CURRICULUM VITAE

Romain COUILLET
Professeur des Universités (HDR) à CentraleSupélec
Titulaire de la chaire IDEX GSTATS à Univ. Grenoble-Alpes
Titulaire de la chaire MIAI LargeDATA à Univ. Grenoble-Alpes

Né le 18 mars 1983 (36 ans). Nationalité française.

Table des Matières

Informations Générales

Expérience professionnelle

Université Grenoble–Alpes, Saint-Martin d'Hères, France.

Titulaire de la chaire IDEX en datascience GSTATS **Avril 2018 - Présent**

Titulaire de la chaire MIAI LargeDATA **Septembre 2019 - Présent**

- Recherche en théorie des matrices aléatoires appliquées à l'apprentissage en grandes dimensions.
- Enseignement dans les masters en science des données.

CentraleSupélec, Université ParisSaclay, Gif sur Yvette, France.

Professeur des Universités **Janvier 2016 - Présent**

Enseignant chercheur **Janvier 2011 - Présent**

- Recherche en probabilités, statistiques, apprentissage, traîtement des données et des signaux.
- Enseignement en cycle ingénieur, Master (SAR et MVA), thèse.

ST-Ericsson, Sophia Antipolis, France.

Ingénieur R&D, étudiant en thèse **Septembre 2007 - Décembre 2010**

- Recherche en théorie des matrices aléatoires.
- Applications aux standards 4G et MIMO.

Langues

Français (langue maternelle), anglais (courant), allemand (scolaire).

Titres universitaires

Diplômes

Université d'Orsay, Saclay, France. **Janvier 2011 - Février 2015**

HDR en Physique

- Titre : Méthodes d'estimation robuste dans le régime des grandes matrices aléatoires
- Jury : A. Hero, L. Pastur, J-Y. Tourneret (rapporteurs), F. Benaych-Georges, P. Bondon, M. McKay, E. Ollila.

CentraleSupélec, Gif sur Yvette, France. **Janvier 2008 - Novembre 2010**

Thèse, spécialité Physique (Télécommunications), Novembre 2010

- Titre : Application de la théorie des matrices aléatoires aux futurs réseaux flexibles de communications sans fils
- Directeur de thèse : Mérouane Debbah
- Jury : P. Loubaton, X. Mestre (rapporteurs), M. Debbah, P. Duhamel, W. Hachem, A. Moustakas, J. Silverstein.

EURECOM, Sophia Antipolis, France. **Septembre 2005 - Juin 2007**

Diplôme d'ingénieur, Ingénierie des télécommunications, Septembre 2007

Communications mobiles, systèmes embarqués, informatique.

Telecom ParisTech, Paris, France. **Septembre 2004 - Juin 2007**

Master Science, Systèmes de Communications, (mention TB), Mars 2008

Communications sans fils, traitement d'image, techniques de détection aveugles.

Cursus antérieur

Lycée Louis le Grand, Paris, France. **Septembre 2001 - Juin 2004**

Classe préparatoire aux grandes écoles

Activités d'enseignement

Service

[2011-2018] Enseignant-chercheur temps-plein à CentraleSupélec **∼240h ETD**
[2018-2020] Enseignant-chercheur (20%) à CS, chaire UGA (80%) **∼80h ETD**

Niveaux thèse et master

ENS ParisSaclay (Saclay, France) **depuis 2013**
- Random matrix theory and machine learning applications (Master MVA, cours magistraux, 24h ETD)

Université Grenoble-Alpes (Grenoble, France) **depuis 2018**
- Introduction to Convex Optimization Theory (Master SIGMA, cours magistraux, 20h ETD)
- Introduction to Scientific Writing (Master SIGMA, cours magistraux, 9h ETD)
- Scientific Writing and Associated Softwares (Niveau thèse, cours & TPs, 24h ETD)

CentraleSupélec (Gif sur Yvette, France) **depuis 2011**
- Techniques of scientific writing (Niveau thèse, cours et TP, 24h ETD)
- Introduction to random matrix theory (Master SAR, séminaires, 18h ETD)
- Theoretical foundations of flexible radio networks (Master SAR, séminaires, 18h ETD)

Niveaux pré-master

CentraleSupélec (Gif sur Yvette, France) **2011-2018**
- Représentation statistique des signaux (TD, 2×12h ETD)
- Signaux et systèmes (TD, 24h ETD)
- Filtrages numérique et analogique (TP, 32h ETD)
- Introduction à la rédaction scientifique (électif, 18h ETD)

Encadrement

ENS Paris Saclay **depuis 2011**
- Stages de master (4 à 6 mois, jusqu'à 4 étudiants/an)

CentraleSupélec (Gif sur Yvette, France) **2011-2018**
- Projets longs de master (master SAR, projets de 3 mois, 2 étudiants/an)
- Projets majeure Telecom (étudiants de 3e année, projets de 3 mois, 2-3 étudiants/an)
- Projets de conception (étudiants de 1ère année, projets de 2 mois, 6 étudiants/an)
- Projets de synthèse (étudiants de 2e année, projets de 2 mois, 4 étudiants/an)

Administration

CentraleSupélec (Gif sur Yvette, France) **2015-2018**
- Définition du nouveau programme ingénieur CentraleSupélec.
- Participation au chantier "stratégie et image de marque" de CentraleSupélec.

Organisation de cours

Grenoble INP (Grenoble, France) **depuis 2018**
- Refonte du cours d'optimisation convexe (niveau master ; cours, TPs, examens)
- Création d'un cours de communication scientifique (niveau master et doctorat).

CentraleSupélec (Gif-sur-Yvette, France) **depuis 2012**
- Mise en place des 2 séminaires du master SAR (cours, examens)
- Mise en place des modules de rédaction scientifique

ENS ParisSaclay (Saclay, France) **depuis 2013**
- Mise en place du cours du master MVA (cours, TP, examen)

Activités Périphériques

Concours CPGE Centrale–Supélec **2016-2018**

- Evaluateur au Concours Centrale–Supélec (80h)

Activités de Recherche

Informations générales

Sujets de recherche : statistique, théorie des matrices aléatoires, apprentissage automatique, statistiques robustes, graphes, traitement des données et du signal, communications mobiles, finance statistique, applications marginales (traitement naturel du langage, virologie, données médicales).

