

Job Scheduling

Master 2 Research Tutorial: High-Performance Architectures

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November 22, 2006

- 1 Task Graphs and Parallel Tasks From Outer Space
- 2 Batch Scheduling
 - Basic idea: FCFS + Backfilling
 - EASY
 - How Good is the Schedule?
- 3 Gang Scheduling as an Alternative
 - Principles
 - Drawbacks
 - Batch Scheduling it is then
 - Batch Scheduling and Grids?
- 4 What about Theory ?
 - Scheduling Definitions and Notions
 - Platform Models and Scheduling Problems
 - Back to job scheduling
- 5 Conclusion

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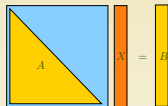
Analyzing a simple code

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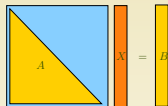
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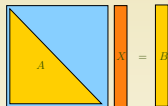
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\prec_{seq} is the **sequential order** :

$$T_{1,1} \prec_{seq} T_{1,2} \prec_{seq} T_{1,3} \prec_{seq} \dots \prec_{seq} T_{1,n} \prec_{seq} T_{2,2} \prec_{seq} T_{2,3} \prec_{seq} \dots \prec_{seq} T_{n,n} .$$

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In the previous example, we have :

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Definition.

Two tasks T and T' are not independent ($T \perp T'$) whenever they share a written variable:

$$T \perp T' \Leftrightarrow \left\{ \begin{array}{l} In(T) \cap Out(T') \neq \emptyset \\ \text{or } Out(T) \cap In(T') \neq \emptyset \\ \text{or } Out(T) \cap Out(T') \neq \emptyset \end{array} \right. .$$

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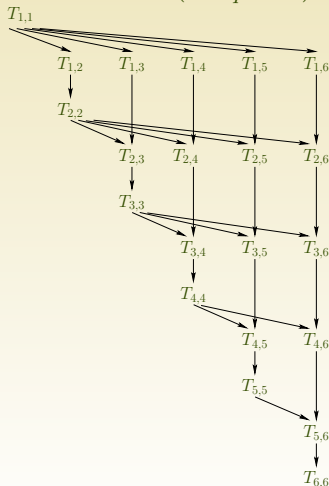
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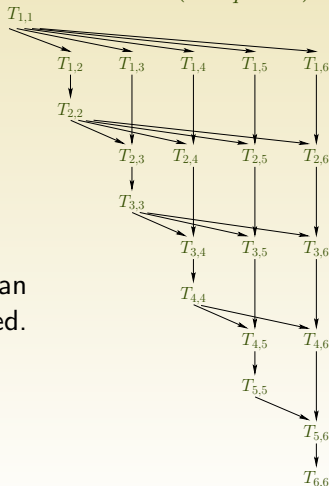
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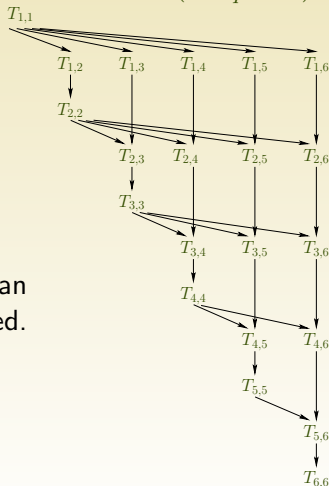
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Transitivity arcs are generally omitted.

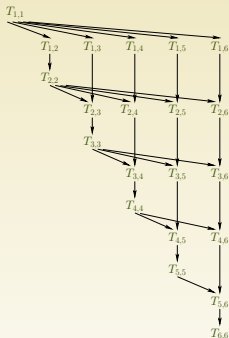


The previous task graph comes from a **low-level** analysis of the code.

It probably makes little sense to do a parallel implementation with MPI with such a low task granularity. Can totally make sense with OpenMP.

Such task graphs can also be used by compilers to do code optimization by exploiting multiple functional units, pipelines functional units, etc.

With **blocking** these tasks could become MPI (parallel) tasks.



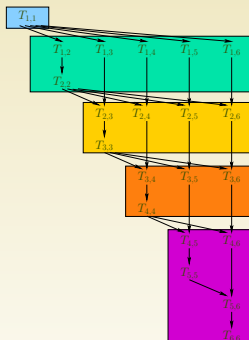
From Coarse-grain Task Graphs...

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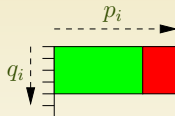
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Hide applications' complexity

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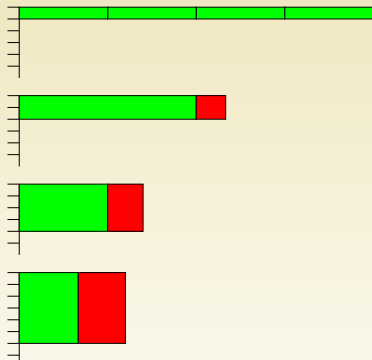
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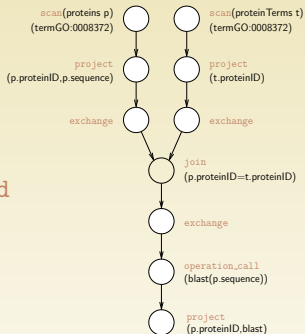
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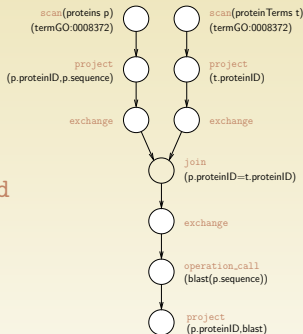
Task-graph do not necessarily come from instruction-level analysis.

