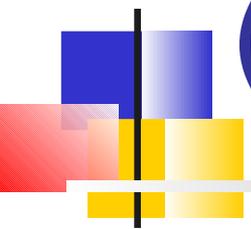
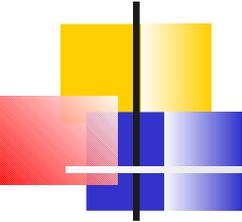


# Principles of High Performance Computing (ICS 632)



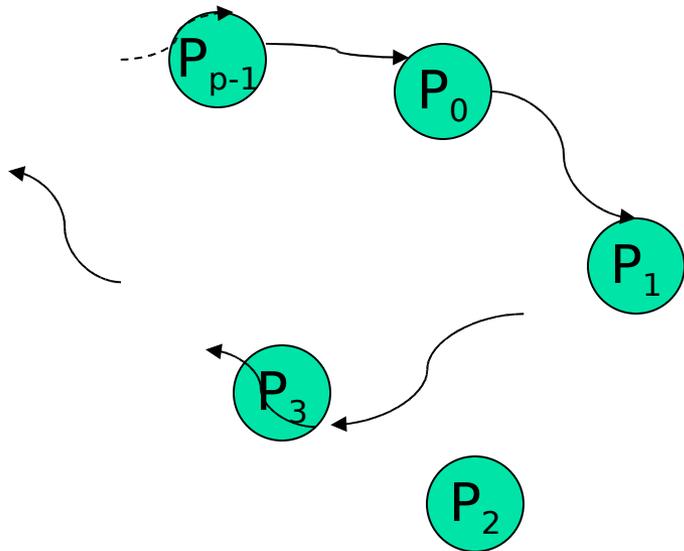
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Communication in a  
Ring Topology

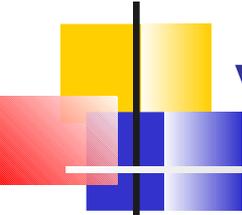


# Ring Topology (Section 3.3)

---



- Each processor is identified by a rank
  - `MY_NUM()`
- There is a way to find the total number of processors
  - `NUM_PROCS()`
- Each processor can send a message to its successor
  - `SEND(addr, L)`
- And receive a message from its predecessor
  - `RECV(addr, L)`
  
- We'll just use the above pseudo-code rather than MPI
- Note that this is much simpler than the example tree topology we saw in the previous set of slides



# Virtual vs. Physical Topology

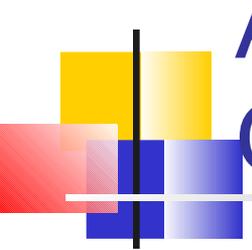
---

- Now that we have chosen to consider a Ring topology we “pretend” our physical topology is a ring topology
- We can always implement a virtual ring topology (see previous set of slides)
  - And read Section 4.6
- So we can write many “ring algorithms”
- It may be that a better virtual topology is better suited to our physical topology
- But the ring topology makes for very simple programs and is known to be reasonably good in practice
- So it’s a good candidate for our first look at parallel algorithms

# Cost of communication (Sect.

## 3.2.1)

- It is actually difficult to precisely model the cost of communication
  - E.g., MPI implementations do various optimizations given the message sizes
- We will be using a simple model
  - Time =  $L + m/B$ 
    - L: start-up cost or *latency*
    - B: bandwidth (b = 1/B)
    - m: message size
- We assume that if a message of length m is sent from  $P_0$  to  $P_q$ , then the communication cost is  $q(L + m b)$
- There are many assumptions in our model, some not very realistic, but we'll discuss them later

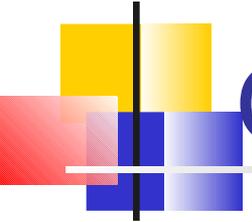


# Assumptions about Communications

---

- Several Options
  - Both Send() and Recv() are blocking
    - Called “rendez-vous”
    - Very old-fashioned systems
  - Recv() is blocking, but Send() is not
    - Pretty standard
    - MPI supports it
  - Both Recv() and Send() are non-blocking
    - Pretty standard as well
    - MPI supports it

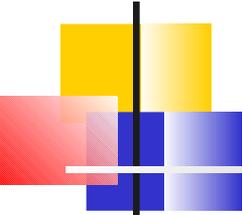




# Collective Communications

---

- To write a parallel algorithm, we will need collective operations
  - Broadcasts, etc.
- Now MPI provide those, and they likely:
  - Do not use the ring logical topology
  - Utilize the physical resources well
- Let's still go through the exercise of writing some collective communication algorithms
- We will see that for some algorithms we really want to do these communications “by hand” on our virtual topology rather than using the MPI collective