Résumé des publications (chiffres Google Scholar de mars 2020)

Global	1 livre, 3 chapitres, 50+ revues, 85+ conférences, 6 brevets/idées innovantes.
Citations	3400+ (cinq meilleures : 723, 542, 145, 110, 88)
h-index	27
i10-index	58

Distinctions

Médaille de bronze du CNRS. **2013**

Médaille de Bronze 2013 du CNRS dans la subsection INS2I.

Récompense mes travaux de jeune chercheur en statistiques, traitement des données, du signal et communications mobiles depuis 2008.

Prix de jeune chercheur IEEE ComSoc **2013**

2013 IEEE ComSoc Outstanding Young Researcher Award for the EMEA Region

Récompense mes travaux de jeune chercheur en matrices aléatoires et communications mobiles depuis 2008.

Prix de meilleure thèse. **2011**

Prix EEA/GdR ISIS/GRETSI 2011 de la meilleure thèse 2010

Prix pour ma thèse de doctorat, "Application de la théorie des matrices aléatoires aux futurs réseaux flexibles de communications sans fil"

Prix de meilleure thèse (d'un étudiant). **2020**

Prix EEA/GdR ISIS/GRETSI 2020 de la meilleure thèse 2019

Prix pour ma thèse de doctorat, "Méthodes des matrices aléatoires pour l'apprentissage en grande dimension"

Meilleur article étudiant. **2019** *Best student paper award de la conférence EUSIPCO 2019* **Meilleur article étudiant.** **2013**

Second prix du 2012-2013 IEEE Australia Council Student Paper Contest

Meilleur article étudiant. **2011**

Finaliste du Best Student Paper Award de la conférence Asilomar 2011

Meilleur article étudiant. **2008**

Meilleur article étudiant de la conférence ValueTools 2008

Orateur en session plénière. **2016**

Conférence ACM RACS 2016 à Odense, Danemark. **Projets en**

Projets

cours.

Projet	Contribution	Pé
ANR MIAI LargeDATA chair	50% (PI)	2019-
IDEX GSTATS chair	100% (PI)	2018-
ANR DARLING	50% (co-PI)	2019-
HUAWEI RMT4AI	100% (PI)	2019-

Projets précédents.

Projet	Contribution	Pé
ANR RMT4GRAPH	100% (PI)	2014-
ERC MORE	50%	2012-
ParisSaclay RMT4ML	100% (PI)	2017-
Fondation Supélec DeepRMT	100% (PI)	2017-
Singapore MERLION	100% (PI)	2015-
Mastodons AGADIR	20%	
HUAWEI RMTin5G	100% (PI)	2015-
ANR DIONISOS	25% (co-PI)	2012-
ANR SESAME	20%	2008-
FP7 NEWCOM#	10%	2012-
FP7 NEWCOM++	10%	2009-

Activités communautaires

Jury CRCN-INRIA **depuis 2018**

Membre du jury CRCN INRIA-GRA (centre de Grenoble) en 2018, 2019, 2020 **Agence Nationale pour la Recherche (ANR, France)** **depuis 2015**

Relecteur de projets ANR (environ 2/an depuis 2015) **Academy of Finland** **2015**

Evaluateur pour l'académie de Finlande (relecture de projets et comité sur 2 jours à Helsinki) **IEEE** (membre senior) **depuis 2007**

Editeur associé pour IEEE TSP (2015–2019) **GRETSI** (membre) **depuis 2011**

Membre de l'association GRETSI (depuis 2011)

Organisation de Conférences

Organisateur des sessions spéciales de la conférence IEEE CAMSAP 2019.

Technical Area Chair de la conférence Asilomar 2016.

Organisateur de sessions spéciales et journées scientifiques :

- *International* : Session spéciale "Random Matrices in Signal Processing and Machine Learning" (IEEE SSP, 2016), Session spéciale "Random Matrix Advances in Signal Processing" (IEEE SSP, 2014), Session spéciale

"Random Matrices and Applications" (IEEE Asilomar, 2013).

Encadrement de thèses

Thèses en cours.

Charles Séjourné **2020-2023**

Henrique Goulart **depuis 2019**

Encadrement de postdocs

Jack W. Silverstein, Professeur à North Carolina State University, expert en théorie des matrices aléatoires.

Alfred O. Hero, Professeur à l'University of Michigan, expert en statistiques, science des données et traitement du signal.

Florent Benaych-Georges, Professeur à l'Université de Paris Descartes et à l'Ecole Polytechnique, expert en théorie des graphes et matrices aléatoires.

Gilles Wainrib, Professeur assistant à l'ENS Paris, expert en apprentissage et réseaux de neurones aléatoires.

Walid Hachem, DR CNRS à l'Université de Marne la Vallée, expert en théorie des matrices aléatoires et applications.

Mérouane Debbah, Professeur at CentraleSupélec, expert en matrices aléatoires et communications mobiles.

Abla Kammoun, Research Scientist à KAUST University, experte en matrices aléatoires, traitement du signal et statistiques.

Visites

Hong-Kong University of Science and Technology, Hong-Kong. **Juin 2014**

Collaboration avec le Professeur M. McKay,

Articles avec comité de relecture

J1. C. Louart, R. Couillet "A Concentration of Measure Approach to Large Dimensional Robust Statistics", (submitted to) Annals of Applied Probability, 2020.

J2. K. Elkhalil, A. Kammoun, R. Couillet, T. Al-Naffouri, M-S. Alouini, "A Large Dimensional Analysis of Regularized Discriminant Analysis Classifiers" (to appear) IEEE Transactions on Signal Processing, 2020.

J3. L. Dall'Amico, R. Couillet, N. Tremblay, "A unified framework for spectral clustering in sparse graphs", (submitted to) Journal of Machine Learning Research, 2020.

J4. C. Louart, R. Couillet, "Concentration of Measure and Large Random Matrices with an application to Sample Covariance Matrices", (submitted to) Random Matrix Theory and Applications, 2019.

J5. X. Mai, R. Couillet, "Consistent Semi-Supervised Graph Regularization for High Dimensional Data", (submitted) Journal of Machine Learning Research, 2019.

J6. R. Couillet, M. Tiomoko, S. Zozor, E. Moisan, "Random matrix-improved estimation of covariance matrix distances", Journal of Multivariate Analysis, vol. 174, pp. 104531, 2019.

J7. X. Mai, R. Couillet, "A Random Matrix Analysis and Improvement of Semi-Supervised Learning for Large Dimensional Data", Journal of Machine Learning Research, vol. 19, no. 79, pp. 1-27, 2018.

J8. A. Kammoun, R. Couillet, "Subspace Kernel Clustering of Large Dimensional Data" (submitted to) Annals of Applied Probability, 2017.

