Application of Random Matrix Theory to Future Wireless Networks

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Outline

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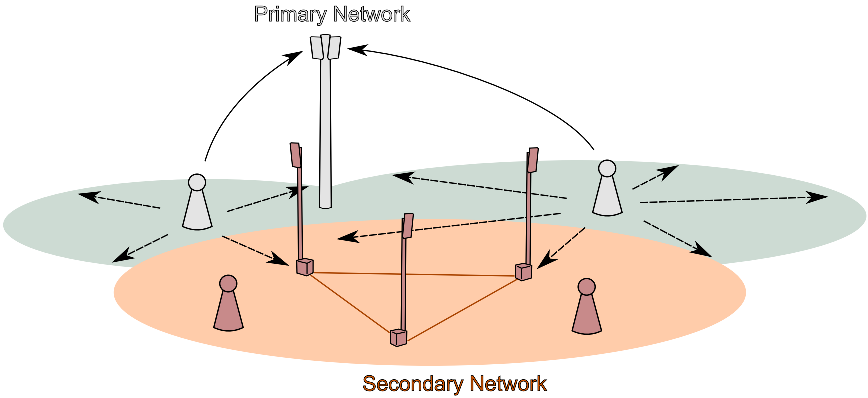
General system model



Exploration and Exploitation

Cognitive Radio Setting:

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We will discuss both exploration and exploitation phases as (possibly large) multivariate systems.

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This information allows for:

- the detection of spectrum holes;
- the evaluation of the optimal secondary coverage;
- *ideally,* the elaboration of a 'map' of space-frequency resources.

Problem formulation

- We consider the model

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- We wish to confront the hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ given the data matrix $\mathbf{Y} \triangleq [\mathbf{y}^{(1)}, \ldots, \mathbf{y}^{(M)}] \in \mathbb{C}^{N \times M}$.

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- We consider, in a Bayesian framework, the Neyman-Pearson test ratio

$$C(\mathbf{Y}) \triangleq \frac{P_{\mathcal{H}_1|\mathbf{Y},I}(\mathbf{Y})}{P_{\mathcal{H}_0|\mathbf{Y},I}(\mathbf{Y})}$$

with prior information I on $\mathbf{H}, \mathbf{x}^{(m)}, \sigma, \ldots$.

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- In the following,
  - we derive the case $P = 1$, σ known and the knowledge about **H** conveys unitary invariance
    - $\mathrm{E}[\mathrm{tr}\, \mathbf{H}\mathbf{H}^{\mathsf{H}}]$ known: this is what we assume here;
    - $\mathrm{E}[\mathbf{H}\mathbf{H}^{\mathsf{H}}] = \mathbf{Q}$ unknown but such that $\mathrm{E}[\mathrm{tr}\, \mathbf{Q}]$ is known;
    - $\mathrm{rank}(\mathbf{H}\mathbf{H}^{\mathsf{H}})$ known.
  - we compare alternative methods when $P = 1$ and σ are unknown.

Evaluation of $P_{\mathbf{Y}|\mathcal{H}_i,I}(\mathbf{Y})$

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From unitary invariance of **H**, denoting $\mathbf{\Sigma} = \mathbf{U}\mathbf{G}\mathbf{U}^{\mathrm{H}}$, $\operatorname{diag}(\mathbf{G}) = (g_1,\ldots,g_n,\sigma^2,\ldots,\sigma^2)$

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- if **H** is Gaussian, $P_{(g_1-\sigma^2,\ldots,g_n-\sigma^2)}$ is the joint eigenvalue distribution of a central Wishart;

Result in the Gaussian case, $n = 1$

R. Couillet, M. Debbah, "A Bayesian Framework for Collaborative Multi-Source Signal Sensing", IEEE Transactions on Signal Processing, vol. 58, no. 10, pp. 5186-5195, 2010.

Theorem (Neyman-Pearson test)

The ratio $C(\mathbf{Y})$ when the receiver knows $n = 1$, $P = 1$, $\mathrm{E}[\frac{1}{N}\operatorname{tr}\mathbf{H}\mathbf{H}^{\mathrm{H}}] = 1$ and σ^2, reads

$$C(\mathbf{Y}) = \frac{1}{N}\sum_{l=1}^{N} \frac{\sigma^{2(N+M-1)} e^{\sigma^2 + \frac{\lambda_l}{\sigma^2}}}{\prod_{\substack{i=1 \\ i \neq l}}^{N} (\lambda_l - \lambda_i)} J_{N-M-1}(\sigma^2, \lambda_l)$$

with $\lambda_1, \ldots, \lambda_N$ the eigenvalues of $\mathbf{Y}\mathbf{Y}^{\mathrm{H}}$ and where

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- integration over σ^2 (or P when $P \neq 1$) is difficult.

Comparison to energy detector

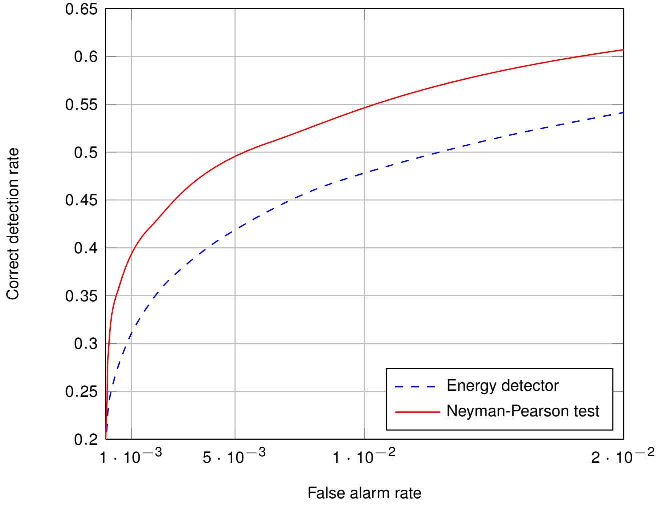

Figure: ROC curve for single-source detection, $K = 1$, $N = 4$, $M = 8$, $\mathrm{SNR} = -3$ dB, FAR range of practical interest, with signal power $E = 0$ dBm, either known or unknown at the receiver.