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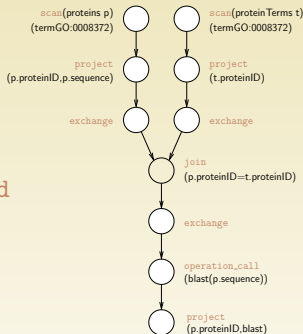
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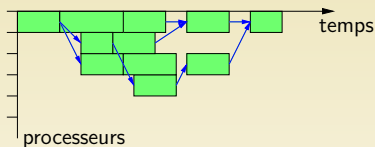


- ▶ Each task may be parallel, preemptable, divisible, ...
- ▶ Each edge depicts a dependency i.e. most of the times some data to transfer.

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Need for Batch Scheduling

- ▶ Parallel Tasks from Scientific Computations (simulation, medical)



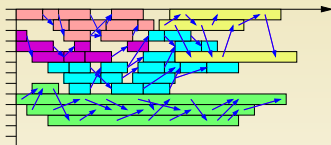
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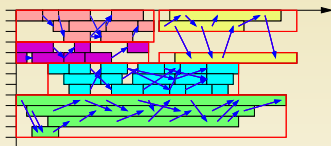
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- ▶ When one purchases a cluster, typically **many users** want to use it.
 - ▶ One cannot let them step on each other's toes
 - ▶ Every user wants to be on a **dedicated** machine
 - ▶ Applications are written assuming some amount of RAM, some notion that all processors go at the same speed, etc.

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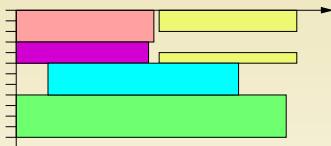
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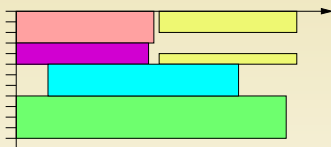
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The **Job Scheduler** is the entity that prevents them from stepping on each other's toes

The Job Scheduler gives out nodes to applications

Batch Scheduling

Each job is defined as a **Number of nodes** (q_i) and a **Time** (p_i):

I want 6 nodes for 1h

Typically users are “charged” against an “allocation”: e.g. “*You only get 100 CPU hours per week*”.

A batch scheduler is a central middleware to manage resources (e.g. processors) of parallel machines:

- ▶ accept jobs (computing tasks) submitted by users
- ▶ decide **when** and **where** jobs are executed
- ▶ start jobs execution

They take into account:

- ▶ **unavailability** of some nodes
- ▶ users jobs **mutual exclusion**
- ▶ **specific needs** for jobs (memory, network, ...)

While trying to :

- ▶ **maximize resources usage**
- ▶ be **fair** among users

Typical wanted features:

- ▶ Interactive mode
- ▶ Batch mode
- ▶ Parallel jobs support
- ▶ Multi-queues with priorities
- ▶ Admission policies (limit on usage, notions of user groups, power users)
- ▶ Resources matching
- ▶ File staging
- ▶ Jobs dependences
- ▶ Backfilling
- ▶ Reservations
- ▶ Best effort jobs
- ▶ Environment reconfiguration

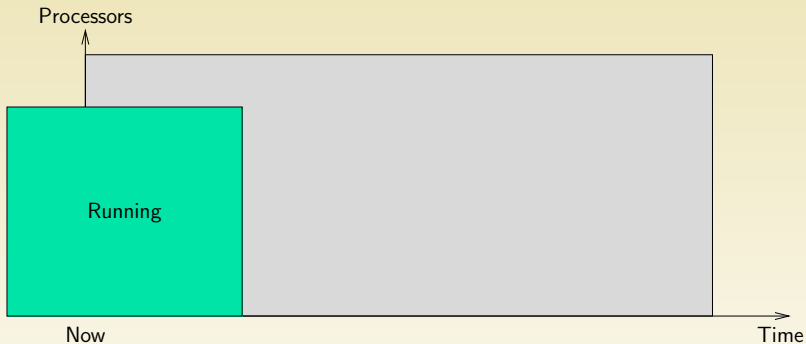
There are many existing batch schedulers : LSF, PBS/Torque, Maui scheduler, Sun Grid Engine, EASY, OAR, ...

These are **complex systems** with many config options !

Main batch schedulers features

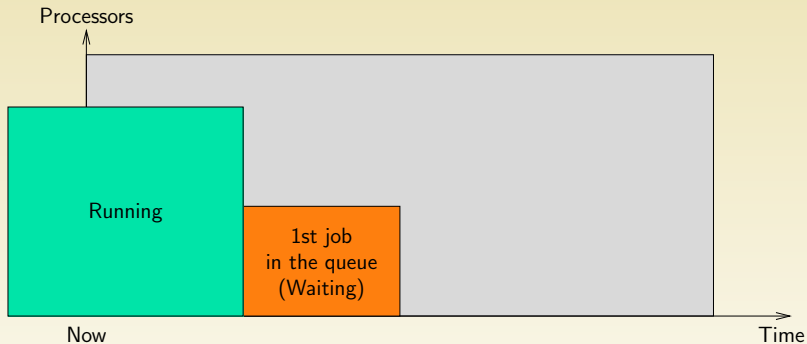
	OpenPBS	SGE	Maui Scheduler (+ OpenPBS)	OAR
Interactive mode	×	×	×	×
Batch mode	×	×	×	×
Parallel jobs support	×	×	×	×
Multi-queues with priorities	×	×	×	×
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General Principle