J9. L. Yang, M. McKay, R. Couillet, "High-Dimensional MVDR Beamforming: Optimized Solutions based on Spiked Random Ma-

trix Models", IEEE Transactions on Signal Processing, vol. 66, no. 1, pp. 1933-1947, 2018.

J10. A. Karadimitrakis, A. L. Moustakas, R. Couillet, "Gallager Bound for MIMO Channels: Large-N Asymptotics" IEEE Transactions on Wireless Communications, vol. 17, no. 2, pp. 1323-1330, 2018.

J11. N. Auguin, D. Morales, M. McKay, R. Couillet, "Large-dimensional behavior of regularized Maronna's M-estimators of covariance matrices" IEEE Transactions on Signal Processing, vol. 66, no. 13, pp. 3529–3542, 2018.

J12. C. Louart, Z. Liao, R. Couillet, "A Random Matrix Approach to Neural Networks" Annals of Applied Probability, vol. 28, no. 2, pp. 1190–1248, 2018.

J13. Z. Liao, R. Couillet, "A Large Dimensional Analysis of Least Square Support Vector Machines" IEEE Transactions on Signal Processing, vol. 67, no. 4, pp. 1065-1074, 2018.

J14. R. Couillet, H. Tiomoko Ali, "Improved spectral community detection in large heterogeneous networks" Journal of Machine Learning Resarch, vol. 18, no. 225, pp. 1–49, 2018.

J15. R. Couillet, M. McKay, "Optimal block-sparse PCA for high dimensional correlated samples" (submitted to) Journal of Multivariate Analysis, 2016.

J16. R. Couillet, G. Wainrib, H. Sevi, H. Tiomoko Ali, "The asymptotic performance of linear echo state neural networks" Journal of Machine Learning Research, vol. 17, no. 178, pp. 1-35, 2016.

J17. R. Couillet, F. Benaych-Georges, "Kernel Spectral Clustering of Large Dimensional Data" Electronic Journal of Statistics, vol. 10, no. 1, pp. 1393-1454, 2016.

J18. F. Benaych-Georges, R. Couillet, "Spectral Analysis of the Gram Matrix of Mixture Models" ESAIM : Probability and Statistics, DOI http ://dx.doi.org/10.1051/ps/20 2016.

J19. R. Couillet, Estimation robuste et matrices aléatoires, revue Traitement du Signal, vol. 33, no. 2-3, pp. 273-320, 2016.

J20. R. Couillet, G. Wainrib, Perspectives en matrices aléatoires et grands réseaux, revue Traitement du Signal, vol. 33, no. 2-3, pp. 351-376, 2016.

J21. M. Sadeghi, L. Sanguinetti, R. Couillet, Y. Chau, "Large System Analysis of Power Normalization Techniques in Massive MIMO", IEEE Transactions on Vehicular Technologies, vol. 66, no. 10, pp. 9005-9017, 2017.

J22. M. Sadeghi, L. Sanguinetti, R. Couillet, Y. Chau, "Reducing the Computational Complexity of Multicasting in Large-Scale Antenna Systems", IEEE Transactions on Wireless Communications, vol. 16, no. 5, pp. 2963-2975, 2017.

J23. L. Sanguinetti, R. Couillet, M. Debbah, "Large System Analysis of Base Station Cooperation for Power Minimization" IEEE Transactions on Wireless Communications, vol. 15, no. 8, pp. 5480-5496, 2016.

J24. A. Abboud, F. Iutzeler, R. Couillet, H. Siguerdidjane, M. Debbah, "Distributed Production-Sharing Optimization and Ap-

plication to Power Grid Networks," IEEE Transactions on Signal and Information Processing over Networks, vol. 2, no. 1, pp. 1628, 2016.

J25. A. Kammoun, R. Couillet, F. Pascal, M.-S. Alouini, "Optimal Design of the Adaptive Normalized Matched Filter Detector using Regularized Tyler Estimator" IEEE Transactions on Aerospace and Electronic Systems, vol. 54, no. 2, pp. 755–769, 2018.

J26. A. Kammoun, R. Couillet, F. Pascal, M.-S. Alouini, "Convergence and Fluctuations of Regularized Tyler Estimators" IEEE Transactions on Signal Processing, vol. 64, no. 4, pp. 1048-1060, 2016.

J27. D. Morales-Jimenez, R. Couillet, M. McKay, "Large Dimensional Analysis of Robust M-Estimators of Covariance with Outliers" IEEE Transactions on Signal Processing, vol. 63, no. 21, pp. 5784-5797, 2015.

J28. L. Yang, R. Couillet, M. McKay, "A Robust Statistics Approach to Minimum Variance Portfolio Optimization" IEEE Transactions on Signal Processing, vol. 63, no. 24, pp. 6684–6697, 2015.

J29. R. Couillet, A. Kammoun, F. Pascal, "Second order statistics of robust estimators of scatter. Application to GLRT detection for elliptical signals" Elsevier Journal of Multivariate Analysis, vol. 143, pp. 249-274, 2015.

J30. A. Müller, R. Couillet, E. Bjørnson, S. Wagner, M. Debbah, "Interference-Aware RZF Precoding for Multi-Cell Downlink Systems" IEEE Transactions on Signal Processing, vol. 63, no. 15, pp. 3959-3973 2015.

J31. R. Couillet, "Robust spiked random matrices and a robust G-MUSIC estimator" Elsevier Journal of Multivariate Analysis, vol. 140, pp. 139-161, 2015.

J32. R. Couillet, M. McKay, "Large Dimensional Analysis and Optimization of Robust Shrinkage Covariance Matrix Estimators" Elsevier Journal of Multivariate Analysis, vol. 131, pp. 99-120, 2014.

J33. Y. Chitour, R. Couillet, F. Pascal "On the convergence of Maronna's M-estimators of scatter" IEEE Signal Processing Letters, vol. 22, no. 6, pp. 709-712, 2014.

J34. R. Couillet, F. Pascal, J. W. Silverstein, "The Random Matrix Regime of Maronna's M-estimator with elliptically distributed samples", vol. 139, pp. 56-78, Elsevier Journal of Multivariate Analysis, 2015.

J35. J. Vinogradova, R. Couillet, W. Hachem, "Estimation of Toeplitz covariance matrices in large dimensional regime with application to source detection large", IEEE Transactions on Signal Processing, vol. 63, no. 18, pp. 4903-4913, 2015.

