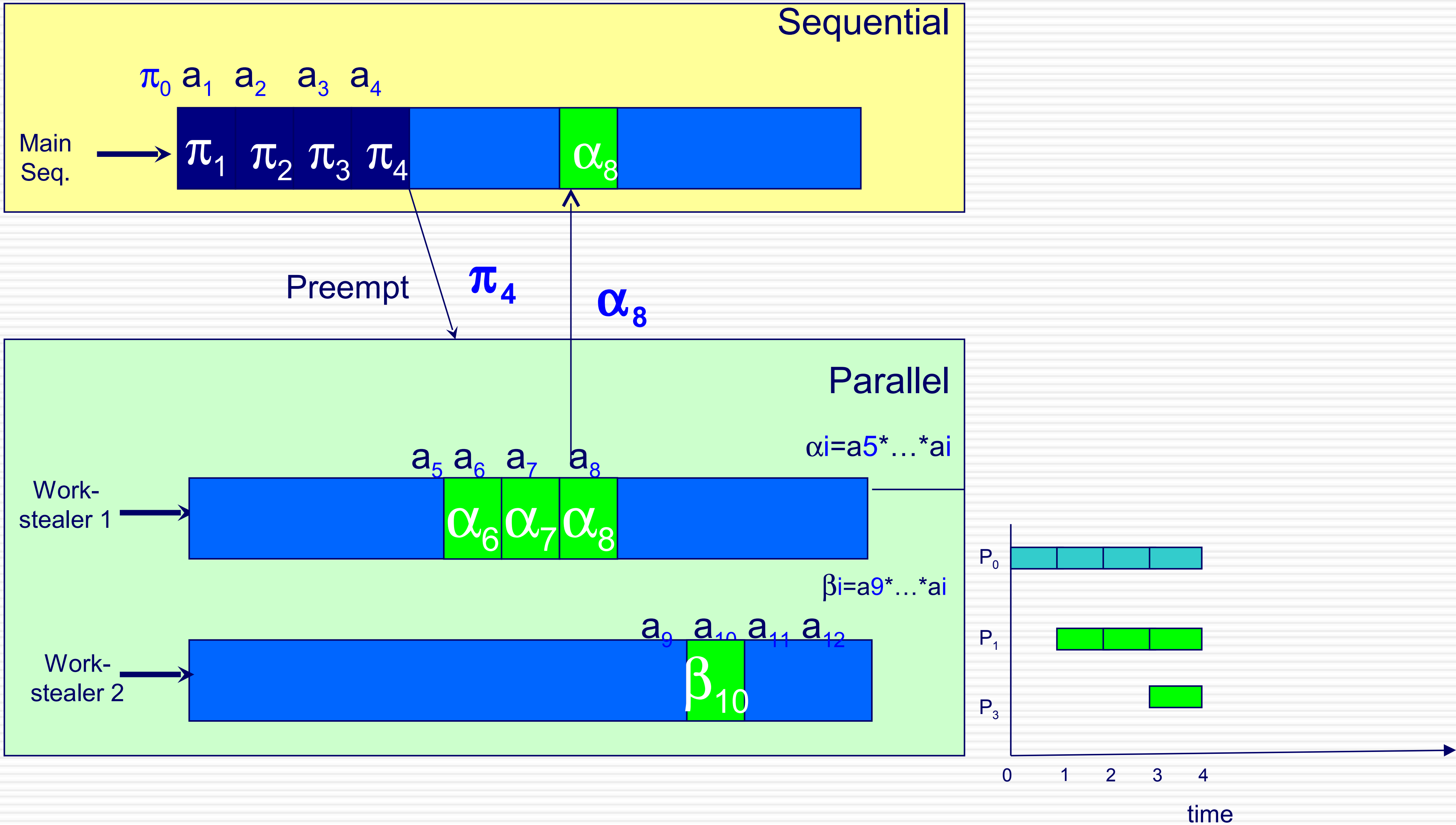

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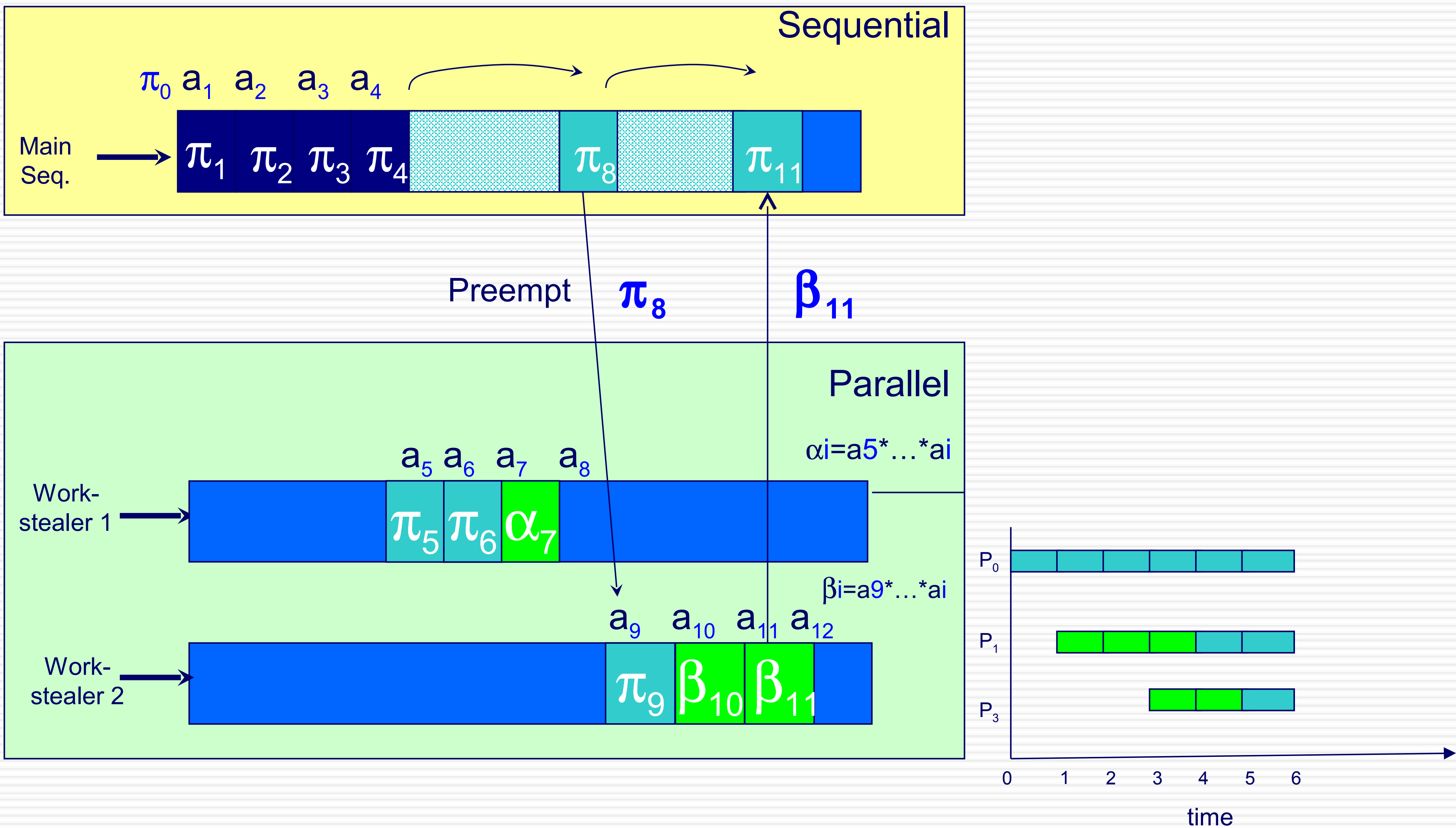

