Lecture 2

- **Remind: Work W and depth D :**
  - With work-stealing schedule:
    - #steals = O(pD)
    - Execution time on p procs = W/p + O(D) w.h.p.
    - Similar bound achieved with processors with changing speed or multiprogrammed systems.
- **How to parallelize ?**
  - 1/ There exists a fine-grain parallel algorithm that is optimal in sequential
    - Work-stealing and Communications
  - 2/ Extra work induced by parallel can be amortized
  - 3/ Work and Depth are related
    - Adaptive parallel algorithms

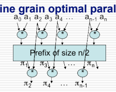
Parallelism induces overhead : e.g. Parallel prefix on fixed architecture

- **Prefix problem** :
  - input : $a_0, a_1, \ldots, a_n$
  - output : $\pi_1, \ldots, \pi_n$ with $\pi_i = \prod_{k=0}^{i} a_k$
- **Sequential** algorithm :
  - for (π[0] = a[0], i = 1 ; i <= n; i++) π[i] = π[i – 1] * a [i] ;

performs only **n** *operations*

- **Fine grain optimal parallel** algorithm :

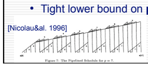
[Ladner-Fisher-81]



Critical time = 2. *log* n
but performs **2.n ops**

- Tight lower bound on **p identical processors**:

[Nicolau&al. 1996]



Optimal time T_p = 2n / (p+1)
but performs **2.n.p/(p+1) ops**

Parallel requires twice more operations than sequential !!

Lower bound(s) for the prefix

Prefix circuit of depth d
⇓ [Fitch80]
#operations > $2n - d$

$$\text{parallel time} \geq \frac{2n}{(p+1).\Pi_{ave}}$$

Overview

- Introduction : interactive computation, parallelism and processor oblivious
  - Overhead of parallelism : parallel prefix
- Machine model and work-stealing
- Scheme 1: **Extended work-stealing : concurently sequential and parallel**

3. Work-first principle and adaptability

- **Work-first principle**: -implicit- dynamic choice between two executions :
  - a **sequential *"depth-first"*** execution of the parallel algorithm (local, default) ;
  - a **parallel "*breadth-first*"** one.
- Choice is performed at runtime, depending on resource idleness: rare event if Depth is small to Work
- **WS adapts parallelism to processors with practical provable performances**
  - Processors with changing speeds / load (data, user processes, system, users,
  - Addition of resources (fault-tolerance [Cilk/Porch, Kaapi, …])
- **The choice is justified only when the sequential execution of the parallel algorithm is an efficient sequential algorithm:**
  - Parallel Divide&Conquer computations
  - …

-> **But,** this may not be general in practice

How to get both optimal work W_1 and W_∞ small?

- General approach**: to** mix **both**
  - a **sequential** algorithm with optimal work W_1
  - and a fine grain **parallel** algorithm with minimal critical time W_∞
- Folk technique **:** *parallel, than sequential*
  - Parallel algorithm until a certain « grain »; then use the sequential one
  - Drawback : W_∞ increases ;o) …and, also, the number of steals
- *Work-preserving speed-up technique* [Bini-Pan94] *sequential, then parallel* **Cascading** [Jaja92] **: Careful interplay of both algorithms to build one with both**

  W_∞ **small** and W_1 = O(W_{seq})

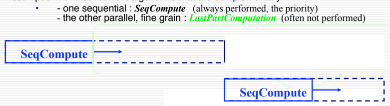
  - Use the work-optimal sequential algorithm to reduce the size
  - Then use the time-optimal parallel algorithm to decrease the time
  - Drawback : sequential at coarse grain and parallel at fine grain ;o(

Extended work-stealing: concurrently sequential and parallel

Based on the work-stealing and the ***Work-first*** *principle* :
Instead of optimizing the **sequential execution** of the **best parallel** algorithm, let optimize the **parallel execution** of the ***best sequential*** algorithm

Execute always a sequential algorithm to reduce parallelism overhead
⇒ parallel algorithm is used only if a processor becomes **idle** (ie *workstealing*) [Roch&al2005,…] to ***extract parallelism*** from the remaining work a sequential computation