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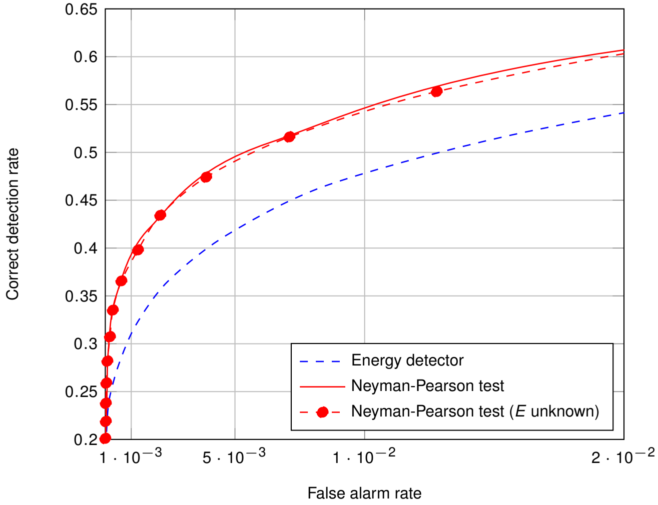

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Unknown power and noise variances

- Bayesian approaches:

$$P_{\mathbf{Y}|\mathcal{H}_i,I}(\mathbf{Y}) = \int_{\mathbb{R}_+^2} P_{\mathbf{Y}|\mathcal{H}_i,\sigma,P}(\mathbf{Y}) P_{(\sigma,P)}(\sigma,P) d(\sigma,P)$$

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- ad-hoc methods, such as conditioning number:

$$C_{\rm cond}(\mathbf{Y}) \triangleq \frac{\max_i \lambda_i}{\min_i \lambda_i}$$

  - based on large random matrix considerations: under $\mathcal{H}_0$, as $N/M \to c$

$$\frac{\max_i \lambda_i}{\min_i \lambda_i} \xrightarrow{\rm a.s.} \frac{(1+\sqrt{c})^2}{(1-\sqrt{c})^2}$$

  - totally empirical.

Performance comparison for unknown σ^2, P

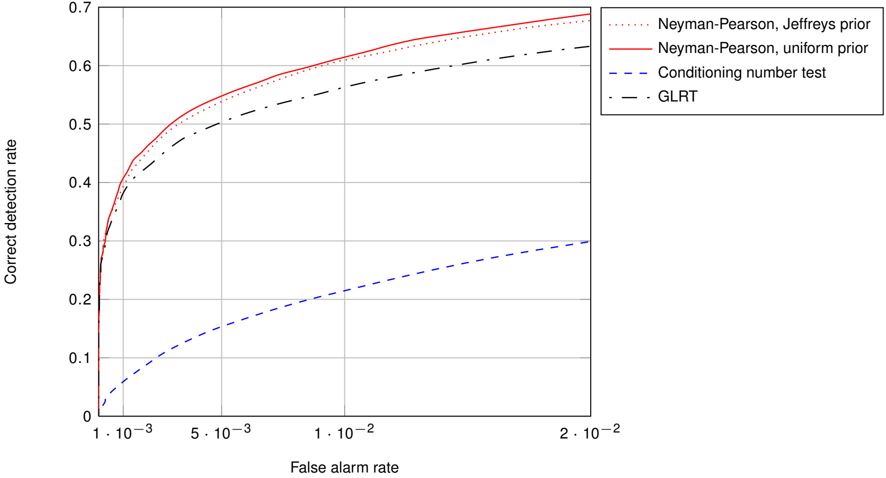

Figure: ROC curve for *a priori* unknown σ^2 of the Neyman-Pearson test, conditioning number method and GLRT, $K = 1$, $N = 4$, $M = 8$, SNR $= 0$ dB. For the Neyman-Pearson test, both uniform and Jeffreys prior, with exponent $\beta = 1$, are provided.

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and wish to infer $P_1, \ldots, P_K$.

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$$\mathbf{y}^{(m)} = \sum_{k=1}^{K} \sqrt{P_k} \mathbf{H}_k \mathbf{x}_k^{(m)} + \sigma \mathbf{w}^{(m)}$$

  and wish to infer $P_1, \ldots, P_K$.

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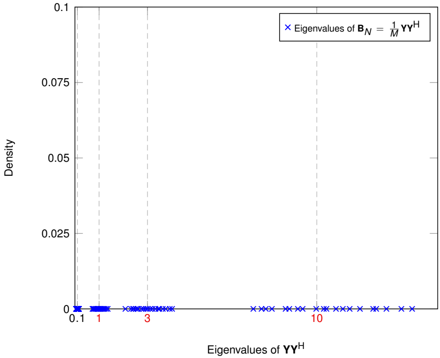
Most information about $P_1, \ldots, P_K$ is contained in the eigenvalues of $\mathbf{B}_N \triangleq \frac{1}{M}\mathbf{Y}\mathbf{Y}^{\mathsf{H}}$.