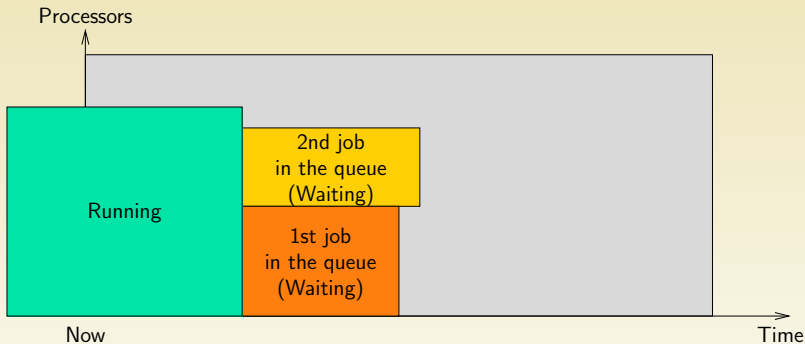
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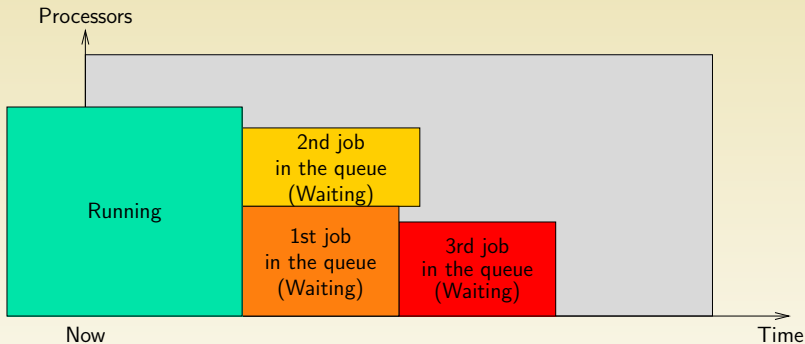
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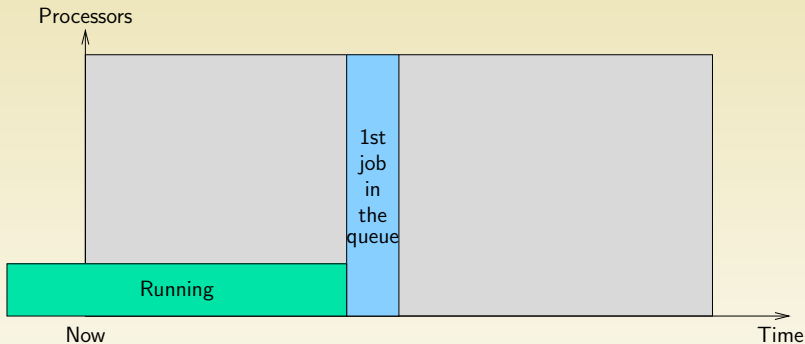
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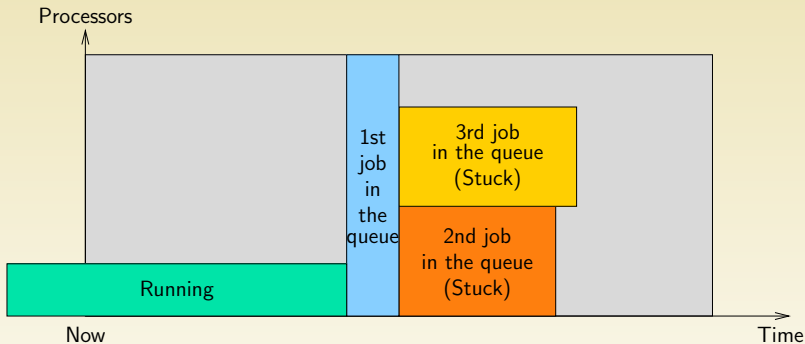
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First Come First Served



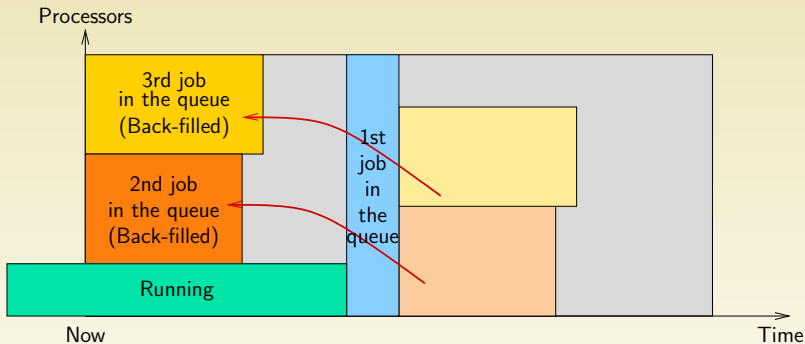
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- ▶ Which job(s) should be picked for promotion through the queue?
- ▶ Many heuristics are possible
- ▶ Two have been studied in detail
 - ▶ EASY
 - ▶ Conservative Back Filling (CBF)
- ▶ In practice EASY (or variants of it) is used, while CBF is not.
- ▶ Although, OAR, a recently proposed batch scheduler implements CBF.