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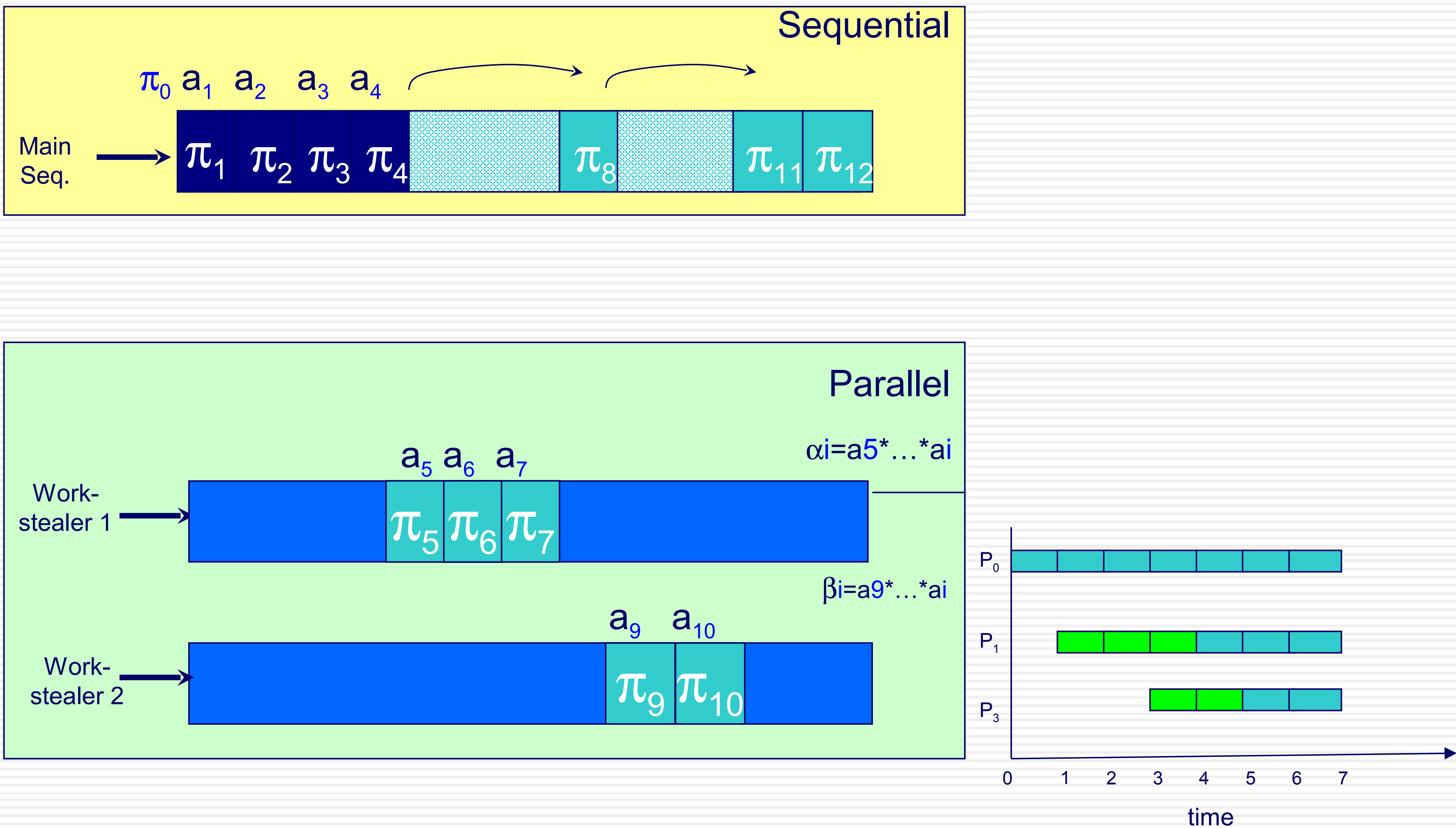