J36. R. Couillet, W. Hachem, "Analysis of the limiting spectral measure of large random matrices of the separable covariance type", Random Matrix Theory and Applications, vol. 3, pp. 1-23, 2014.

J37. J. Hoydis, R. Couillet, P. Piantanida, "The Second-Order Coding Rate of the MIMO Rayleigh Block-Fading Channel," IEEE Transactions on Information Theory, vol. 61, no. 12, pp. 6591-6622, 2015.

J38. J. Vinogradova, R. Couillet, W. Hachem, "Statistical Inference in Large Antenna Arrays under Unknown Noise Pattern," IEEE Transactions on Signal Processing, vol. 61, no. 22, pp. 5633-5645, 2013.

J39. F. Chapon, R. Couillet, W. Hachem, X. Mestre, "The outliers among the singular values of large rectangular random matrices with additive fixed rank deformation," Markov Processes and Related Fields, vol. 20, pp. 183-228, 2014.

J40. R. Couillet, F. Pascal, J. W. Silverstein, "Robust Estimates of Covariance Matrices in the Large Dimensional Regime," IEEE Transactions on Information Theory, vol. 60, no. 11, 2014.

J41. G. Geraci, R. Couillet, J. Yuan, M. Debbah, I. B. Collings, "Large System Analysis of Linear Precoding in MISO Broadcast Channels with Confidential Messages," IEEE Journal on Selected Area in Communications, vol. 31, no. 9, pp. 1660-1671, 2013. **Second prize of the 2012-2013 IEEE Australia Council Student Paper Contest.**

J42. J. Hoydis, R. Couillet, M. Debbah, "Iterative Deterministic Equivalents for the Capacity Analysis of Communication Systems," Technical Report.

J43. R. Couillet, S. Medina Perlaza, H. Tembine, M. Debbah, "Electrical Vehicles in the Smart Grid: A Mean Field Game Analysis," IEEE Journal on Selected Areas in Communications : Smart Grid Communications Series, vol. 30, no. 6, pp. 1086-1096, 2012.

J44. J. Yao, R. Couillet, J. Najim, M. Debbah, "Fluctuations of an Improved Population Eigenvalue Estimator in Sample Covariance Matrix Models," IEEE Transactions on Information Theory, vol. 59, no. 2, pp. 1149-1163, 2013.

J45. R. Couillet, M. Debbah, "Signal Processing in Large Systems: a New Paradigm," IEEE Signal Processing Magazine, vol. 30, no. 1, pp. 24-39, 2013.

J46. R. Couillet, W. Hachem, "Fluctuations of spiked random matrix models and failure diagnosis in sensor networks," IEEE Transactions on Information Theory, vol. 59, no. 1, pp. 509-525, 2013.

J47. A. Kammoun, R. Couillet, J. Najim, M. Debbah, "Performance of capacity inference methods under colored interference," IEEE Transactions on Information Theory, vol. 59, no. 2, pp. 1129-1148, 2013.

J48. R. Couillet, J. Hoydis, M. Debbah, "Random beamforming over quasi-static and fading channels: A deterministic equivalent approach," IEEE Transactions on Information Theory, vol. 58, no. 10, pp. 6392-6425, 2012.

J49. S. Wagner, R. Couillet, M. Debbah, D. T. M. Slock, "Large System Analysis of Linear Precoding in MISO Broadcast Channels with Limited Feedback", IEEE Transactions on Information Theory, vol. 58, no. 7, pp. 4509-4537, 2012.

J50. R. Couillet, J. W. Silverstein, Z. Bai, M. Debbah, "Eigen-Inference for Energy Estimation of Multiple Sources", IEEE Transactions on Information Theory, vol. 57, no. 4, pp. 2420-2439, 2011.

J51. R. Couillet, M. Debbah, J. W. Silverstein, "A Deterministic Equivalent for the Analysis of Correlated MIMO Multiple Access Channels", IEEE Transactions on Information Theory, vol. 57, no. 6, pp. 3493-3514, 2011.

J52. R. Couillet, M. Debbah, "A Bayesian Framework for Collaborative Multi-Source Signal Sensing", IEEE Transactions on Signal Processing, vol. 58, no. 10, pp. 5186-5195, 2010.

J53. R. Couillet, A. Ancora, M. Debbah, "Bayesian Foundations of Channel Estimation for Cognitive Radios", Advances in Electronics and Telecommunications, vol. 1, no. 1, pp. 41-49, 2010.

J54. R. Couillet, M. Debbah, "Le téléphone du futur : plus intelligent pour une exploitation optimale des fréquences" Revue de l'Electricité et de l'Electronique, no. 6, pp. 71-83, 2010.

J55. R. Couillet, M. Debbah, "Mathematical foundations of cognitive radios", Journal of Telecommunications and Information Technologies, no. 4, 2009.

J56. R. Couillet, M. Debbah, "Outage performance of flexible OFDM schemes in packet-switched transmissions", Eurasip Journal on Advances on Signal Processing, Volume 2009, Article ID 698417, 2009.

B2. R. Couillet, M. Debbah, **Mathematical Foundations for Signal Processing, Communications and Networking**, CRC Press, Taylor & Francis Group, 2011 [chapitre de livre]

Chapitre "Random matrix theory" sur les matrices aléatoires

et plus précisément les méthodes d'inférence statistique.

B3. R. Couillet, M. Debbah, **Orthogonal Frequency Division Multiple Access Fundamentals and Applications**, Auerbach Publications, CRC Press, Taylor & Francis Group, 2010 [chapitre de livre]

Chapitre "Fundamentals of OFDMA Synchronization" sur les considérations théoriques et les outils appliqués en synchronisation pour l'OFDM et l'OFDMA.

- R. Couillet, M. Debbah, **No. 08368023.1** "Method for short-time OFDM transmission and apparatus for performing flexible OFDM modulation"
- R. Couillet, S. Wagner, **No. 09368025.4** "Precoding process for a transmitter of a MU-MIMO communication system"
- R. Couillet, **No. 09368030.4** "Process for estimating the channel in an OFDM communication system, and receiver for doing the same"

II1. R. Couillet, **Idée Innovante** "User Subspace Clustering"

TUTORIAUX

T1. R. Couillet, M. Debbah, "Eigen-Inference Statistical methods for Cognitive Radio", European Wireless, Lucca, Italy, 2010.