Extensible Argonne Scheduling System

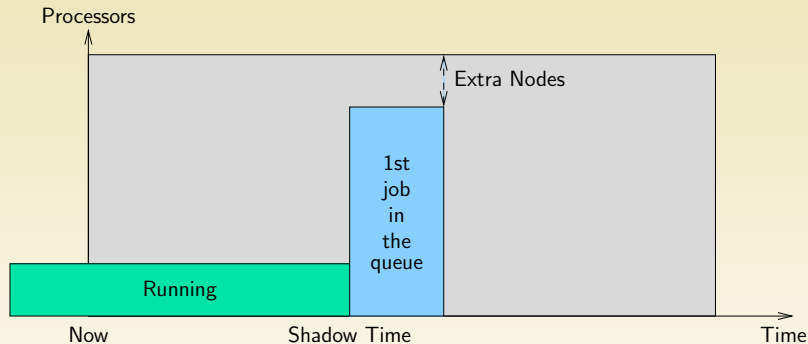
Maintain only one *reservation*, for the first job in the queue.

Definitions:

Shadow time time at which the first job in the queue starts execution

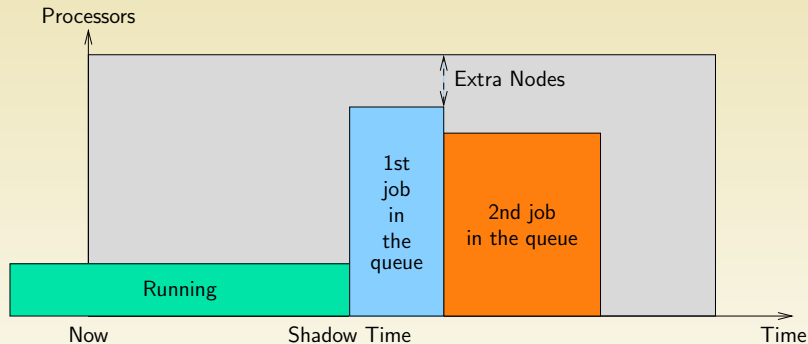
Extra nodes number of nodes idle when the first job in the queue starts execution

- 1 Go through the queue in order starting with the 2nd job.
- 2 Backfill a job if it will terminate by the shadow time, **or** it needs less than the extra nodes.



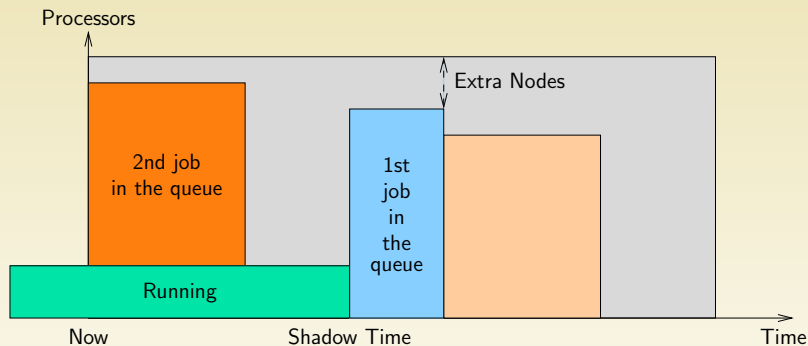
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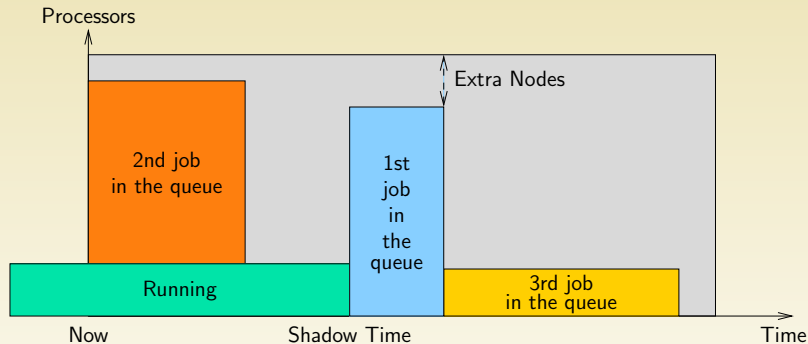
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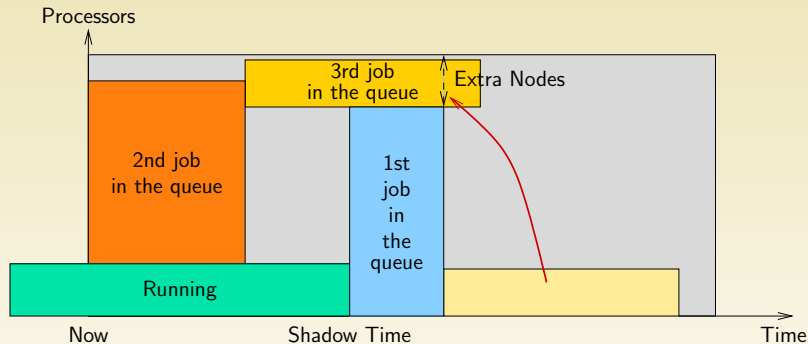
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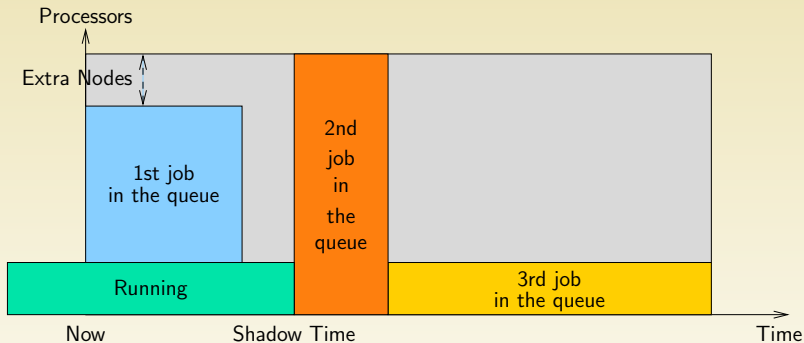
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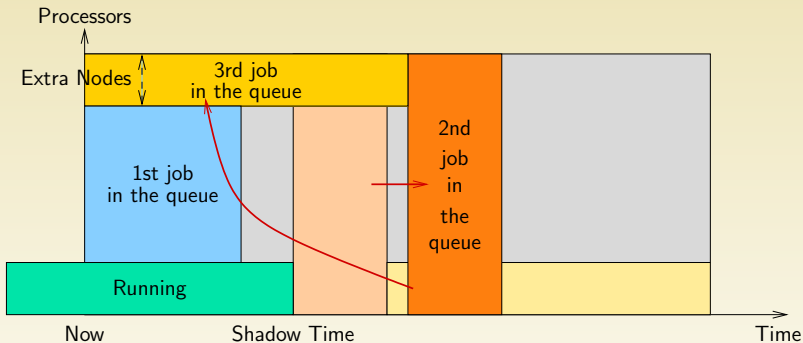
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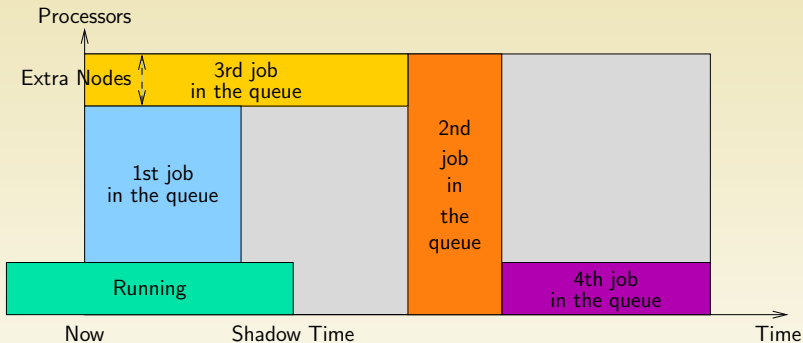
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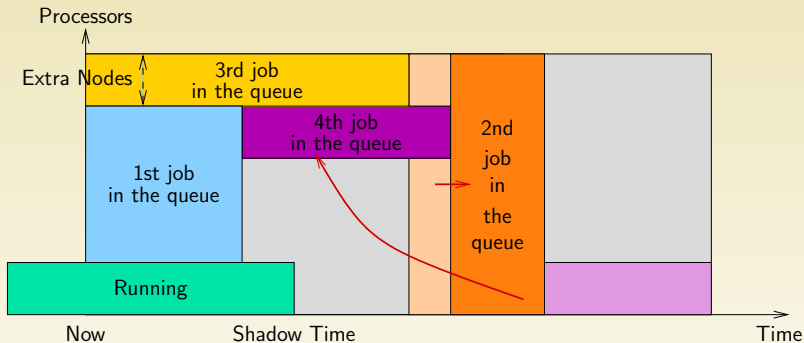
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Unbounded Delay.