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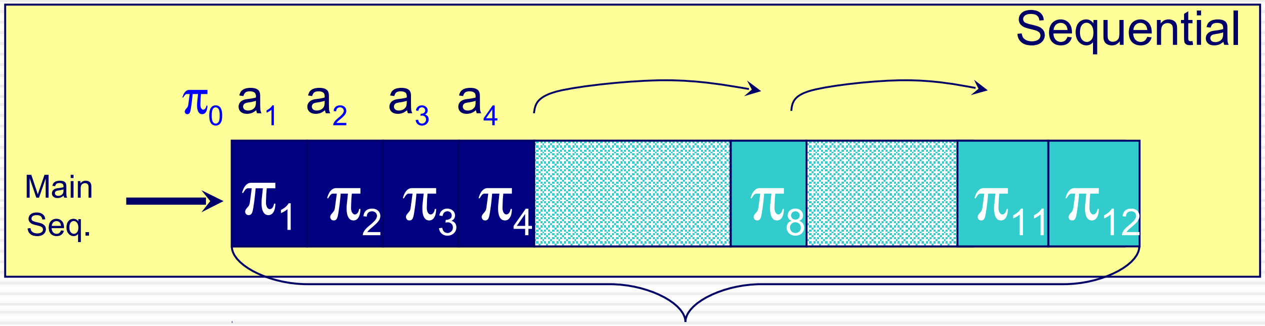
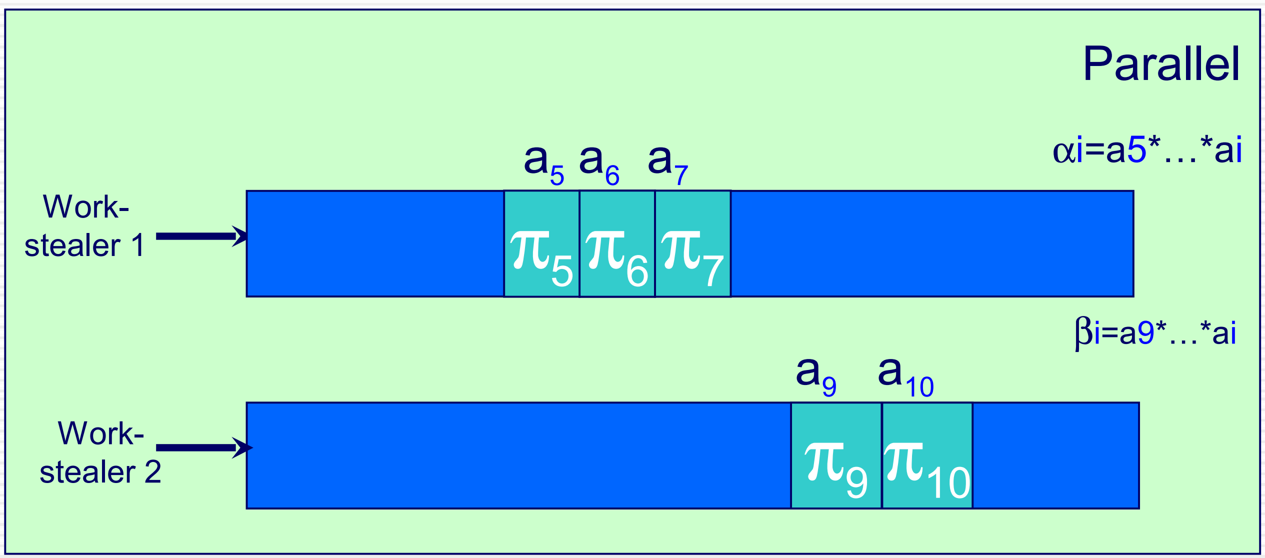
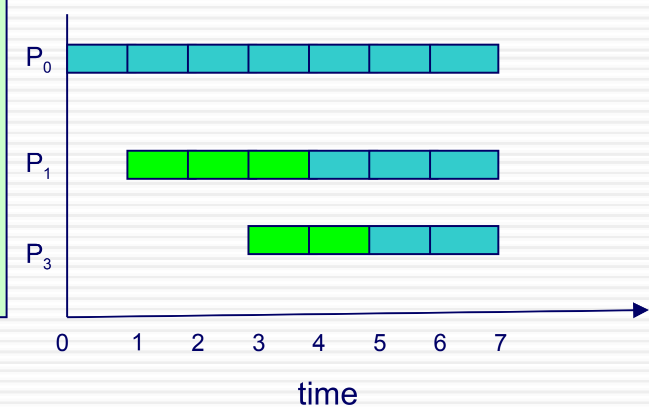


Implicit critical path on the sequential process

$T_p = 7$ $T_p^* = 6$





Analysis of the algorithm

- Execution time $\leq \frac{2n}{(p+1).\Pi_{ave}} + O\left(\frac{\log n}{\Pi_{ave}}\right)$

  Lower bound

- Sketch of the proof :

  Dynamic coupling of two algorithms that complete simultaneously:

  - Sequential: (optimal) number of operations S on one processor
  - Extract_par : work stealer perform X operations on other processors
    - dynamic splitting always possible till finest grain BUT local sequential
      - Critical path small (eg : log X with a W= n / log* n macroloop)
      - Each non constant time task can potentially be splitted (variable speeds)

$$T_s = \frac{S}{\Pi_{ave}} \text{ and } T_p = \frac{X}{(p-1).\Pi_{ave}} + O\left(\frac{\log X}{\Pi_{ave}}\right)$$

  - Algorithmic scheme ensures $T_s = T_p + O(\log X)$

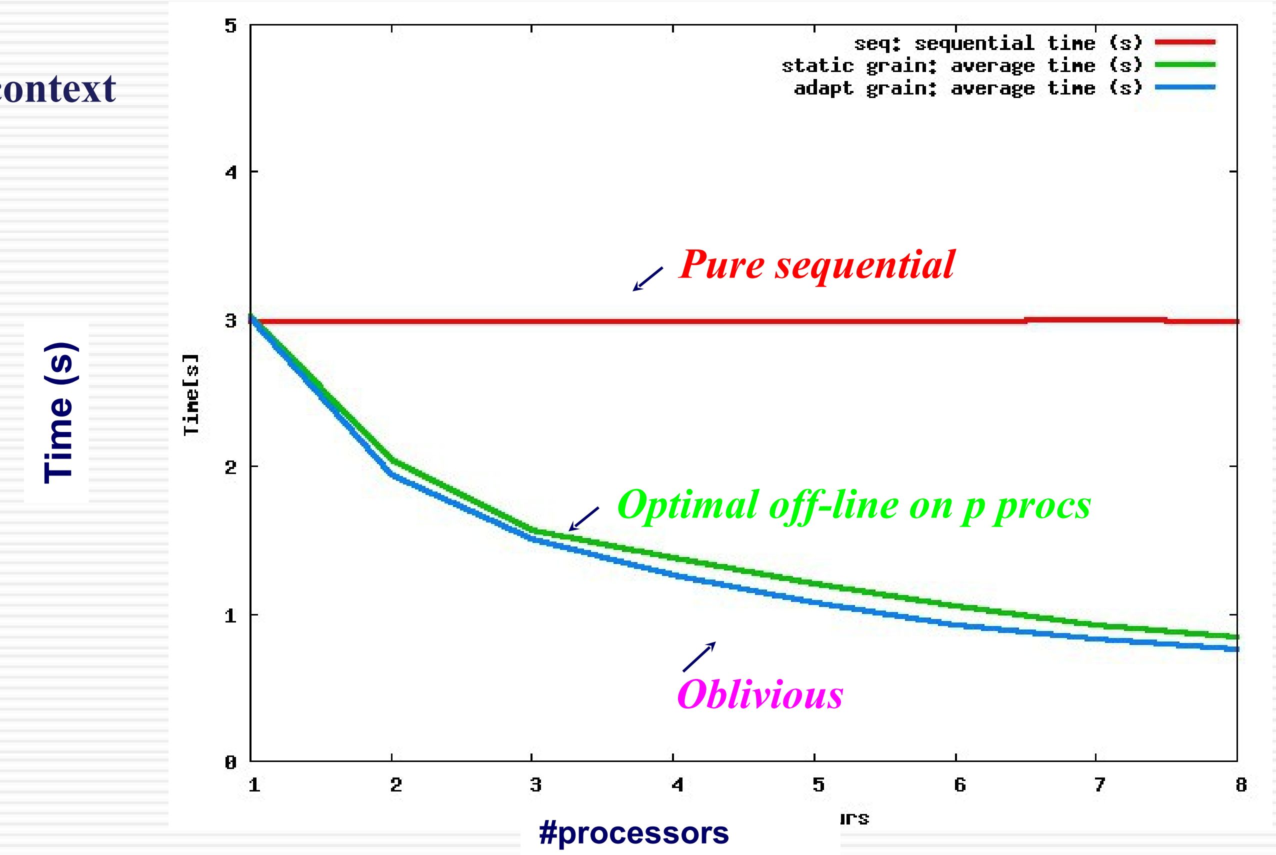
    => enables to bound the whole number X of operations performed
    and the overhead of parallelism = (s+X) - #ops_optimal

Results 1/2

[D Traore]

Prefix sum of 8.10^6 double on a SMP 8 procs (IA64 1.5GHz/ linux)

Single user context



Single-usercontext : processor-oblivious prefix achieves near-optimal performance :