Présentation Analytique des Travaux

Domaines de recherche

Mathématiques appliquées :

- *théorie des matrices aléatoires* : modèles hermitiens, modèles spiked, modèles non standard à entrées non indépendantes ou non linéaires, matrices à noyaux, etc.
- *mathématiques* : probabilités, analyse complexe, algèbre
- *statistique* : estimation robuste, méthodes à noyau, analyse en composantes principales, etc.

Graphes et apprentissage automatisé :

- *graphes* : méthodes spectrales de détection de communauté
- *réseaux de neurones* : réseaux "echo-state", "extreme learning machines" et "random feature maps", réseaux profonds, rétropropagation du gradient
- *apprentissage automatisé* : méthodes spectrales à noyaux (algorithmes type Ng–Weiss–Jordan), support vector machines, méthodes semi-supervisées

Traitement du signal :

- *traitement d'antennes* : détection et estimation, méthodes sous-espaces, estimation et régression robustes
- *réseaux de capteurs* : détection de changements d'état, algorithmes distribués

Télécommunications mobiles :

- *traitement des signaux et données* : inférence statistique pour les réseaux cognitifs, classification non supervisée d'utilisateurs
- *théorie de l'information* : performances de systèmes MIMO larges, communications MIMO à paquets courts
- *communications mobiles* :

design de précodeurs massive MIMO, multi-cellulaires, systèmes réalistes

Finances statistiques :

- *optimisation de porte-feuilles* : problème de Markowtiz et ses dérivés en grandes dimensions
- *inférence de séries temporelles financières* : modèles de réseaux de neurones récurrents (echo-state) pour les données financières
- *apprentissage et séries financières* : méthodes de classification et regroupement d'actifs par méthodes à noyaux modernes.

Mobile communications :

- *information theory* : large MIMO system performance, finite blocklength communications

Résumé des activités de recherche

Mes activités de recherche sont centrées sur l'analyse théorique des performances et l'amélioration d'algorithmes et méthodes pour les systèmes de grandes dimensions. L'outil au cœur de ma recherche est la théorie des matrices aléatoires, et plus généralement les probabilités, l'analyse complexe et l'algèbre pour des vecteurs et matrices de grandes dimensions.

Chronologiquement, mon intérêt s'est d'abord porté sur l'analyse des performances de systèmes complexes de communications mobiles (de 2007 à 2012 principalement). Parmi mes travaux notables, (J ?(?) ?51) établit pour la première fois la région de capacité ergodique d'un système multi-utilisateur MIMO en voie montante, grâce à des résultats

nouveaux en théorie des matrices aléatoires. Les performances de précodeurs linéaires de ce même système mais en voie descendante ont été ensuite étudiées finement et sous des hypothèses réalistes dans (J ?(?) ?49), qui généralise à cette occasion de nombreux travaux. D'autres travaux, techniquement plus poussés, étendent ces résultats, notamment dans (J ?(?) ?48,J ?(?) ?42). Un travail bien plus récent et techniquement plus ardu (J ?(?) ?37) analyse les probabilités d'erreurs asymptotiques d'un système MIMO dans le cas de communications à paquets courts. Sur la période 2010–2013, l'intérêt de mes recherches s'est déplacé vers le traitement du signal pour les communications mobiles, avec en particulier des articles sur l'estimation rapide de débits et positions d'utilisateurs dans un contexte de radio intelligente (J ?(?) ?52,J ?(?) ?50,. Tous ces travaux, effectués dans le cadre de ma thèse et au-delà, ont notamment été menés en collaboration avec Jack Silverstein et Zhidong Bai, professeurs en mathématiques et experts en théorie des matrices aléatoires.

Suite à cette première vague de travaux, restreints mathématiquement à l'analyse de fonctionnelles du spectre de matrices de Gram, j'ai déplacé mes centres d'intérêts vers l'analyse statistique de grands systèmes pour des applications au traitement du signal, et notamment au traitement d'antennes. Ici on s'intéresse, non plus à des fonctionnelles de spectres, mais à des couples valeurs-vecteurs

propres isolés dans le spectre (modèles dits spiked). Mes travaux dans cette direction ont permis d'établir des résultats mathématiquement plus généraux pour des modèles en traitement d'antennes plus réalistes, comme en particulier les travaux (J ?(?) ?38,J ?(?) ?3 sur les modèles avec corrélation spatiale et temporelle ou encore (J ?(?) ?46) dans un contexte de réseau de capteurs.

Néanmoins, toutes ces études, aussi utiles et techniquement poussées soient-elles, venaient naturellement à la suite d'études préalables. Il m'a alors tenu à cœur d'attaquer des problèmes nouveaux qui requièrent une plus grande ouverture d'esprit. C'est ainsi que j'ai engagé, sur une période de trois ans, l'étude purement statistique d'estimateurs robustes dans le régime des grands systèmes. Les objets et outils mathématiques impliqués dans ce travail diffèrent fortement des techniques de matrices aléatoires classiques qui ne sont pas adaptables ici. Parmi les résultats marquants, les articles successifs (J ?(?) ?40,J ?(?) ?34,J ?(? ont permis d'établir une compréhension fine du comportement de ces estimateurs et d'en déduire une méthode générique pour une utilisation améliorée dans les problèmes de grandes tailles ; des applications spécifiques au traitement d'antennes (J ?(?) ?31) ou encore à l'analyse robuste de données financières (J ?(?) ?28) en ont découlé. Une grande partie de ces travaux a été menée conjointement avec Matthew McKay et Frédéric

Pascal, professeurs spécialisés en traitement du signal. L'ensemble des travaux mentionnés jusqu'ici a donné lieu à l'écriture du livre (B**??**), à l'obtention des brevets (P**??**–P ?(?) ? ?) et a été récompensé du prix IEEE Outstanding Young Researcher Award et de la médaille de Bronze du CNRS en 2013. Ces résultats ont également constitué le cœur de ma thèse d'HDR avec pour rapporteurs les professeurs Leonid Pastur, mathématicien expert en matrices aléatoires, ainsi que les professeurs Jean-Yves Tourneret et Alfred Hero tout deux experts mondiaux en traitement des données et signaux et en statistique.