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- ▶ BUT, other jobs may be delayed infinitely!

No Starvation.

- ▶ Delay of first job is bounded by runtime of current jobs
- ▶ When the first job finishes, the second job becomes the first job in the queue
- ▶ Once it is the first job, it cannot be delayed further

Other approach.

- ▶ **Conservative Backfilling.** *EVERY* job has a *reservation*. A job may be backfilled only if it does not delay any other job ahead of it in the queue.
- ▶ Fixes the unbounded delay problem that EASY has. More complicated to implement (The algorithm must find holes in the schedule) though.
- ▶ EASY favors small long jobs and harms large short jobs.

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Possibly when

- ▶ A new job arrives

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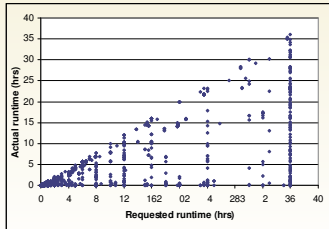
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Are estimates accurate?



How good is the schedule ?

All of this is great, but how do we know what a “good” schedule is? FCFS, EASY, CFB, Random?

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Job 1 needs 1h of compute time and waits 1s

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Job 1 asks for 1 nodes and waits 1 h

Job 2 asks for 512 nodes and waits 1h

Again, Job 1 is unhappy while Job 2 is probably sort of happy.

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- 3 Slowdown or **Stretch** (turn-around time divided by turn-around time if alone in the system)

Doesn't really take care of the small/large problem. Could think of some scaling, but unclear !

Now What ?

Now we have a few metrics we can consider

We can run simulations of the scheduling algorithms, and see how they fare.

We need to test these algorithms in representative scenarios

Supercomputer/cluster traces. Collect the following for long periods of time:

- ▶ Time of submission
- ▶ How many nodes asked
- ▶ How much time asked
- ▶ How much time was actually used
- ▶ How much time spent in the queue

Uses of the traces:

- 1 Drive simulations
- 2 Come up with models of user behaviors

Sample Results

A type of experiments that people have done: replace user estimate by f times the actual run time

Possible to improve performance by multiplying user estimates by 2!

	EASY	CBF
Mean Slowdown		
KTH	-4.8%	-23.0%
CTC	-7.9%	-18.0%
SDSC	+4.6%	-14.2%
Mean Response time		
KTH	-3.3%	-7.0%
CTC	-0.9%	-1.6%
SDSC	-1.6%	-10.9%

- ▶ These are all **heuristics**.
- ▶ They are not specifically designed to optimize the metrics we have designed.
- ▶ It is difficult to truly understand the reasons for the results.
- ▶ But one can derive some empirical wisdom.
- ▶ One of the reasons why one is stuck with possibly obscure heuristics is that we're dealing with an *on-line* problem: We don't know what happens next.
- ▶ We cannot wait for all jobs to be submitted to make a decision. But we can wait for a while, accumulate jobs, and schedule them together.