- close to the lower bound both on 1 proc and on p processors

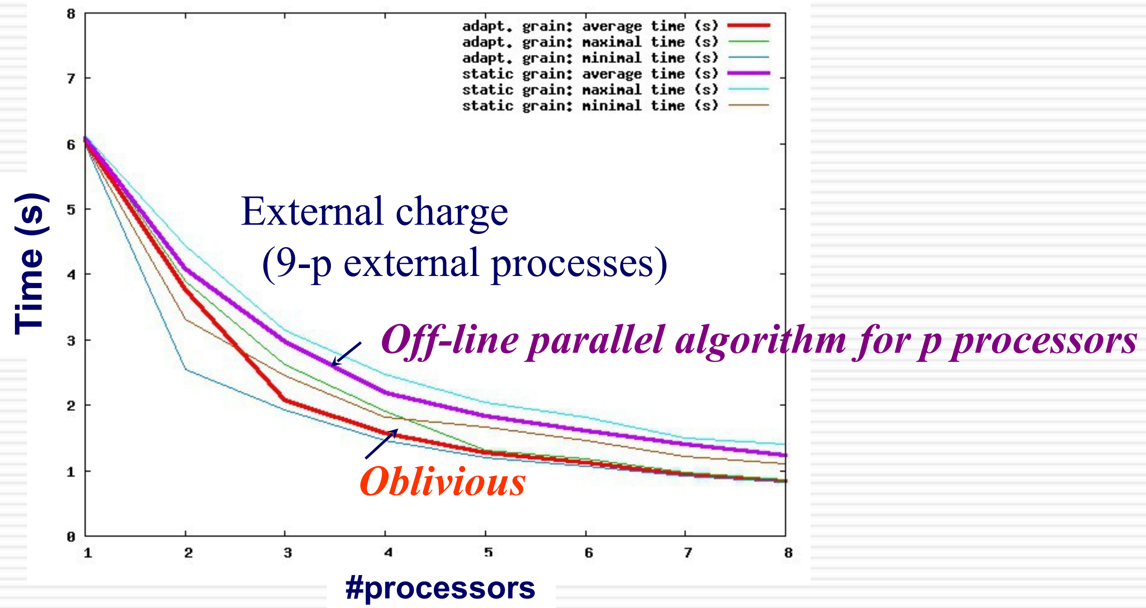
- Less sensitive to system overhead : even better than the theoretically "optimal" off-line parallel algorithm on p processor

Results 2/2

[D Traore]

Prefix sum of 8.10^6 double on a SMP 8 procs (IA64 1.5GHz/ linux)

Multi-user context :



Multi-user context :
Additional external charge: (9-p) additional external dummy processes are concurrently executed

Processor-oblivious prefix computation is always the fastest
15% benefit over a parallel algorithm for p processors with off-line schedule,

Conclusion

- **Fine grain parallelism enables efficient execution on a small number of processors**
  - Interest : portability ; mutualization of code ;
  - Drawback : needs work-first principle => algorithm design

- **Efficiency of classical work stealing relies on *work-first principle* :**
  - Implicitly defenerates a parallel algorithm into a sequential efficient ones ;
  - Assumes that parallel and sequential algorithms perform about the same amount of operations

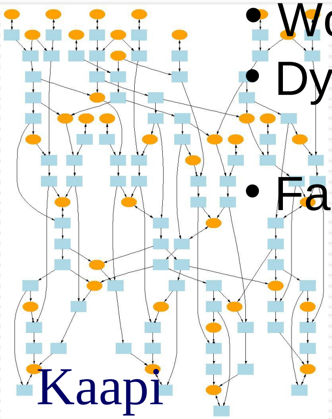
- **Processor Oblivious algorithms based on *work-first principle***
  - Based on anytime extraction of parallelism from any sequential algorithm (may execute different amount of operations) ;
  - Oblivious: near-optimal whatever the execution context is.

- **Generic scheme for stream computations :**

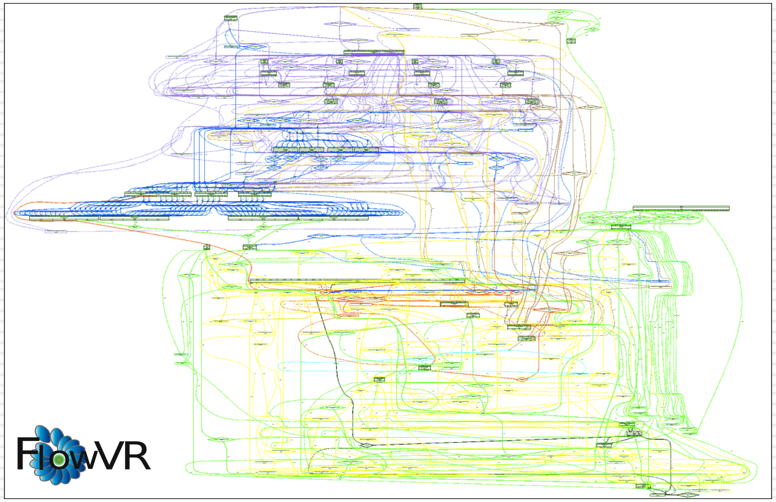
  parallelism introduce a copy overhead from local buffers to the output

  gzip / compression, MPEG-4 / H264

Kaapi (kaapi.gforge.inria.fr)



- Work stealing / work-first principle
- Dynamics Macro-dataflow :
  partitioning (Metis, …)
- Fault Tolerance (add/del resources)



FlowVR (flowvr.sf.net)

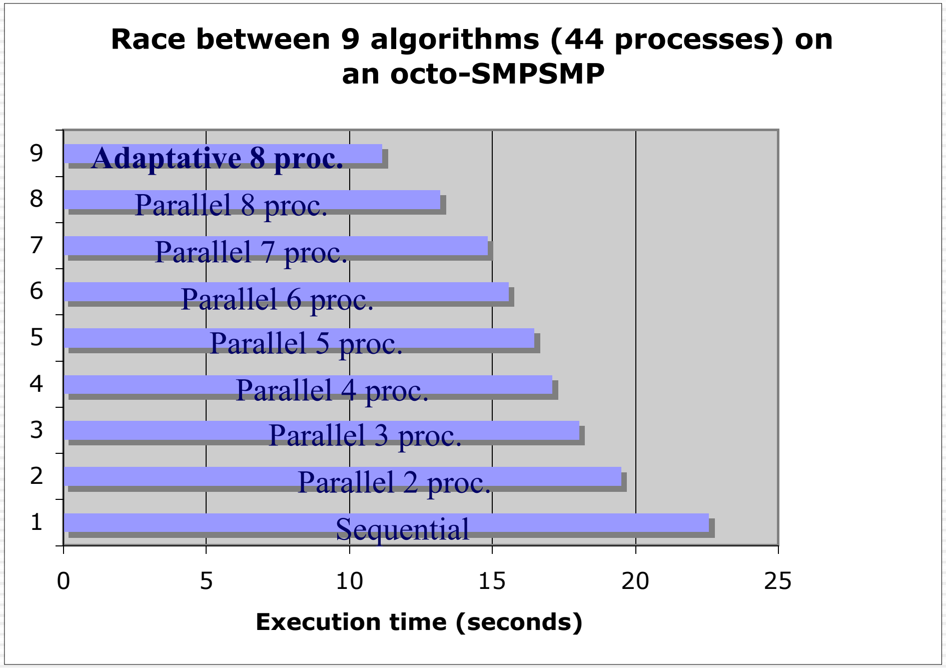
- Dedicated to interactive applications
- Static Macro-dataflow
- Parallel Code coupling