Aujourd'hui, l'intérêt croissant porté par les méthodes d'apprentissage automatisé pour le traitement des grandes données (BigData) motive l'analyse et l'amélioration de ces méthodes dans le régime des grandes matrices aléatoires. Dans le cadre du projet ANR Jeunes Chercheurs (ANR RMT4GRAPH) que je dirige, nous étudions aujourd'hui au sein de mon équipe (composée d'une dizaine de doctorants, post-doctorants et stagiaires) les algorithmes non linéaires (à base de noyaux) de classification supervisée, semi-supervisée, ou non supervisée, les réseaux de neurones aléatoires non linéaires, récurrents ou non, profonds ou non, ainsi que l'inférence sur graphes. Nos résultats depuis 2014 sont extrêmement prometteurs et permettent en particulier une meilleure compréhension des méthodes

de classification non supervisée à noyaux (J ?(?) ?17,J ?(?) ?18), de classification semi-supervisée (J ?(?) ?7,J ?(?) ?5), à l'apprentissage par transfert (C**??**), des approches apparentées au SVM (J ?(?) ?13), des méthodes de détection de communauté sur graphes (J ?(?) ?14) utilisant en particulier des outils puissants de la physique statistique (C**??**), mais également permettent des avancées nouvelles dans l'étude des réseaux de neurones grâce au développement d'outils novateurs dans ce domaine (J ?(?) ?12). De manière plus fondamentale, la plupart de ces résultats sont démontrés valables pour des modèles de données et algorithmes extrêmement réalistes, notamment grâce à l'idée novatrice d'une extension de la théorie des matrices aléatoires à l'aide de l'outil de la concentration de la mesure (J ?(?) ?4,C**??**,C**??**). Ces travaux ont par ailleurs donné lieu à des idées innovantes pratiques développées au cours de projets industriels (I ?(?) ?1).
L'objectif à moyen et long termes de ces études est de développer de nouvelles méthodes d'apprentissage mieux adaptées aux grandes données ainsi qu'un nouveau paradigme d'analyse et d'amélioration des performances de ces méthodes au moyen de la théorie des grandes matrices aléatoires.

Une description détaillée de tous les articles publiés depuis 2007 est présentée ci-après.

Articles avec comité de relecture

J1. C. Louart, R. Couillet "A Concentration of Mea-

sure Approach to Large Dimensional Robust Statistics", (submitted to) Annals of Applied Probability, 2020.

Abstract. This article studies the robust covariance matrix estimation of a data collection $X = (x_1, ..., x_n)$ with $x_i = \tau_i z_i + m$, where $z_i \in \mathbb{R}^p$ is a concentrated vector (e.g., an elliptical random vector), $m \in \mathbb{R}^p$ a deterministic signal and $\tau_i \in \mathbb{R}$ a scalar perturbation of possibly large amplitude, under the assumption where both n and p are large. This estimator is defined as the fixed point of a function which we show is contracting for a so-called stable semi-metric. We exploit this semi-metric along with concentration of measure arguments to prove the existence and uniqueness of the robust estimator as well as evaluate its limiting spectral distribution.

J2. K. Elkhalil, A. Kammoun, R. Couillet, T. Al-Naffouri, M-S. Alouini, "A Large Dimensional Analysis of Regularized Discriminant Analysis Classifiers" (to appear) IEEE Transactions on Signal Processing, 2020.

Abstract. Abstract—In this paper, we conduct a large dimensional study of regularized discriminant analysis classifiers with its two popular variants known as regularized LDA and re-

gularized QDA. The analysis is based on the assumption that the data samples are drawn from a Gaussian mixture model with different means and covariances and relies on tools from random matrix theory (RMT). We consider the regime in which both the data dimension and training size within each class tends to infinity with fixed ratio. Under mild assumptions, we show that the probability of misclassification converges to a deterministic quantity that describes in closed form the performance of these classifiers in terms of the class statistics as well as the problem dimension. The result allows for a better understanding of the underlying classification algorithms in terms of their performances in practical large but finite dimensions. Further exploitation of the results permits to optimally tune the regularization parameter with the aim of minimizing the probability of misclassification. The analysis is validated with numerical results involving synthetic as well as real data from the USPS dataset yielding a high accuracy in predicting the performances and hence making an interesting connection between theory and practice.

J3. L. Dall'Amico, R. Couillet, N. Tremblay, "A uni-

fied framework for spectral clustering in sparse graphs", (submitted to) Journal of Machine Learning Research, 2020.

Abstract. This article considers spectral community detection in the regime of sparse networks with heterogeneous degree distributions, for which we devise an algorithm to efficiently retrieve communities. Specifically, we demonstrate that a conveniently parametrized form of regularized Laplacian matrix can be used to perform spectral clustering in sparse networks, without suffering from its degree heterogeneity. Besides, we exhibit important connections between this proposed matrix and the now popular non-backtracking matrix, the Bethe-Hessian matrix, as well as the standard Laplacian matrix. Interestingly, as opposed to competitive methods, our proposed improved parametrization inherently accounts for the hardness of the classification problem. These findings are summarized under the form of an algorithm capable of both estimating the number of communities and achieving high-quality community reconstruction.

J4. C. Louart, R. Couillet, "Concentration of Measure and Large Random Matrices with an application to Sample Covariance Matrices",

(submitted to) Random Matrix Theory and Applications, 2019.

Abstract. The present work provides an original framework for random matrix analysis based on revisiting the concentration of measure theory from a probabilistic point of view. By providing various notions of vector concentration (q-exponential, linear, Lipschitz, convex), a set of elementary tools is laid out that allows for the immediate extension of classical results from random matrix theory involving random concentrated vectors in place of vectors with independent entries. These findings are exemplified here in the context of sample covariance matrices but find a large range of applications in statistical learning and beyond, thanks to the broad adaptability of our hypotheses.

J5. X. Mai, R. Couillet, "Consistent Semi-Supervised Graph Regularization for High Dimensional Data", (submitted) Journal of Machine Learning Research, 2019.

Abstract. Semi-supervised Laplacian regularization, a standard graph-based approach for learning from both labelled and unlabelled data, is demonstrated by the recent work of (Mai and Couillet, 2017) to have an insignificant high dimensional learning efficiency with respect to unla-

belled data, causing it to be outperformed by its unsupervised counterpart, spectral clustering, given sufficient unlabelled data. Following a detailed discussion on the origin of this inconsistency problem, a novel regularization approach is proposed as solution, which is shown both theoretically and empirically to have a superior performance over Laplacian regularization.

J6. R. Couillet, M. Tiomoko, S. Zozor, E. Moisan, "Random matrix-improved estimation of covariance matrix distances", Journal of Multivariate Analysis, vol. 174, pp. 104531, 2019.