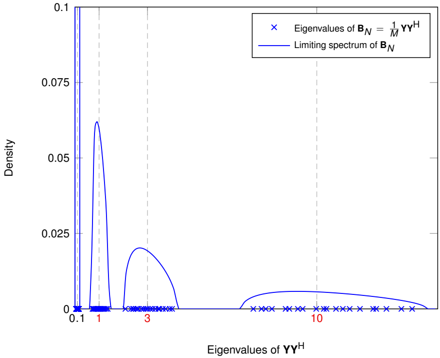
From small to large system analysis



The classical approach requires to evaluate $P_{P_1,\ldots,P_K|Y}$

- assuming Gaussian parameters, this is similar to previous calculus
- leads to a sum of two-dimensional integrals
- prohibitively expensive to evaluate even for small N, n_k, M

From small to large system analysis



Assuming dimensions N, n_k, M grow large, large dimensional random matrix theory provides

- a link between:
  - **the "observation":** the limiting spectral distribution (l.s.d.) of $\mathbf{B}_N$;
  - **the "hidden parameters":** the powers $P_1, \ldots, P_K$, i.e. the l.s.d. of $\mathbf{P}$.
- consistent estimators of the hidden parameters.

Limiting spectrum of the sample covariance matrix

- **Definition.** The Stieltjes transform $m_F(z)$, of a distribution function F is defined as

$$m_F(z) = \int \frac{1}{\omega - z} dF(\omega).$$

Knowing the Stieltjes transform of F is equivalent to knowing F (similar to Fourier transform).

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- for simplicity, consider the *sample covariance matrix* model

$$\mathbf{Y} \triangleq \mathbf{T}^{\frac{1}{2}}\mathbf{X} \in \mathbb{C}^{N\times n}, \ \mathbf{B}_N = \frac{1}{n}\mathbf{Y}\mathbf{Y}^{\mathrm{H}} \in \mathbb{C}^{N\times N}, \ \underline{\mathbf{B}}_N = \frac{1}{n}\mathbf{Y}^{\mathrm{H}}\mathbf{Y} \in \mathbb{C}^{n\times n}$$

  where $\mathbf{T} \in \mathbb{C}^{N\times N}$ has eigenvalues $t_1, \ldots, t_K$, t_k with multiplicity N_k and $\mathbf{X} \in \mathbb{C}^{N\times n}$ is i.i.d. zero mean, variance 1.

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- If $F^{\mathbf{T}} \Rightarrow T$, then $m_{F^{\mathbf{B}_N}}(z) = m_{\mathbf{B}_N}(z) \xrightarrow{\text{a.s.}} m_F(z)$ such that

$$m_T\left(-1/m_{\underline{F}}(z)\right) = -z m_{\underline{F}}(z) m_F(z)$$

with $m_{\underline{F}}(z) = c m_F(z) + (c-1)\frac{1}{z}$ and $N/n \to c$.

Complex integration

- From Cauchy integral formula, denoting $\mathcal{C}_k$ a contour enclosing **only** t_k,

$$t_k = \frac{1}{2\pi i} \oint_{\mathcal{C}_k} \frac{\omega}{\omega - t_k} d\omega$$

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- After the variable change $\omega = -1/m_{\underline{F}}(z)$,

$$t_k = \frac{N}{N_k}\frac{1}{2\pi i}\oint_{\mathcal{C}_{\underline{F},k}} z m_F(z) \frac{m'_{\underline{F}}(z)}{m^2_{\underline{F}}(z)} dz,$$

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$$m_F(z) \simeq m_{\mathbf{B}_N}(z) \triangleq \frac{1}{N}\sum_{k=1}^{N} \frac{1}{\lambda_k - z}, \quad \text{with} \quad (\lambda_1, \ldots, \lambda_N) = \mathrm{eig}(\mathbf{B}_N) = \mathrm{eig}(\mathbf{Y}\mathbf{Y}^{\mathsf{H}}).$$

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$$t_k - \hat{t}_k \xrightarrow{\text{a.s.}} 0 \quad \text{with} \quad \hat{t}_k = \frac{N}{N_k}\frac{1}{2\pi i}\oint_{\mathcal{C}_{\underline{F},k}} z m_{\mathbf{B}_N}(z) \frac{m'_{\underline{\mathbf{B}}_N}(z)}{m^2_{\underline{\mathbf{B}}_N}(z)} dz$$

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with $\mathcal{N}_k$ the indexes of cluster k and $\mu_1 < \ldots < \mu_N$ are the ordered eigenvalues of the matrix $\mathrm{diag}(\boldsymbol{\lambda}) - \frac{1}{n}\sqrt{\boldsymbol{\lambda}}\sqrt{\boldsymbol{\lambda}}^{\mathsf{T}}$, $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_N)^{\mathsf{T}}$.

Application to the current model

R. Couillet, J. W. Silverstein, Z. Bai, M. Debbah, "Eigen-Inference for Energy Estimation of Multiple Sources," *to appear in* IEEE Transactions on Information Theory, 2010.