[E Boyer, B Raffin 2006]

Thank you !

Back slides

The Prefix race: sequential/parallel fixed/ adaptive

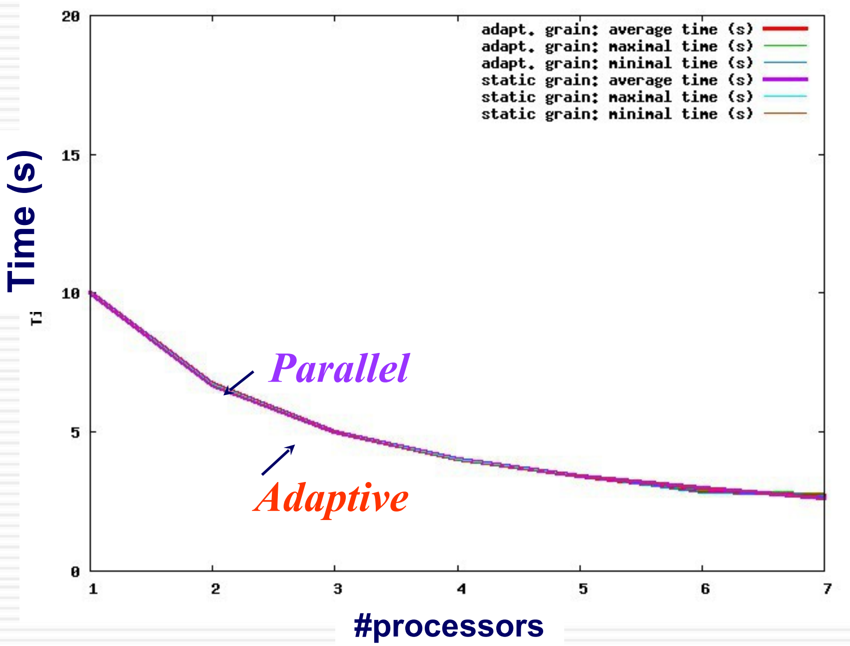


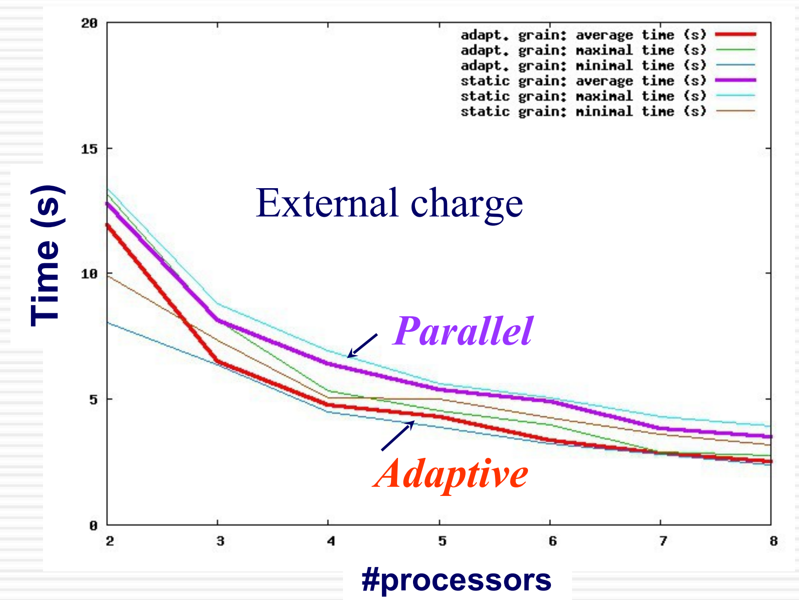
	Sequentiel	Statique					Adaptatif
		p=2	p=4	p=6	p=7	p=8	p=8
Minimum	21,83	18,16	15,89	14,99	13,92	12,51	8,76
Maximum	23,34	20,73	17,66	16,51	15,73	14,43	12,70
Moyenne	22,57	19,50	17,10	15,58	14,84	13,17	11,14
Mediane	22,58	19,64	17,38	15,57	14,63	13,11	11,01

On each of the 10 executions, adaptive completes first

Adaptive prefix : some experiments

Prefix of 10000 elements on a SMP 8 procs (IA64 / linux)





Single user context

Adaptive is equivalent to:

- sequential on 1 proc
- optimal parallel-2 proc. on 2 processors
- …
- optimal parallel-8 proc. on 8 processors