Assumption : two concurrent algorithms that are complementary:
- - one sequential : ***SeqCompute*** (always performed, the priority)
  - the other parallel, fine grain : ***LastPartComputation*** (often not performed)



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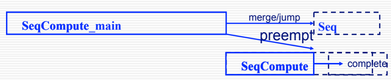

Note:
- **merge and jump** operations to ensure non-idleness of the victim
- Once *SeqCompute_main* completes, it becomes a work-stealer

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- **Scheme 2: Amortizing the overhead of synchronization (Nano-loop)**

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Extended work-stealing and granularity

- **Scheme of the sequential process : nanoloop**

```
While (not completed(Wrem) ) and (next_operation hasn't been stolen)
{
    atomic { extract_next k operations ; Wrem -= k  ; }
    process the k operations extracted ;
}
```

- ***Processor-oblivious* algorithm**
  - Whatever *p* is, it performs O(*p.D*) preemption operations (« continuation faults »)
    -> ***D*** should be as small as possible to maximize both speed-up and locality
  - If no steal occurs during a (sequential) computation, then its *arithmetic work* is optimal to the one W_{opt} of the sequential algorithm (no spawn/fork/copy)
    -> ***W*** should be as close as possible to W_{opt}
- Choosing **$k = Depth(W_{rem})$** does not increase the depth of the parallel algorithm while ensuring ***O(W / D)*** *atomic operations :*
  since $D > log_2 W_{rem}$, **then if $p = 1$: $W \sim W_{opt}$**
- **Implementation :** atomicity in nano-loop based without lock
  - Efficient mutual exclusion between sequential process and parallel work-stealer
- ***Self-adaptive granularity***

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Interactive application with time constraint

Anytime Algorithm:
- Can be stopped at any time (with a result)
- Result quality improves as more time is allocated

In Computer graphics, anytime algorithms are common:
Level of Detail algorithms (time budget, triangle budget, etc…)
Example: Progressive texture loading, triangle decimation (Google Earth)

Anytime processor-oblivious algorithm:
On ***p*** processors with average speed Π_{ave}, it outputs in a fixed time ***T*** a result with the same quality than a sequential processor with speed Π_{ave} in time $p.\Pi_{ave}$.

Example: Parallel Octree computation for 3D Modeling

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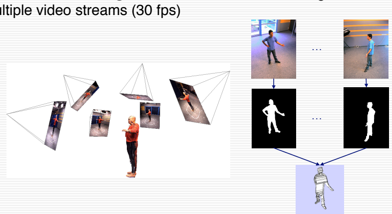
Parallel 3D Modeling

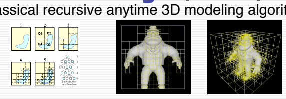
3D Modeling :
build a 3D model of a scene from a set of calibrated images

On-line 3D modeling for interactions: 3D modeling from multiple video streams (30 fps)

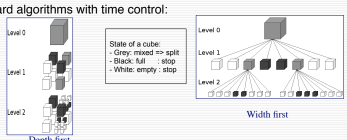


Octree Carving [L. Soares 06]

A classical recursive anytime 3D modeling algorithm.



Standard algorithms with time control:



Depth first
+ iterative deepening

Width first

At termination: quick test to decide all grey cubes time control

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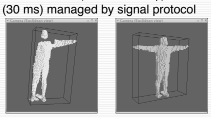
Width first parallel octree carving

Well suited to work-stealing
-Small critical path, while huge amount of work (eg. D = 8, W = 164 000)
- non-predictable work, non predictable grain :

For cache locality, each level is processed by a self-adaptive grain :
"sequential iterative" / "parallel recursive split-half"

Octree needs to be "balanced" when stopping:
- Serially computes each level *(with small overlap)*
- Time deadline (30 ms) managed by signal protocol

Unbalanced  Balanced

Theorem: W.r.t the adaptive in time T on p procs., the sequential algorithm:
- goes at most one level deeper : **$| d_s - d_p | \leq 1$** ;
- computes at most : **$n_s \leq n_p + O(\log n_s)$** .