Batch Schedulers are what we're stuck with at the moment.
They are often hated by users.

- ▶ I submit to the queue asking for 10 nodes for 1 hour.
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A completely different approach is **gang scheduling**, which we discuss next.

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- ▶ All processes belonging to a job run at the same time (the term **gang** denotes all processors within a job).
- ▶ Each process runs alone on each processor.
- ▶ BUT: there is rapid **coordinated** context switching.
- ▶ It is possible to **suspend/preempt** jobs arbitrarily

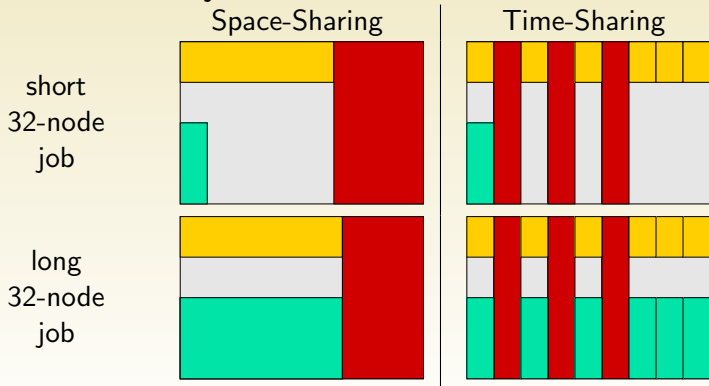
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- ▶ It is possible to **suspend/preempt** jobs arbitrarily \rightsquigarrow May allow more flexibility to optimize some metrics.
- ▶ If processing times are not known in advance (or grossly erroneous), preemption can help short jobs that would be “stuck” behind a long job.
- ▶ Should improve machine utilization.

Gang Scheduling: an Example

- ▶ A 128 node cluster.
- ▶ A running 64-node job.
- ▶ A 32-node job and a 128-node job are queued.

Should the 32-node job be started ?



More uniform slowdown, better resource usage.

- ▶ Overhead for context switching (trade-off between overhead and fine grain).

Gang Scheduling: Drawbacks

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- ▶ Some implementations (MOSIX, Kerighed).

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Why don't we like Batch Scheduling? Because queue waiting times are difficult to predict.

- ▶ depends on the status of the queue
- ▶ depends on the scheduling algorithm used
- ▶ depends on all sorts of configuration parameters set by system administrator
- ▶ depends on future job completions!
- ▶ etc.

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That is why there is more and more demand for **reservation** support. Users build (badly?) the schedule by themselves.

Batch Scheduling and Grids

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Other issues:

- ▶ File Staging ?
- ▶ Load Balancing between sites ?

Sequential Job Scheduling for Grids

A set unrelated processors P_1, \dots, P_n and a set of sequential jobs J_1, \dots, J_n (processing time $p_{i,j}$).

Let's try a few natural scheduling strategies. We denote by a_i the time at which P_i is available (at the beginning $a_i = 0$ for all P_i):

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Problem: How do you get an estimate of $p_{i,j}$?

So Where are we ?

- ▶ Batch schedulers are complex pieces of software that are used in practice.
- ▶ A lot of experience on how they work and how to use them.
- ▶ But ultimately everybody knows they are an imperfect solution.
- ▶ Many view the lack of theoretical foundations as a big problem.
- ▶ Let's look at what theoreticians think of job scheduling.
- ▶ The first step is to define the scheduling problem (On-line vs. Off-line, Preemption vs. No preemption).

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The Job Scheduling Problem

- ▶ When do jobs “arrive”?
 - On-line We know when they arrive (periodic, aperiodic, etc.)
 - We don't: batch scheduling, gang scheduling.
 - We only get upper bounds on the real processing times (kind of **non-clairvoyant**).
 - Off-line more related to application scheduling but should be studied before everything else.
- ▶ Control of the resources
 - ▶ With preemption: Gang Scheduling
 - ▶ Without preemption: Batch Scheduling
- ▶ The practical implementations (batch and gang) are only heuristics and do not consider the problem at a theoretical level. In fact, they don't optimize any metric each individual user cares about.

Definition: Task system.

A task system is an directed graph $G = (V, E, w)$ where :

- ▶ V is the set of tasks (V is finite)
- ▶ E represent the dependence constraints:

$$e = (u, v) \in E \text{ iff } u \prec v$$

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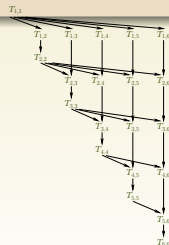
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We could set $w(T_{i,j}) = 1$ but also decide that performing a division is more expensive than a multiplication followed by an addition.



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A schedule of a task system $G = (V, E, w)$ is a time function $\sigma : V \rightarrow \mathbb{N}^*$ such that:

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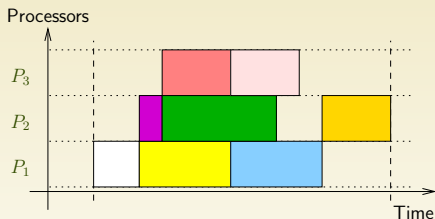
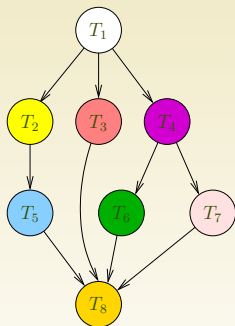
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Depending on the application and platform model, much more complex definitions can be proposed.