- Extending $\mathbf{Y}$ with zeros, our model is a "double sample covariance matrix"

$$\underbrace{\mathbf{Y}}_{(N+n)\times M} = \underbrace{\begin{bmatrix} \mathbf{H}\mathbf{P}^{\frac{1}{2}} & \sigma \mathbf{I}_N \\ 0 & 0 \end{bmatrix}}_{(N+n)\times(N+n)} \underbrace{\begin{bmatrix} \mathbf{X} \\ \mathbf{W} \end{bmatrix}}_{(N+n)\times M} .$$

- Limiting distribution of $\frac{1}{M}\mathbf{Y}\mathbf{Y}^{\mathrm{H}}$

Theorem (l.s.d. of $\mathbf{B}_N$)

Let $\mathbf{B}_N = \frac{1}{M}\mathbf{Y}\mathbf{Y}^{\mathrm{H}}$ with eigenvalues $\lambda_1, \ldots, \lambda_N$. Denote $m_{\underline{\mathbf{B}}_N}(z) \triangleq \frac{1}{M}\sum_{k=1}^{M} \frac{1}{\lambda_k - z}$, with $\lambda_i = 0$ for $i > N$. Then, for $M/N \to c$, $N/n_k \to c_k$, $N/n \to c_0$, for any $z \in \mathbb{C}^+$,

$$m_{\underline{\mathbf{B}}_N}(z) \xrightarrow{\text{a.s.}} m_{\underline{F}}(z)$$

with $m_{\underline{F}}(z)$ the unique solution in $\mathbb{C}^+$ of

$$\frac{1}{m_{\underline{F}}(z)} = -\sigma^2 + \frac{1}{f(z)}\left[\frac{c_0 - 1}{c_0} + m_P\left(-\frac{1}{f(z)}\right)\right], \quad \text{with } f(z) = (c-1)m_{\underline{F}}(z) - c z m_{\underline{F}}(z)^2.$$

Application to the current model (2)

R. Couillet, J. W. Silverstein, Z. Bai, M. Debbah, "Eigen-Inference for Energy Estimation of Multiple Sources," *to appear in* IEEE Trans. on Inf. Theory, 2010.

- estimator calculus

Theorem (Estimator of $P_1, \ldots, P_K$)

Let $\mathbf{B}_N \in \mathbb{C}^{N \times N}$ be defined as in Theorem 2, and $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_N)$, $\lambda_1 < \ldots < \lambda_N$. Assume that asymptotic cluster separability condition is fulfilled for some k. Then, as $N, n, M \to \infty$,

$$\hat{P}_k - P_k \xrightarrow{\text{a.s.}} 0,$$

where

$$\hat{P}_k = \frac{NM}{n_k(M-N)} \sum_{i \in \mathcal{N}_k} (\eta_i - \mu_i)$$

with $\mathcal{N}_k$ the set indexing the eigenvalues in cluster k of F, $\eta_1 < \ldots < \eta_N$ the eigenvalues of $\mathrm{diag}(\boldsymbol{\lambda}) - \frac{1}{N}\sqrt{\boldsymbol{\lambda}}\sqrt{\boldsymbol{\lambda}}^{\mathsf{T}}$ and $\mu_1 < \ldots < \mu_N$ the eigenvalues of $\mathrm{diag}(\boldsymbol{\lambda}) - \frac{1}{M}\sqrt{\boldsymbol{\lambda}}\sqrt{\boldsymbol{\lambda}}^{\mathsf{T}}$.

Remarks

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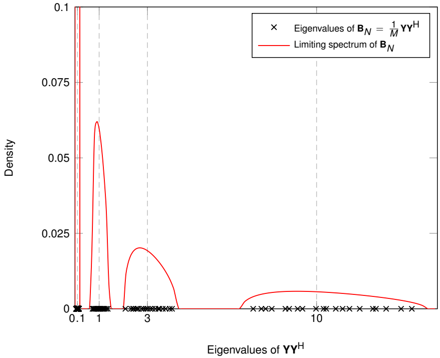
- solution is computationally simple, explicit, and the final formula compact.
- cluster separability condition is fundamental. This requires
  - for all other parameters fixed, the P_k cannot be too close top one another: source separation problem.
  - for all other parameters fixed, σ^2 must be kept low: low SNR undecidability problem.
  - for all other parameters fixed, M/N cannot be too low: sample deficiency issue (not such an issue though).
  - for all other parameters fixed, N/n cannot be too low: diversity issue.
- exact spectrum separability is an essential ingredient (known for very few models to this day).



Simulations

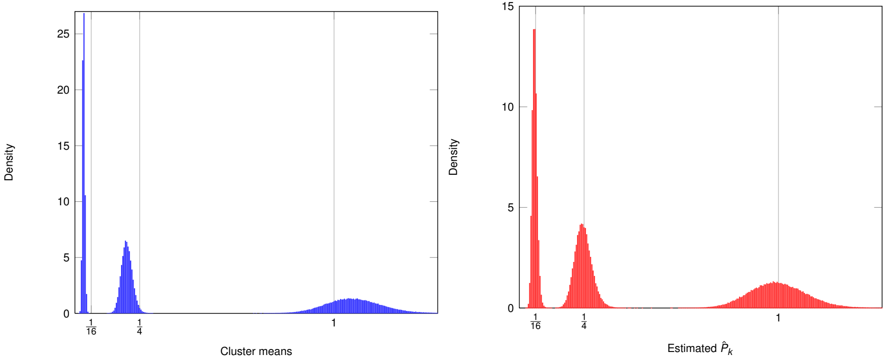


Figure: Histogram of the cluster-mean approach and of $\hat{P}_k$ for $k \in \{1, 2, 3\}$, $P_1 = 1/16$, $P_2 = 1/4$, $P_3 = 1$, $n_1 = n_2 = n_3 = 4$ antennas per user, $N = 24$ sensors, $M = 128$ samples and $\mathrm{SNR} = 20$ dB.