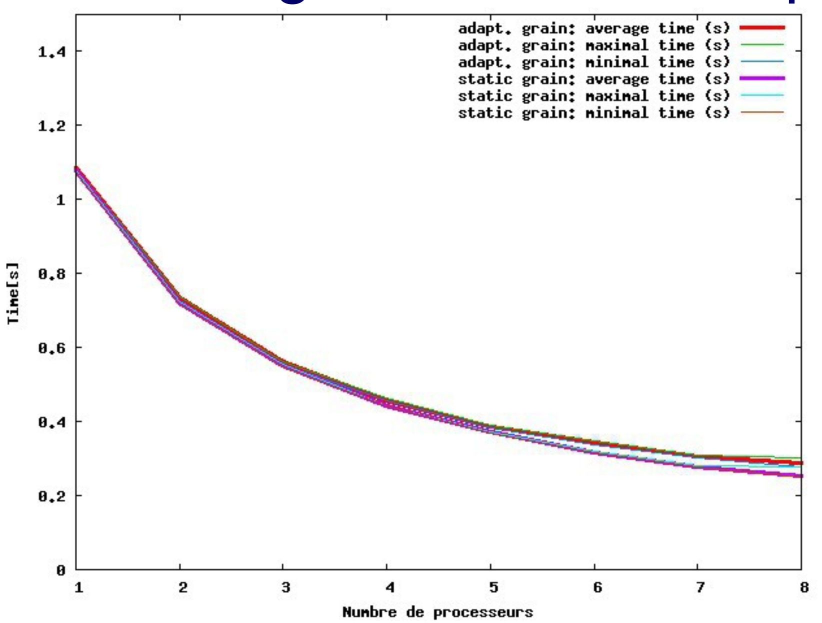
Multi-user context

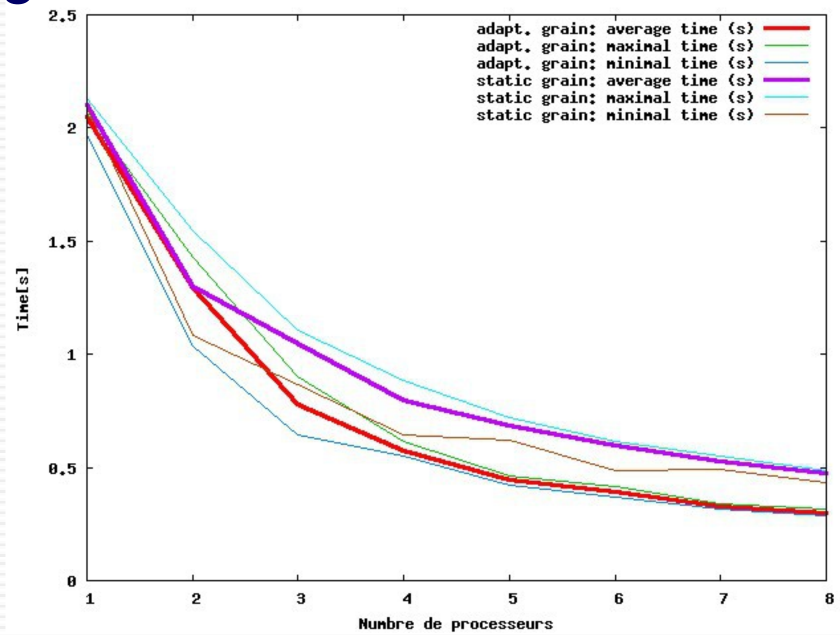
Adaptive is the fastest

15% benefit over a static grain algorithm

With * = double sum (r[i]=r[i-1] + x[i])