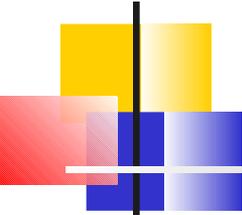
## Broadcast (Section 3.3.1)

---

- We want to write a program that has  $P_k$  send the same message of length  $m$  to all other processors

Broadcast( $k, addr, m$ )

- On the ring, we just send to the next processor, and so on, with no parallel communications whatsoever
- This is of course not the way one should implement a broadcast in practice if the physical topology is not merely a ring
  - MPI uses some type of tree topology



# Broadcast (Section 3.3.1)

---

*Broadcast (k, addr, m)*

*q = MY\_NUM()*

*p = NUM\_PROCS()*

*if (q == k)*

*SEND (addr, m)*

*else*

*if (q == k-1 mod p)*

*RECV (addr, m)*

*else*

*RECV (addr, m)*

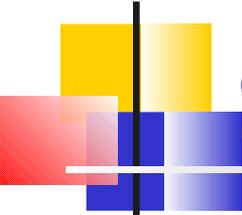
*SEND (addr, m)*

*endif*

*endif*

- Assumes a blocking receive
- Sending may be non-blocking
- The broadcast time is

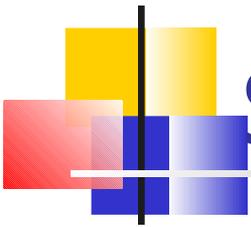
$$(p-1)(L+m b)$$



# Scatter (Section 3.2.2)

---

- Processor  $k$  sends a different message to all other processors (and to itself)
  - $P_k$  stores the message destined to  $P_q$  at address  $\text{addr}[q]$ , including a message at  $\text{addr}[k]$
- At the end of the execution, each processor holds the message it had received in  $\text{msg}$
- The principle is just to pipeline communication by starting to send the message destined to  $P_{k-1}$ , the most distant processor



# Scatter (Section 3.3.2)

```
Scatter(k, msg, addr, m)
```

```
q = MY_NUM()
```

```
p = NUM_PROCS()
```

```
if (q == k)
```

```
  for i = 0 to p-2
```

```
    SEND(addr[k+p-1-i mod p], m)
```

```
  msg ← addr[k]
```

```
else
```

```
  RECV(tempR, L)
```

```
  for i = 1 to k-1-q mod p
```

```
    tempS ↔ tempR
```

```
    SEND(tempS, m) || RECV(tempR, m)
```

```
  msg ← tempR
```

Same execution time as the broadcast  
 $(p-1)(L + m b)$

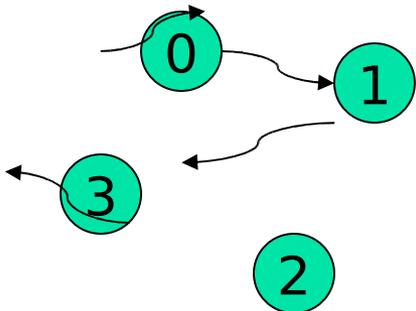
Swapping of send buffer  
and receive buffer (pointer)

Sending and  
Receiving  
in Parallel, with a  
non blocking Send

# Scatter (Section 3.3.2)

*Scatter(k, msg, addr, m)*

```
q = MY_NUM()
p = NUM_PROCS()
if (q == k)
  for i = 0 to p-2
    SEND(addr[k+p-1-i mod p], m)
  msg ← addr[k]
else
  RECV(tempR, L)
  for i = 1 to k-1-q mod p
    tempS ↔ tempR
    SEND(tempS, m) || RECV(tempR, m)
  msg ← tempR
```



$k = 2, p = 4$

Proc q=2

```
send addr[2+4-1-0 % 4 = 1]
send addr[2+4-1-1 % 4 = 0]
send addr[2+4-1-2 % 4 = 3]
msg = addr[2]
```

Proc q=3

```
recv (addr[1])
// loop 2-1-3 % 4 = 2 times
send (addr[1]) || recv (addr[0])
send (addr[0]) || recv (addr[3])

msg = addr[3]
```

Proc q=0

```
recv (addr[1])
// loop 2-1-2 % 4 = 1 time
send (addr[1]) || recv (addr[0])

msg = addr[0]
```