Performance comparison

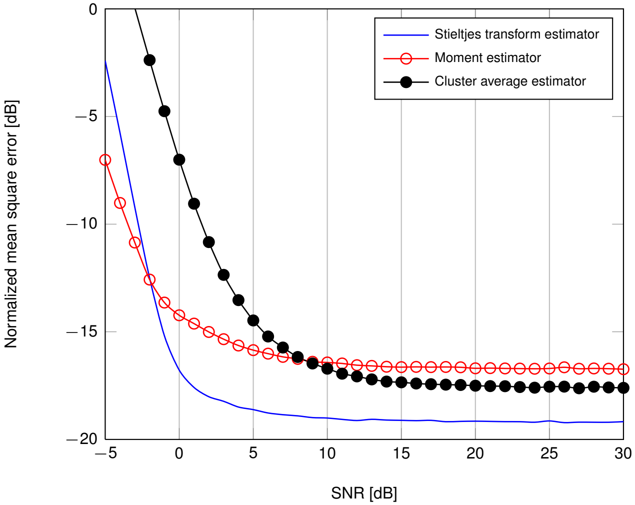
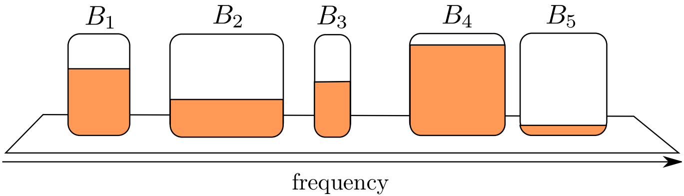


Figure: Normalized mean square error of largest estimated power $\hat{P}_3$, $P_1 = 1/16$, $P_2 = 1/4$, $P_3 = 1$, $n_1 = n_2 = n_3 = 4$,$N = 24$, $M = 128$. Comparison between classical, moment and Stieltjes transform approaches.

Outline

Problem statement

- We assume, from the exploration phase, that power Q_f can be transmitted in bandwidth B_f.



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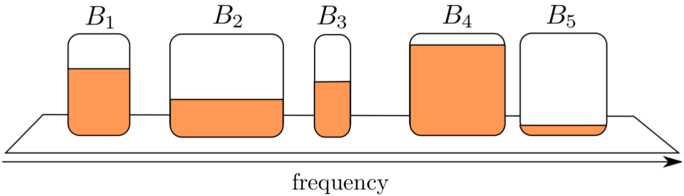

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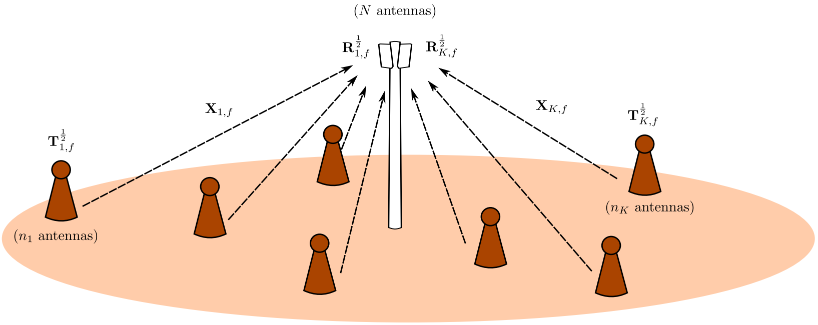
- We consider the uplink scenario of
  - an N-antenna base station;
  - K users equipped with $n_1, \ldots, n_K$ antennas;
  - Kronecker channels at all pairs $\mathbf{H}_{k,f} = \mathbf{R}_{k,f}^{\frac{1}{2}} \mathbf{X}_{k,f} \mathbf{T}_{k,f}^{\frac{1}{2}} \in \mathbb{C}^{N \times n_k}$ at frequency B_f;
  - colored noise with covariance $\boldsymbol{\Sigma}$.



Maximizing ergodic sum rate

- Due to mobility, we wish to optimize the uplink ergodic sum rate (per antenna),

$$\mathcal{I}(\mathbf{P}_{1,1},\ldots,\mathbf{P}_{K,F}) \triangleq \frac{1}{N}\sum_{f=1}^{F}\frac{|B_f|}{\sum_{f'}|B_{f'}|}\mathrm{E}\left[\log\det\left(\mathbf{I}_N+\sum_{k=1}^{K}\mathbf{\Sigma}_f^{-\frac{1}{2}}\mathbf{H}_{k,f}\mathbf{P}_{k,f}\mathbf{H}_{k,f}^{\mathrm{H}}\mathbf{\Sigma}_f^{-\frac{1}{2}}\right)\right].$$

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- Determine the sum rate maximizing precoders $\mathbf{P}_{k,f}^{\star}$

$$(\mathbf{P}_{1,1}^{\star},\ldots,\mathbf{P}_{K,F}^{\star}) = \arg\max_{\substack{\{\mathbf{P}_{k,f}\}\\ \sum_{k=1}^{K}\operatorname{tr}\mathbf{P}_{k,f}\leq Q_f}} \mathcal{I}(\mathbf{P}_{1,1},\ldots,\mathbf{P}_{K,F}).$$

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- Simplifying assumptions:
  - Problem can be treated for each f independently. We then assume $F=1$.
  - Taking $\mathbf{\Sigma}=\sigma^2\mathbf{I}_N$ does not restrict generality.