Finest "grain" limited to 1 page = 16384 octets = 2048 double



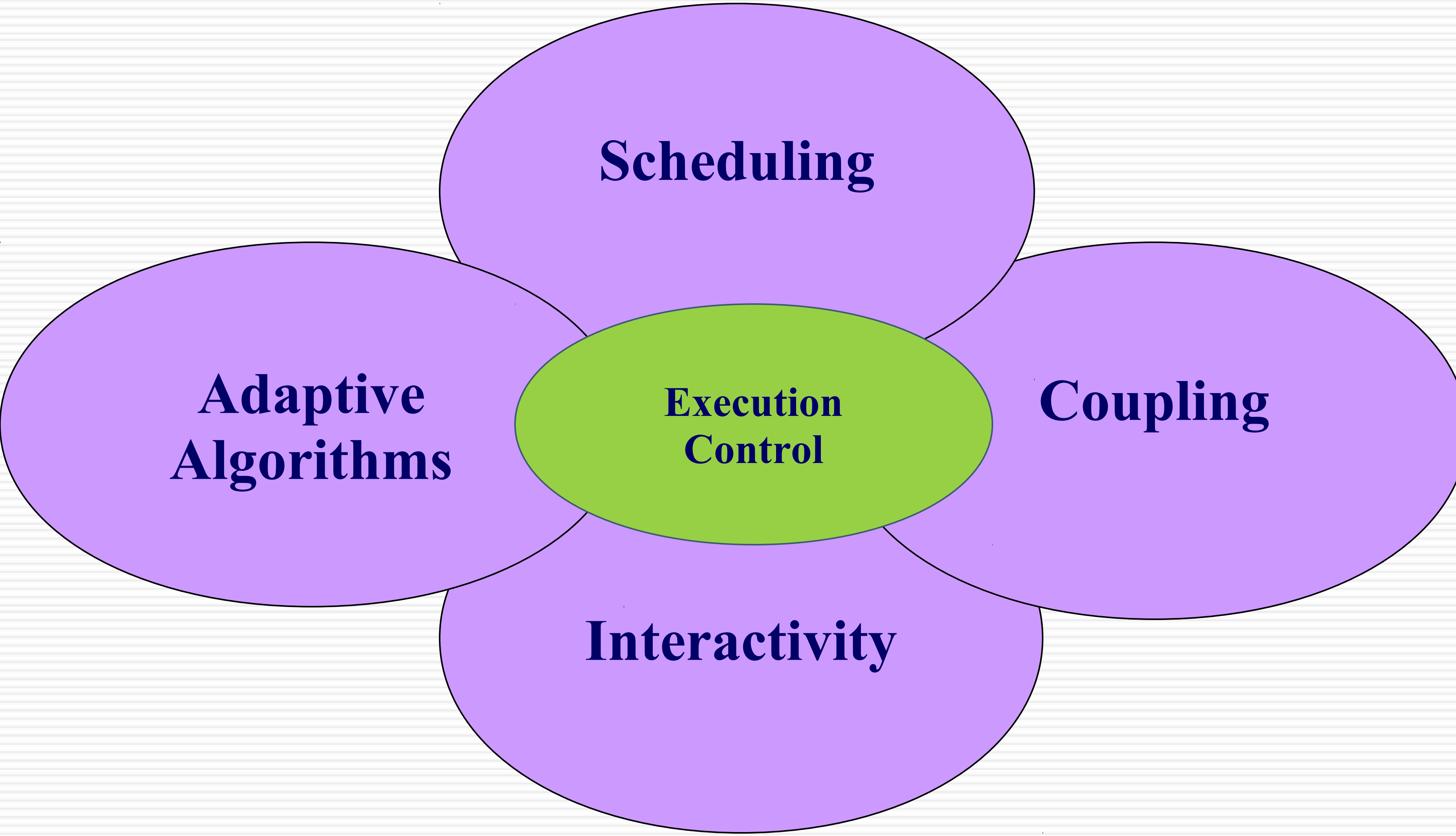


Single user Processors with variable speeds

Remark for n=4.096.000 doubles :

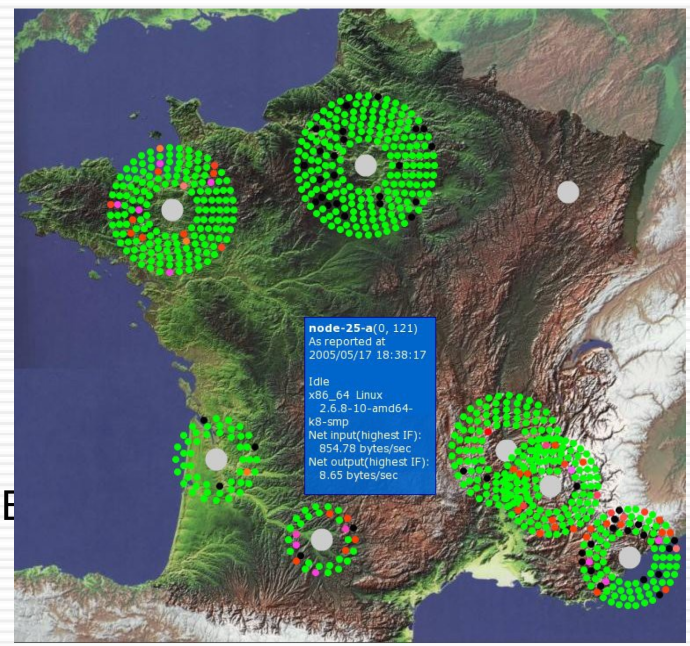
- "pure" sequential : 0,20 s
- minimal "grain" = 100 doubles : 0.26s on 1 proc and 0.175 on 2 procs (close to lower bound)

The Moais Group



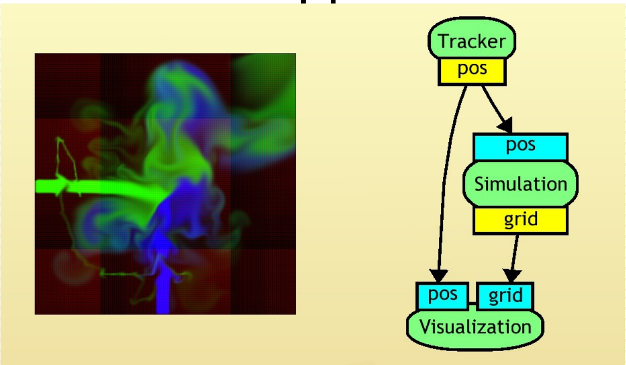
Moais Platforms

- Icluster 2 :
  - 110 dual Itanium bi-processors with Myrinet network
- GrImage (“Grappe” and Image):
  - Camera Network
  - 54 processors (dual processor cluster)
  - Dual gigabits network
  - 16 projectors display wall
- Grids:
  - Regional: Ciment
  - National: Grid5000
    - Dedicated to CS experiments
- SMPs:
  - 8-way Itanium (Bull novascale)
  - 8-way dual-core Opteron + 2 GPUs
- MPSoCs
  - Collaborations with ST Microelectronics on STE



Parallel Interactive App.

- Human in the loop
- Parallel machines (cluster) to enable large interactive applications



- Two main performance criteria:
  - Frequency (refresh rate)
    - Visualization: 30-60 Hz
    - Haptic : 1000 Hz
  - Latency (makespan for one iteration)
    - Object handling: 75 ms
- A classical programming approach: data-flow model
  - Application = static graph
    - Edges: FIFO connections for data transfert
    - Vertices: tasks consuming and producing data
    - Source vertices: sample input signal (cameras)
    - Sink vertices: output signal (projector)
- One challenge:

  Good mapping and scheduling of tasks on processors