Manipulating functions is generally not very convenient. That is why **Gantt-chart** are used to depict schedules and allocations.



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Sketch of the proof.

\Rightarrow Assume that G has a cycle $v_1 \rightarrow v_2 \rightarrow \dots \rightarrow v_k \rightarrow v_1$. Then $v_1 \prec v_1$ and a valid schedule σ should hold $\sigma(v_1) + w(v_1) \leq \sigma(v_1)$ true, which is impossible because $w(v_1) > 0$.

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More precisely, we sort topologically the vertexes and schedule them one after the other on the same processor. Dependences are then fulfilled. \square

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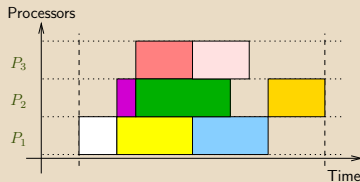
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Therefore all task systems we will be considering in the following are **Directed Acyclic Graphs**.

Definition: **Makespan.**

The **makespan** of a schedule is the total execution time :

$$MS(\sigma) = \max_{v \in V} \{ \sigma(v) + w(v) \} - \min_{v \in V} \{ \sigma(v) \} .$$



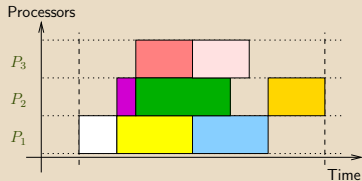
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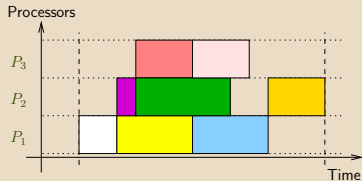
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- ▶ $Pb(p)$: find a schedule with the smallest possible makespan, using at most p processors. $MS_{opt}(p)$ denotes the optimal makespan using only p processors.
- ▶ $Pb(\infty)$: find a schedule with the smallest makespan when the number of processors that can be used is not bounded. We note $MS_{opt}(\infty)$ the corresponding makespan.

Graham Notation

Many parameter can change in a scheduling problem. Graham has then proposed the following classification : $\langle \alpha | \beta | \gamma \rangle$ [BruckerBook]

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By reduction to 2-Partition: can $\mathcal{A} = \{a_1, \dots, a_n\}$ be partitioned into two sets $\mathcal{A}_1, \mathcal{A}_2$ such $\sum_{a \in \mathcal{A}_1} a = \sum_{a \in \mathcal{A}_2} a$?



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List scheduling

When simple problems are hard, we should try to find good **approximation** heuristics.

Natural idea: using greedy strategy like trying to allocate the most possible task at a given time-step. However at some point we may face a choice (when there is more ready tasks than available processors).

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Any strategy that does not let on purpose a processor idle is efficient [Coffman76]. Such a schedule is called **list-schedule**.

Theorem 2: Coffman.

Let $G = (V, E, w)$ be a DAG, p the number of processors, and σ_p a list-schedule of G .

$$MS(\sigma_p) \leq \left(2 - \frac{1}{p}\right) MS_{opt}(p).$$

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Most of the time, list-heuristics are based on the **critical path**.

List Scheduling for Parallel Rigid Tasks

Let us assume we have $J_1 = (p_1, q_1), \dots, J_n = (p_n, q_n)$ and m machines.

Let us denote by T^* the optimal makespan for this instance.

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We have $\forall t_1, t_2 \in [0, T] : t_1 \leq t_2 - T^* \Rightarrow q(t_1) + q(t_2) > m$ (otherwise, the tasks running at time t_2 could have been run at time t_1).

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$$\begin{aligned} mT^* &\geq \sum_i q_i p_i = \int_0^T q(t) = \int_0^{2T^*} q(t) + \int_{2T^*}^T q(t) \\ &\geq \underbrace{\int_0^{T^*} q(t) + q(t + T^*)}_{> mT^*} + \underbrace{\int_{2T^*}^T q(t)}_{\geq 0}, \text{ which is absurd.} \end{aligned}$$

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Theorem 3.

List-scheduling has a approximation factor of 2 for minimizing the C_{\max} of Parallel Rigid Tasks.

Taking communications into account

A very simple model (things are already complicated enough): the **macro-data flow** model. If there is some data-dependence between T and T' , the communication cost is

$$c(T, T') = \begin{cases} 0 & \text{if } \text{alloc}(T) = \text{alloc}(T') \\ c(T, T') & \text{otherwise} \end{cases}$$

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Definition.

A DAG with communication cost (say cDAG) is a directed acyclic graph $G = (V, E, w, c)$ where vertexes represent tasks and edges represent dependence constraints. $w : V \rightarrow \mathbb{N}^*$ is the computation time function and $c : E \rightarrow \mathbb{N}^*$ is the communication time function. Any valid schedule has to respect the dependence constraints.

$$\forall e = (v, v') \in E,$$

$$\begin{cases} \sigma(v) + w(v) \leq \sigma(v') & \text{if } \text{alloc}(v) = \text{alloc}(v') \\ \sigma(v) + w(v) + c(v; v') \leq \sigma(v') & \text{otherwise.} \end{cases}$$

Even $P_b(\infty)$ is NP-complete !!!

You constantly have to figure out whether you should use more processor (but then pay more for communications) or not. Finding the good trade-off is a real challenge.

$4/3$ -approximation if all communication times are smaller than computation times.

Finding guaranteed approximations for other settings is really hard, but really useful (file staging).