Deterministic Equivalents of the Sum Rate

- The stochastic character of $\mathbf{H}_{k,f}$ makes things difficult.
- We instead find a deterministic approximation $\mathcal{I}^\circ(\mathbf{P}_1, \ldots, \mathbf{P}_K)$ for $\mathcal{I}(\mathbf{P}_1, \ldots, \mathbf{P}_K)$ such that

$$\mathcal{I}^\circ(\mathbf{P}_1, \ldots, \mathbf{P}_K) - \mathcal{I}(\mathbf{P}_1, \ldots, \mathbf{P}_K) \to 0$$

as $N, n_1, \ldots, n_K \to \infty$, and denote

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- We can show

$$\mathcal{I}(\mathbf{P}_1^\star, \ldots, \mathbf{P}_K^\star) - \mathcal{I}(\mathbf{P}_1^\circ, \ldots, \mathbf{P}_K^\circ) \to 0.$$

Deterministic Equivalents: Strategy

- With $\mathbf{B}_N \triangleq \sum_{k=1}^{K} \mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^{\mathsf{H}}$ ($\mathbf{H}_k = \mathbf{R}_k^{\frac{1}{2}} \mathbf{X}_k \mathbf{T}_k^{\frac{1}{2}}$), notice that

$$\begin{aligned}\mathcal{I}(\mathbf{P}_1, \ldots, \mathbf{P}_K) &= \mathrm{E}\left[\frac{1}{N} \log\det\left(\mathbf{I}_N + \frac{1}{\sigma^2}\mathbf{B}_N\right)\right] \\ &= \mathrm{E}\left[\int_{\sigma^2}^{\infty}\left(\frac{1}{\omega} - \frac{1}{N}\operatorname{tr}\left(\mathbf{B}_N + \omega \mathbf{I}_N\right)^{-1}\right) d\omega\right] \\ &= \mathrm{E}\left[\int_{\sigma^2}^{\infty}\left(\frac{1}{\omega} - m_{\mathbf{B}_N}(-\omega)\right) d\omega\right].\end{aligned}$$

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- It suffices to find a deterministic equivalent for $m_{\mathbf{B}_N}(z)$.

Final Results

R. Couillet, M. Debbah and J. W. Silverstein, "A Deterministic Equivalent for the Analysis of Correlated MIMO Multiple Access Channels", *to appear in* IEEE Transactions on Information Theory, arXiv Preprint 0906.3667.

Theorem (Deterministic equivalent of the Stieltjes transform)

Under some mild conditions on the $\mathbf{R}_k$ *and* $\mathbf{T}_k$ *matrices, as* $N, n_1, \ldots, n_K \to \infty$

$$m_{\mathbf{B}_N}(z) - m_N(z) \xrightarrow{\text{a.s.}} 0$$

where

$$m_N(z) = \frac{1}{N} \operatorname{tr} \left(-z \left[\sum_{k=1}^{K} \bar{e}_k(z) \mathbf{R}_k + \mathbf{I}_N \right] \right)^{-1}$$

and $\{\bar{e}_i(z)\}$, $i \in \{1, \ldots, K\}$, *form the unique solution to*

$$e_i(z) = \frac{1}{n_i} \operatorname{tr} \mathbf{R}_i \left(-z \left[\sum_{k=1}^{K} \bar{e}_k(z) \mathbf{R}_k + \mathbf{I}_N \right] \right)^{-1}$$

$$\bar{e}_i(z) = \frac{1}{n_i} \operatorname{tr} \mathbf{T}_i^{\frac{1}{2}} \mathbf{P}_i \mathbf{T}_i^{\frac{1}{2}} \left(-z \left[e_i(z) \mathbf{T}_i^{\frac{1}{2}} \mathbf{P}_i \mathbf{T}_i^{\frac{1}{2}} + \mathbf{I}_{n_i} \right] \right)^{-1}$$

Final results (2)

R. Couillet, M. Debbah and J. W. Silverstein, “A Deterministic Equivalent for the Analysis of Correlated MIMO Multiple Access Channels”, *to appear in* IEEE Transactions on Information Theory, arXiv Preprint 0906.3667.

Theorem (Deterministic equivalents of the sum rate)

Under similar conditions, with $\mathcal{I}(\mathbf{P}_1, \ldots, \mathbf{P}_K) = \frac{1}{N}\mathrm{E} \log\det\left(\mathbf{I}_N + \frac{1}{\sigma^2}\mathbf{B}_N\right)$ *and* $z = -\sigma^2$*,*

$$\mathcal{I}(\mathbf{P}_1, \ldots, \mathbf{P}_K) - \mathcal{I}^\circ(\mathbf{P}_1, \ldots, \mathbf{P}_K) \to 0$$

where

$$\mathcal{I}^\circ(\mathbf{P}_1, \ldots, \mathbf{P}_K) = \frac{1}{N}\log\left|\mathbf{I}_N + \sum_{k=1}^{K} \bar{e}_k \mathbf{R}_k\right| + \sum_{k=1}^{K} \frac{1}{N}\log\left|\mathbf{I}_{n_k} + e_k \mathbf{T}_k^{\frac{1}{2}} \mathbf{P}_k \mathbf{T}_k^{\frac{1}{2}}\right| - \sigma^2 \sum_{k=1}^{K} \frac{n_k}{N} \bar{e}_k e_k.$$

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- It remains to maximize $\mathcal{I}^\circ(\mathbf{P}_1,\ldots,\mathbf{P}_K)$ over $\mathbf{P}_1,\ldots,\mathbf{P}_K$ with $\sum_k \operatorname{tr}\mathbf{P}_k \leq Q$.