Abstract. Given two sets $x_1^{(1)}, \ldots, x_{n_1}^{(1)}$ and $x_1^{(2)}, \ldots, x_{n_2}^{(2)} \in \mathbb{R}^p$ (or $\mathbb{C}^p$) of random vectors with zero mean and positive definite covariance matrices C_1 and $C_2 \in \mathbb{R}^{p \times p}$ (or $\mathbb{C}^{p \times p}$), respectively, this article provides novel estimators for a wide range of distances between C_1 and C_2 (along with divergences between some zero mean and covariance C_1 or C_2 probability measures) of the form $\frac{1}{p} \sum_{i=1}^{n} f(\lambda_i(C_1^{-1} C_2))$ (with $\lambda_i(X)$ the eigenvalues of matrix X). These estimators are derived using recent advances in the field of random matrix theory and are asymptotically consistent as $n_1, n_2, p \to \infty$ with non trivial ratios $p/n_1 < 1$ and $p/n_2 < 1$ (the case $p/n_2 > 1$

is also discussed). A first "generic" estimator, valid for a large set of f functions, is provided under the form of a complex integral. Then, for a selected set of f's of practical interest (namely, $f(t) = t$, $f(t) = \log(t)$, $f(t) = \log(1 + st)$ and $f(t) = \log^2(t)$), a closed-form expression is provided. Beside theoretical findings, simulation results suggest an outstanding performance advantage for the proposed estimators when compared to the classical "plug-in" estimator $\frac{1}{p}\sum_{i=1}^{n} f(\lambda_i(\hat{C}_1^{-1}\hat{C}_2))$ (with $\hat{C}_a = \frac{1}{n_a}\sum_{i=1}^{n_a} x_i^{(a)} x_i^{(a)\mathsf{T}}$), and this even for very small values of n_1, n_2, p.

J7. X. Mai, R. Couillet, "A Random Matrix Analysis and Improvement of Semi-Supervised Learning for Large Dimensional Data", Journal of Machine Learning Research, vol. 19, no. 79, pp. 1-27, 2018.

Abstract. This article provides an original understanding of the behavior of a class of graph-oriented semi-supervised learning algorithms in the limit of large and numerous data. It is demonstrated that the intuition at the root of these methods collapses in this limit and that, as a result, most of them become inconsistent. Corrective measures and a new data-driven parametrization scheme are proposed along with a theoretical ana-

lysis of the asymptotic performances of the resulting approach. A surprisingly close behavior between theoretical performances on Gaussian mixture models and on real datasets is also illustrated throughout the article, thereby suggesting the importance of the proposed analysis for dealing with practical data. As a result, significant performance gains are observed on practical data classification using the proposed parametrization.

J8. A. Kammoun, R. Couillet, "Subspace Kernel Clustering of Large Dimensional Data" (submitted to) Annals of Applied Probability, 2017.

Abstract. Let $x_1, \ldots, x_n$ be independent observations of size p, each of them belonging to one of c distinct classes. We assume that observations within class a are characterized by their distribution $\mathcal{N}(0, \frac{1}{p}C_a)$ where here $C_1, \ldots, C_c$ are some non-negative definite $p \times p$ matrices. This paper studies the asymptotic behavior of the symmetric matrix $\tilde{\Phi}_{kl} = \sqrt{p}((x_k^T x_l)^2 \delta_{k \neq l})$ when p and n grow to infinity with $n/p \to c_0$. Particularly, we prove that, if the class covariance matrices are sufficiently close in a certain sense, the matrix $\tilde{\Phi}$ behaves as a low-rank perturbation of a Wigner matrix, presenting possibly some iso-

lated eigenvalues outside the bulk of the semicircular law. We carry out a careful analysis of some of the isolated eigenvalues and eigenvectors of matrix Φ, and illustrate how these results can help understand spectral clustering methods that use $\tilde{\Phi}$ as a kernel matrix.

J9. L. Yang, M. McKay, R. Couillet, "High-Dimensional MVDR Beamforming: Optimized Solutions based on Spiked Random Matrix Models", IEEE Transactions on Signal Processing, vol. 66, no. 1, pp. 1933-1947, 2018.

Abstract. Minimum variance distortionless response (MVDR) beamforming (or Capon beamforming) is among the most popular adaptive array processing strategies due to its ability to provide noise resilience while nulling out interferers. A practical challenge with this beamformer is that it involves the inverse covariance matrix of the received signals, which must be estimated from data. Under modern high-dimensional applications, it is well-known that classical estimators can be severely affected by sampling noise, which compromises beamformer performance. Here we propose a new approach to MVDR beamforming which is suited to high-dimensional settings. In particular, by drawing an analogy with the MVDR pro-

blem and the so-called "spiked models" in random matrix theory, we propose robust beamforming solutions which are shown to outperform classical approaches (e.g., matched filters and sample matrix inversion techniques), as well as more robust solutions, such as methods based on diagonal loading. The key to our method is the design of an optimized inverse covariance estimator which applies eigenvalue clipping and shrinkage functions that are tailored to the MVDR application. Our proposed MVDR solution is simple, in closed form, and easy to implement.

J10. A. Karadimitrakis, A. L. Moustakas, R. Couillet, "Gallager Bound for MIMO Channels: Large-N Asymptotics" IEEE Transactions on Wireless Communications, vol. 17, no. 2, pp. 1323-1330, 2018.

Abstract. The use of multiple antenna arrays in transmission and reception has become an integral part of modern wireless communications. To quantify the performance of such systems, the evaluation of bounds on the error probability of realistic finite length codewords is important. In this paper, we analyze the standard Gallager error bound for both constraints of maximum average power and maximum instantaneous power. Ap-

plying techniques from random matrix theory, we obtain analytic expressions of the error exponent when the length of the codeword increases to infinity at a fixed ratio with the antenna array dimensions. Analyzing its behavior at rates close to the ergodic rate, we find that the Gallager error bound becomes asymptotically close to an upper error bound obtained recently by Hoydis et al. 2015. We also obtain an expression for the Gallager exponent in the case when the codelength spans several Rayleigh fading blocks, hence taking into account the situation when the channel varies during each transmission.

J11. N. Auguin, D. Morales, M. McKay, R. Couillet, "Large-dimensional behavior of regularized Maronna's M-estimators of covariance matrices" IEEE Transactions on Signal Processing, vol. 66, no. 13, pp. 3529–3542, 2018.