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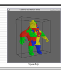
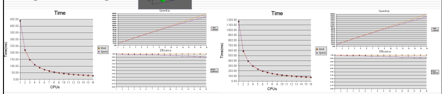
Results

[L. Soares 06]



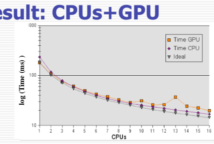
- 16 core Opteron machine, 64 images
- Sequential: 269 ms, 16 Cores: 24 ms
- 8 cores: about 100 steals (167 000 grey cells)



8 cameras, levels 2 to 10   64 cameras, levels 2 to 7

Preliminary result: CPUs+GPU

- 1 GPU + 16 CPUs
- GPU programmed in OpenGL
- efficient coupling till 8 but does not scale



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- **Scheme 3: Amortizing the overhead of parallelism (Macro-loop)**

4. Amortizing the arithmetic overhead of parallelism

Adaptive scheme : extract_seq/nanoloop // extract_par
• ensures an optimal number of operation on 1 processor
• but no guarantee on the work performed on p processors

Eg (C++ STL): find_if (first, last, predicate)
locates the first element in [First, Last) verifying the predicate

This may be a drawback (unneeded processor usage) :
• undesirable for a library code that may be used in a complex application, with many components
• (or not fair with other users)
• increases the time of the application :
  • *any parallelism that may increase the execution time should be avoided*

Motivates the building of **work-optimal** parallel adaptive algorithm (**processor oblivious**)

4. Amortizing the arithmetic overhead of parallelism (cont'd)

Similar to nano-loop for the sequential process :
• that balances the -atomic- local work by the depth of the remaindering one

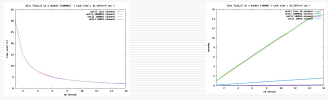
Here, by **amortizing** the work induced by the extract_par operation, ensuring this **work to be *small*** enough :
• Either w.r.t the -useful- **work already performed**
• Or with respect to the - useful - **work yet to performed** (if known)
• or both.

Eg : find_if (first, last, predicate) :
• only the work already performed is known (on-line)
• then prevent to assign more than $\alpha(W_{done})$ operations to work-stealers
• Choices for α(n) :
  • **n/2 : similar to Floyd's iteration** (approximation ratio = 2)
  • **n/log* n :** to ensure optimal usage of the work-stealers

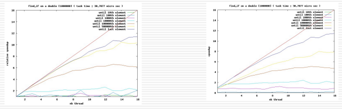
Results on find_if

[S. Guelton]

N doubles : time predicate ~ 0.31 ms



With no amortization macroloop



With amortization macroloop

5. Putting things together *processor-oblivious prefix computation*

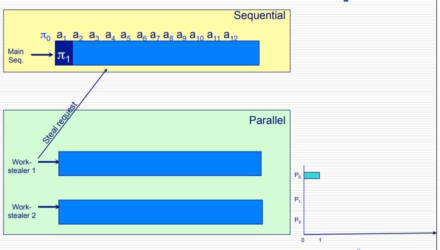
Parallel algorithm based on :

- compute-seq / extract-par scheme

- nano-loop for compute-seq

- macro-loop for extract-par

P-Oblivious Prefix on 3 proc.



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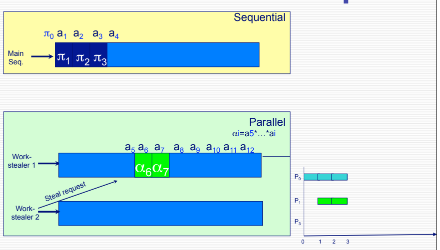

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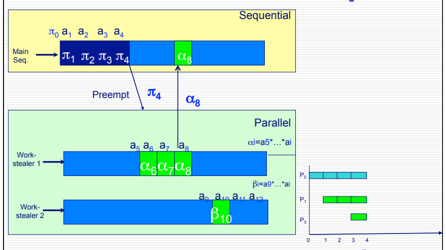