Proc q=1

```
// loop 2-1-1 % 4 = 0 time
recv (addr[1])

msg = addr[1]
```

# All-to-all (Section 3.3.3)

```
All2All(my_addr, addr, m)
```

```
  q = MY_NUM()
```

```
  p = NUM_PROCS()
```

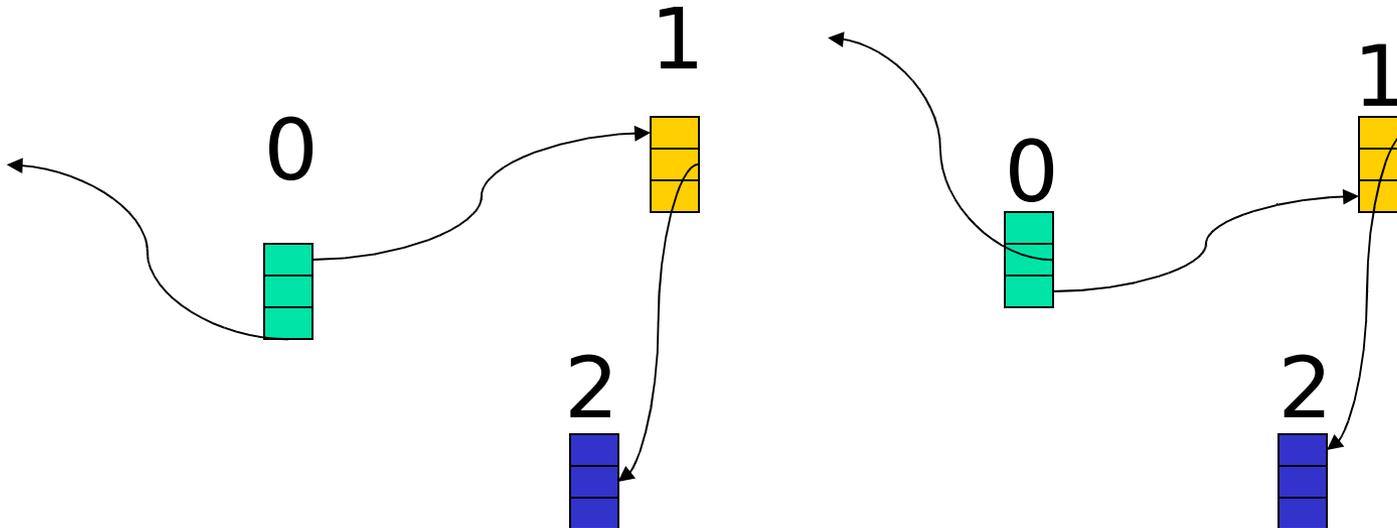
```
  addr[q] ← my_addr
```

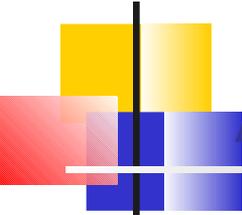
```
  for i = 1 to p-1
```

```
    SEND(addr[q-i+1 mod p], m)
```

```
  || RECV(addr[q-i mod p], m)
```

Same execution time as the scatter  
 $(p-1)(L + m b)$

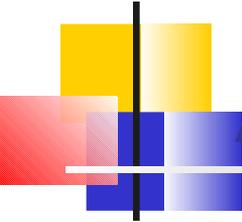




# A faster broadcast?

---

- How can we improve performance?
- One can cut the message in many small pieces, say in  $r$  pieces where  $m$  is divisible by  $r$ .
- The root processor just sends  $r$  messages
- The performance is as follows
  - Consider the last processor to get the last piece of the message
  - There need to be  $p-1$  steps for the first piece to arrive, which takes  $(p-1)(L + m b / r)$
  - Then the remaining  $r-1$  pieces arrive one after another, which takes  $(r-1)(L + m b / r)$
  - For a total of:  $(p - 2 + r) (L + m b / r)$



# A faster broadcast?

---

- The question is, what is the value of  $r$  that minimizes  
 $(p - 2 + r) (L + m b / r)$  ?
- One can view the above expression as  $(c+ar)(d+b/r)$ ,  
with four constants  $a, b, c, d$
- The non-constant part of the expression is then  $ad.r +$   
 $cb/r$ , which must be minimized
- It is known that this value is minimized for

$$\sqrt{cb / ad}$$

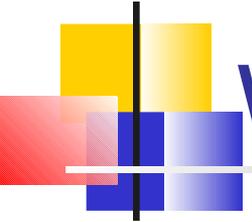
and we have

$$r_{\text{opt}} = \sqrt{m(p-2) b / L}$$

with the optimal time

$$(\sqrt{(p-2) L} + \sqrt{m b})^2$$

which tends to  $mb$  when  $m$  is large, which is independent  
of  $p$ !



# Well-known Network Principle

---

- We have seen that if we cut a (large) message in many (small) messages, then we can send the message over multiple hops (in our case  $p-1$ ) almost as fast as we can send it over a single hop
- This is a fundamental principle of IP networks
  - We cut messages into IP frames
  - Send them over many routers
  - But really go as fast as the slowest router