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- It remains to maximize $\mathcal{I}^\circ(\mathbf{P}_1,\ldots,\mathbf{P}_K)$ over $\mathbf{P}_1,\ldots,\mathbf{P}_K$ with $\sum_k \operatorname{tr}\mathbf{P}_k \leq Q$.
- This is obtained by iterative waterfilling
  - $\mathbf{P}_k$ and $\mathbf{T}_k$ have the same eigenspaces
  - with $\operatorname{eig}(\mathbf{P}_k^\circ) = (p_{k,1}^\circ,\ldots,p_{k,n_k}^\circ)$

$$p_{k,i}^\circ = \left(\mu - \frac{1}{e_k^\circ t_{k,i}}\right)^+,\quad e_k^\circ = e_k(\mathbf{P}_1^\circ,\ldots,\mathbf{P}_K^\circ),\quad \mu \text{ set to satisfy } \sum_k \operatorname{tr}\mathbf{P}_k = Q.$$

Simulation results

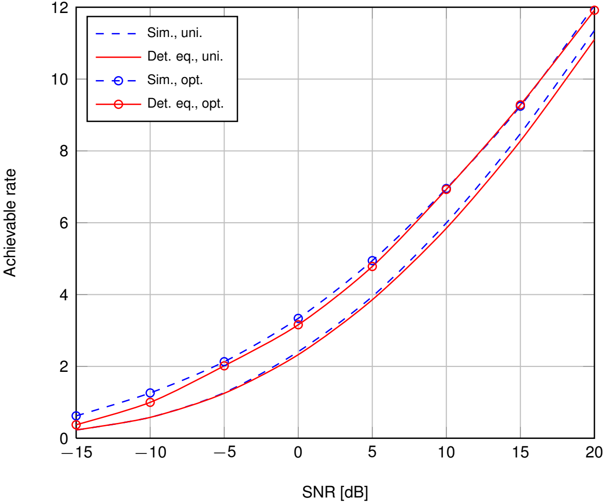

Figure: Ergodic MAC sum rate for an $N = 4$ antenna receiver and $K = 4$ single-antenna transmitters under sum power constraint. Every user transmit signal has different correlation patterns at the receiver, and different path losses. Deterministic equivalents (det. eq.) against simulation (sim.), with uniform (uni.) or optimal (opt.) power allocation.

Outline

The road ahead

- **signal sensing**:
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    - Stieltjes transform approach seems the most powerful;
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Future cognitive radio communications with

- open-access communications,
- secondary networks coordination,

will demand

- a characterization of **what information to share**,
- a merger between **random matrix theory and game theory**,
- a stronger effort on **graph-oriented random matrix theory**.

List of Publications (1)

- **Publications in Journals: (7 accepted / 2 submitted)**
  - **R. Couillet**, J. Hoydis, M. Debbah, “A deterministic equivalent approach to the performance analysis of isometric random precoded systems,” *submitted to* IEEE Transactions on Information Theory, arXiv Preprint XXXX.XXXX.
  - S. Wagner, **R. Couillet**, M. Debbah, D. T. M. Slock, “Large System Analysis of Linear Precoding in MISO Broadcast Channels with Limited Feedback”, *submitted to* IEEE Transactions on Information Theory, arXiv Preprint 0906.3682.
  - **R. Couillet**, J. W. Silverstein, Z. Bai, M. Debbah, “Eigen-Inference for Energy Estimation of Multiple Sources”, *to appear in* IEEE Transactions on Information Theory, arXiv Preprint 1001.3934.
  - **R. Couillet**, M. Debbah, J. W. Silverstein, “A Deterministic Equivalent for the Analysis of Correlated MIMO Multiple Access Channels”, *to appear in* IEEE Transactions on Information Theory, arXiv Preprint 0906.3667.
  - **R. Couillet**, M. Debbah, “A Bayesian Framework for Collaborative Multi-Source Signal Sensing”, IEEE Transactions on Signal Processing, vol. 58, no. 10, pp. 5186-5195, 2010, arXiv Preprint 0811.0764.
  - **R. Couillet**, A. Ancora, M. Debbah, “Bayesian Foundations of Channel Estimation for Cognitive Radios”, Advances in Electronics and Telecommunications, vol. 1, no. 1, pp. 41-49, 2010.
  - **R. Couillet**, M. Debbah, “Le téléphone du futur : plus intelligent pour une exploitation optimale des fréquences” Revue de l’Electricité et de l’Electronique, no. 6, pp. 71-83, 2010.
  - **R. Couillet**, M. Debbah, “Mathematical foundations of cognitive radios”, Journal of Telecommunications and Information Technologies, no. 4, 2009.
  - **R. Couillet**, M. Debbah, “Outage performance of flexible OFDM schemes in packet-switched transmissions”, Eurasip Journal on Advances on Signal Processing, Volume 2009, Article ID 698417, 2009.

List of Publications (2)