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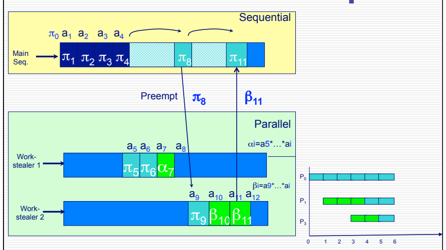

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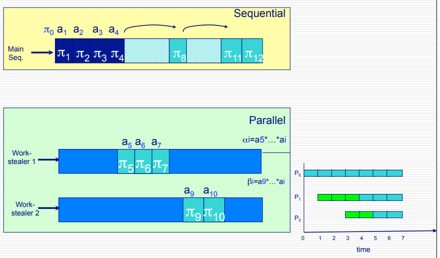

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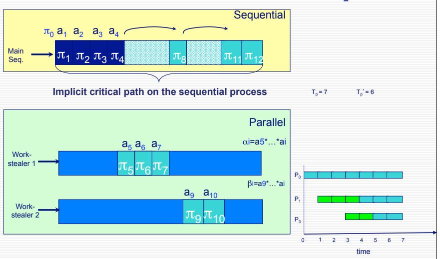

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Analysis of the algorithm

- Execution time $\leq \frac{2n}{(p+1).\Pi_{ave}} + O\left(\frac{\log n}{\Pi_{ave}}\right)$ (Lower bound)
- Sketch of the proof :
  Dynamic coupling of two algorithms that complete simultaneously:
  - Sequential: (optimal) number of operations S on one processor
  - Extract_par : work stealer perform X operations on other processors
    - dynamic splitting always possible till finest grain BUT local sequential
      - Critical path small (eg : log X with a W= n / log* n macroloop)
      - Each non constant time task can potentially be splitted (variable speeds)

$$T_s = \frac{S}{\Pi_{ave}} \text{ and } T_p = \frac{X}{(p-1).\Pi_{ave}} + O\left(\frac{\log X}{\Pi_{ave}}\right)$$

  - Algorithmic scheme ensures $T_s = T_p + O(\log X)$

    => enables to bound the whole number X of operations performed
    and the overhead of parallelism = (s+X) - #ops_optimal

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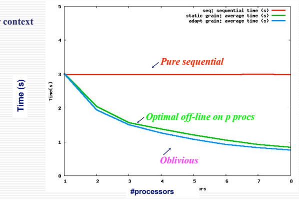
Results 1/2

[D Traore]

Prefix sum of 8.10^6 double on a SMP 8 procs (IA64 1.5GHz/ linux)

Single user context



Single-usercontext : processor-oblivious prefix achieves near-optimal performance :
- close to the lower bound both on 1 proc and on p processors

- Less sensitive to system overhead : even better than the theoretically "optimal" off-line parallel algorithm on p processors

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Results 2/2

[D Traore]

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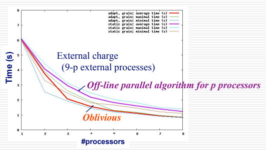
Multi-user context :



Multi-user context :
Additional external charge: (9-p) additional external dummy processes are concurrently executed

Processor-oblivious prefix computation is always the fastest
15% benefit over a parallel algorithm for p processors with off-line schedule,

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Conclusion

- **Fine grain parallelism enables efficient execution on a small number of processors**
  - Interest : portability ; mutualization of code ;
  - Drawback : needs work-first principle => algorithm design
- **Efficiency of classical work stealing relies on *work-first principle* :**
  - Implicitly defenerates a parallel algorithm into a sequential efficient ones ;
  - Assumes that parallel and sequential algorithms perform about the same amount of operations
- **Processor Oblivious algorithms based on *work-first principle***
  - Based on anytime extraction of parallelism from any sequential algorithm (may execute different amount of operations) ;
  - Oblivious: near-optimal whatever the execution context is.
- **Generic scheme for stream computations :**
  parallelism introduce a copy overhead from local buffers to the output
  gzip / compression, MPEG-4 / H264