Results More Related to Job Scheduling

	$model = \emptyset$	$model = pmtn$
$\langle 1 r_j; model \max w_j F_j \rangle$	NP ([Bender-SODA98])	↓
$\langle P r_j; model \max w_j F_j \rangle$	↑	↓
$\langle Q r_j; model \max w_j F_j \rangle$	↑	↓
$\langle R r_j; model \max w_j F_j \rangle$	↑	P (Lin. Prog)
$\langle 1 r_j; model \sum F_j \rangle$	NP ([LenstraRiBr1977])	P (SRPT [Baker1974])
$\langle P r_j; model \sum F_j \rangle$	↑	NP (Numerical-3DM [baptiste-2006])
$\langle Q r_j; model \sum F_j \rangle$	↑	↑
$\langle R r_j; model \sum F_j \rangle$	↑	↑
$\langle 1 r_j; model \sum S_j \rangle$	NP	?
$\langle P r_j; model \sum S_j \rangle$	↑	?
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$\langle 1 r_j; model \sum w_j F_j \rangle$	NP ([LenstraRiBr1977])	NP (Numerical-3DM [LabetoulleLaLeRi84])
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Significance of these results

- ▶ In the previous table we saw that with preemption many problems become “easier”.

This is probably a good indication that the only hope to optimize a “user centric” performance metric is to allow preemption.

Gang scheduling does preemption! Perhaps one can do just a little bit of preemption and be ok?

- ▶ Also, all the previous results are for off-line situations, when we know EVERYTHING about the stream of tasks/jobs.

What about the on-line case?

Competitive ratio: How close does an on-line scheduling algorithm come to the optimal offline algorithm in the worst case.

$\langle 1 | r_j; pmtn | \sum F_j \rangle$ One processor, preemption is allowed, release dates, minimize average flow-time.

Shortest Remaining Processing Time is optimal: Upon job arrival/ departure, ensure that the job with the shortest remaining processing time has the processor (\rightsquigarrow use preemption).

NP-complete for multiple processors or with no preemption.

$\langle 1|r_j; pmtn|F_{\max} \rangle$ One processor, preemption is allowed, release dates, minimize maximum flow-time.

First Come First Served is optimal (\rightsquigarrow preemption is not needed).

NP-complete for multiple processors when preemption is not allowed.

Stretch minimization (Max Stretch)

$\langle 1|r_j; pmtn|S_{\max}\rangle$ One processor, preemption is allowed, release dates, minimize maximum slowdown.

Offline algorithm based on linear programming and/or deadlines (preemption is needed).

Online algorithm There is no $\frac{1}{2}\Delta^{\sqrt{2}-1}$ -competitive algorithms for max-stretch (where Δ is the ratio between largest processing time and the smallest processing time).

There are deadline-based online algorithms that are $O(\sqrt{\Delta})$ -competitive for max-stretch [Bender-SODA98, BenderMuRa02].

FCFS is Δ competitive for S_{\max}

Two job-sizes then the best known competitive ratio is $\frac{1+\sqrt{5}}{2}$ and $\sqrt{2}$ is an upper bound on the competitive ratio.

Stretch minimization (Sum Stretch)

$\langle 1|r_j; pmtn|S_{\max} \rangle$ One processor, preemption is allowed, release dates, minimize average slowdown.

Complexity is open (offline)

SRPT is 2-competitive.

FCFS is Δ^2 -competitive.

NP-complete when preemption is not allowed.

On a single processor minimizing sum-flow is easier than minimizing sum-stretch.

On multiple processors SRPT is 14-competitive.

And so on...

A large literature with results here and there. Max-stretch/Max-flow is kind of about “fairness”, Sum-stretch/Sum-flow is kind of about “performance” \rightsquigarrow It would be nice to sort of optimize both.

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Depressing result:

Theorem 4.

Any $\rho(\Delta)$ -competitive algorithm for AF such that $\rho(\Delta) < \Delta$ (i.e. more clever than FCFS) leads to starvation.

Theorem 5.

Any $\rho(\Delta)$ -competitive algorithm for AS such that $\rho(\Delta) < \Delta^2$ (i.e. more clever than FCFS) leads to starvation.

And so on...

A large literature with results here and there. Max-stretch/Max-flow is kind of about “fairness”, Sum-stretch/Sum-flow is kind of about “performance” \rightsquigarrow It would be nice to sort of optimize both.

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Theorem 5.

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In practice

Being good for a sum-based metric is easy (smaller or weighted smaller first).

Relaxed deadline-based approaches are good for max-based metrics.

- 1 Task Graphs and Parallel Tasks From Outer Space
- 2 Batch Scheduling
 - Basic idea: FCFS + Backfilling
 - EASY
 - How Good is the Schedule?
- 3 Gang Scheduling as an Alternative
 - Principles
 - Drawbacks
 - Batch Scheduling it is then
 - Batch Scheduling and Grids?
- 4 What about Theory ?
 - Scheduling Definitions and Notions
 - Platform Models and Scheduling Problems
 - Back to job scheduling
- 5 Conclusion

Theory Most of the time, the only thing we can do is to compare heuristics. There are three ways of doing that:

- ▶ Theory: being able to guarantee your heuristic;
- ▶ Experiment: Generating random graphs and/or typical application graphs along with platform graphs to compare your heuristics.
- ▶ Smart: proving that your heuristic is optimal for a particular class of graphs (fork, join, fork-join, bounded degree, ...).

However, remember that the first thing to do is to look whether your problem is NP-complete or not. Who knows? You may be lucky...

Practice We do batch scheduling, which completely disregards all this. But theory says that preemption is key.

As usual there is a major disconnect. Only a few authors have read both types of work.

Great opportunity for research is there anything from the theory that should guide the practice?



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