- **Publications in International Conferences: (16 accepted / 3 submitted)**
  - A. Kammoun, **R. Couillet**, J. Najim, M. Debbah, "A G-estimator for rate adaption in cognitive radios," *submitted to* IEEE Dynamic Spectrum Access Networks, Aachen, Germany, 2011.
  - J. Yao, **R. Couillet**, J. Najim, E. Moulines, M. Debbah, "CLT for eigen-inference methods in cognitive radios," *submitted to* IEEE International Conference on Acoustics, Speech and Signal Processing, Prague, Czech Republic, 2011.
  - J. Hoydis, **R. Couillet**, M. Debbah, "Deterministic Equivalents for the Performance Analysis of Isometric Random Precoded Systems," *submitted to* IEEE International Conference on Communications, Kyoto, Japan, 2011.
  - J. Hoydis, J. Najim, **R. Couillet**, M. Debbah, "Fluctuations of the Mutual Information in Large Distributed Antenna Systems with Colored Noise," Forty-Eighth Annual Allerton Conference on Communication, Control, and Computing, Allerton, IL, USA, 2010.
  - **R. Couillet**, H. V. Poor, M. Debbah, "Self-organized spectrum sharing in large MIMO multiple-access channels," IEEE International Symposium on Information Theory, Austin TX, USA, 2010.
  - **R. Couillet**, J. W. Silverstein, M. Debbah, "Eigen-inference for multi-source power estimation," IEEE International Symposium on Information Theory, Austin TX, USA, 2010.
  - S. Wagner, **R. Couillet**, D. T. M. Slock, M. Debbah, "Optimal Training in Large TDD Multi-user Downlink Systems under Zero-forcing and Regularized Zero-forcing Precoding," *submitted to* IEEE Global Communication Conference, Miami, 2010.
  - S. Wagner, **R. Couillet**, D. T. M. Slock, M. Debbah, "Large System Analysis of Zero-Forcing Precoding in MISO Broadcast Channels with Limited Feedback" IEEE International Workshop on Signal Processing Advances for Wireless Communications, Marrakech, Morocco, 2010.
  - **R. Couillet**, M. Debbah, "Information theoretic approach to synchronization: the OFDM carrier frequency offset example", Advanced International Conference on Telecommunications, Barcelona, Spain, 2010.
  - **R. Couillet**, M. Debbah, "Uplink capacity of self-organizing clustered orthogonal CDMA networks in flat fading channels" IEEE Information Theory Workshop Fall'09, Taormina, Sicily, 2009.
  - **R. Couillet**, M. Debbah, J. W. Silverstein, "Asymptotic Capacity of Multi-User MIMO Communications" IEEE Information Theory Workshop Fall'09, Taormina, Sicily, 2009.
  - **R. Couillet**, M. Debbah, J. W. Silverstein, "Rate region of correlated MIMO multiple access channel and broadcast channel" IEEE Workshop on Statistical Signal Processing, Cardiff, Wales, UK, 2009.
  - **R. Couillet**, M. Debbah, "Mathematical foundations of cognitive radios" U.R.S.I.'09, Warsaw, Poland, 2009.
  - **R. Couillet**, M. Debbah, "A maximum entropy approach to OFDM channel estimation", IEEE International Workshop on Signal Processing Advances for Wireless Communications, Perugia, Italy, 2009.
  - **R. Couillet**, M. Debbah, "Bayesian inference for multiple antenna cognitive receivers", IEEE Wireless Communications & Networking Conference, Budapest, Hungary, 2009.
  - **R. Couillet**, M. Debbah, "Flexible OFDM schemes for bursty transmissions", IEEE Wireless Communications & Networking Conference, Budapest, Hungary, 2009.
  - **R. Couillet**, S. Wagner, M. Debbah, "Asymptotic Analysis of Correlated Multi-Antenna Broadcast Channels", IEEE Wireless Communications & Networking Conference, Budapest, Hungary, 2009.
  - **R. Couillet**, S. Wagner, M. Debbah, A. Silva, "The Space Frontier: Physical Limits of Multiple Antenna Information Transfer", ValueTools, Inter-Perf Workshop, Athens, Greece, 2008. **BEST STUDENT PAPER AWARD**
  - **R. Couillet**, M. Debbah, "Free deconvolution for OFDM multicell SNR detection", IEEE Personal, Indoor and Mobile Radio Communications Symposium, Cognitive Radio Workshop, Cannes, France, 2008.

List of Publications (3)

- **Patents: (4 owned by ST-Ericsson)**
  - **R. Couillet**, M. Debbah, *Application no. 08368028.0* “Process and apparatus for performing initial carrier frequency offset in an OFDM communication system”
  - **R. Couillet**, M. Debbah, *Application no. 08368023.1* “Method for short-time OFDM transmission and apparatus for performing flexible OFDM modulation”
  - **R. Couillet**, S. Wagner, *Application no. 09368025.4* “Precoding process for a transmitter of a MU-MIMO communication system”
  - **R. Couillet**, *Application no. 09368030.4* “Process for estimating the channel in an OFDM communication system, and receiver for doing the same”

List of Publications (4)

- **Books and book chapters: (1 book / 2 book chapters)**
  - **Random Matrix Methods for Wireless Communications** [book]
    Theoretical random matrix tools (finite dimensional analysis, limiting spectral laws, free probability, deterministic equivalents, statistical inference) and applications to wireless communications (SU-MIMO, MU-MIMO, CDMA, detection, estimation, channel modelling).
    - *Authors:* **R. Couillet** and M. Debbah
    - *Publisher:* Cambridge University Press
    - *Year:* 2011 (to appear)
  - **Mathematical Foundations for Signal Processing, Communications and Networking** [book chapter XX]
    Chapter “Random matrix theory” on reminders of random matrix theory and especially statistical inference methods.
    - *Chapter authors:* **R. Couillet** and M. Debbah
    - *Publisher:* CRC Press, Taylor & and Francis Group
    - *Year:* 2011 (to appear)
  - **Orthogonal Frequency Division Multiple Access Fundamentals and Applications** [book chapter 13]
    Chapter “Fundamentals of OFDMA Synchronization” on theoretical considerations and application tools for time offset and frequency offset regulation in OFDM and OFDMA systems.
    - *Chapter authors:* **R. Couillet** and M. Debbah
    - *Publisher:* Auerbach Publications, CRC Press, Taylor & and Francis Group
    - *ISBN:* 978-1-4200-8824-3
    - *Year:* 2010

Thank you for your attention.