Abstract. Robust estimators of large covariance matrices are considered, comprising regularized (linear shrinkage) modifications of Maronna's classical M-estimators. These estimators provide robustness to outliers, while simultaneously being well-defined when the number of samples does not exceed the number of variables. By applying

tools from random matrix theory, we characterize the asymptotic performance of such estimators when the numbers of samples and variables grow large together. In particular, our results show that, when outliers are absent, many estimators of the regularized-Maronna type share the same asymptotic performance, and for these estimators we present a data-driven method for choosing the asymptotically optimal regularization parameter with respect to a quadratic loss. Robustness in the presence of outliers is then studied : in the non-regularized case, a large-dimensional robustness metric is proposed, and explicitly computed for two particular types of estimators, exhibiting interesting differences depending on the underlying contamination model. The impact of outliers in regularized estimators is then studied, with remarkable differences with respect to the non-regularized case, leading to new practical insights on the choice of particular estimators.

J12. C. Louart, Z. Liao, R. Couillet, "A Random Matrix Approach to Neural Networks" Annals of Applied Probability, vol. 28, no. 2, pp. 1190–1248, 2018.

Abstract. This article studies the Gram random matrix model $G =$

$\frac{1}{T}\Sigma^{\mathrm{T}}\Sigma$, $\Sigma = \sigma(WX)$, classically found in the analysis of random feature maps and random neural networks, where $X = [x_1, \ldots, x_T] \in \mathbb{R}^{p\times T}$ is a (data) matrix of bounded norm, $W \in \mathbb{R}^{n\times p}$ is a matrix of independent zero-mean unit variance entries, and $\sigma : \mathbb{R} \to \mathbb{R}$ is a Lipschitz continuous (activation) function — $\sigma(WX)$ being understood entry-wise. We prove that, as n, p, T grow large at the same rate, the resolvent $Q = (G+\gamma I_T)^{-1}$, for $\gamma > 0$, has a similar behavior as that met in sample covariance matrix models, involving notably the moment $\Phi = \frac{T}{n}\mathrm{E}[G]$, which provides in passing a deterministic equivalent for the empirical spectral measure of G. This result, established by means of concentration of measure arguments, enables the estimation of the asymptotic performance of single-layer random neural networks. This in turn provides practical insights into the underlying mechanisms into play in random neural networks, entailing several unexpected consequences, as well as a fast practical means to tune the network hyperparameters.

J13. Z. Liao, R. Couillet, "A Large Dimensional Analysis of Least Square Support Vector Machines" IEEE Transactions on Signal Processing,

vol. 67, no. 4, pp. 1065-1074, 2018.

Abstract. In this article, a large dimensional performance analysis of kernel least squares support vector machines (LS-SVMs) is provided under the assumption of a two-class Gaussian mixture model for the input data. Building upon recent random matrix advances, when both the dimension of data p and their number n grow large at the same rate, we show that the LS-SVM decision function converges to a normal-distributed variable, the mean and variance of which depend explicitly on a local behavior of the kernel function. This theoretical result is then applied to real data sets which, despite their non-Gaussianity, exhibit a surprisingly similar behavior. Our analysis provides a deeper understanding of the mechanism into play in SVM-type methods and in particular of the impact on the choice of the kernel function as well as some of their theoretical limits.

J14. R. Couillet, H. Tiomoko Ali, "Improved spectral community detection in large heterogeneous networks" Journal of Machine Learning Resarch, vol. 18, no. 225, pp. 1–49, 2018.

Abstract. In this article, we study spectral methods for community detection based on α-parametrized normali-

zed modularity matrix hereafter called L_α in heterogeneous graph models. We show, in a regime where community detection is not asymptotically trivial, that L_α can be well approximated by a more tractable random matrix which falls in the family of spiked random matrices. The analysis of this equivalent spiked random matrix allows us to improve spectral methods for community detection and assess their performances in the regime under study. In particular, we prove the existence of an optimal value α_{opt} of the parameter α for which the detection of communities is best ensured and we provide an on-line estimation of α_{opt} only based on the knowledge of the graph adjacency matrix. Unlike classical spectral methods for community detection where clustering is performed on the eigenvectors associated with extreme eigenvalues, we show through our theoretical analysis that a regularization should instead be performed on those eigenvectors prior to clustering in heterogeneous graphs. Finally, through a deeper study of the regularized eigenvectors used for clustering, we assess the performances of our new algorithm for community detection. Numerical simulations in the course of the article show that

our methods outperform state-of-the-art spectral methods on dense heterogeneous graphs.

J15. R. Couillet, M. McKay, "Optimal block-sparse PCA for high dimensional correlated samples" (submitted to) Journal of Multivariate Analysis, 2016.
Abstract. A new principal component analysis (PCA) method is proposed which is performed on a subset of blocks of consecutive entries of the population data vectors. This block-based dimensionality reduction introduces a trade-off by which the accuracy of the dominant eigenvector of the dimension-reduced sample covariance matrix is enhanced while some population entries are discarded. This scheme is particularly suited (but not restricted) to population eigenvectors with localized energy and rather sparse structures. Unlike many sparse PCA algorithms, the originality of our scheme lies in its providing an online selection of the subset of blocks which, in the large dimensional regime where both population and sample sizes grow large, provably ensures optimal alignment between population and sample eigenvectors. Moreover, our method inherently handles (a priori unknown) linear correlation between sample data.

J16. R. Couillet, G. Wain-

rib, H. Sevi, H. Tiomoko Ali, "The asymptotic performance of linear echo state neural networks" Journal of Machine Learning Research, vol. 17, no. 178, pp. 1-35, 2016.

Abstract. In this article, a study of the mean-square error (MSE) performance of linear echo-state neural networks is performed, both for training and testing tasks. Considering the realistic setting of noise present at the network nodes, we derive deterministic equivalents for the aforementioned MSE in the limit where the number of input data T and network size n both grow large. Specializing then the network connectivity matrix to specific random settings, we further obtain simple formulas that provide new insights on the performance of such networks.

J17. R. Couillet, F. Benaych-Georges, "Kernel Spectral Clustering of Large Dimensional Data" Electronic Journal of Statistics, vol. 10, no. 1, pp. 1393-1454, 2016.

Abstract. This article proposes a first analysis of kernel spectral clustering methods in the regime where the dimension p of the data vectors to be clustered and their number n grow large at the same rate. We demonstrate, under a k-class Gaussian mixture model, that the normalized Laplacian matrix associated

with the kernel matrix asymptotically behaves similar to a so-called spiked random matrix. Some of the isolated eigenvalue-eigenvector pairs in this model are shown to carry the clustering information upon a separability condition classical in spiked matrix models. We evaluate precisely the position of these eigenvalues and the content of the eigenvectors, which unveil important properties concerning spectral clustering, in particular in simple toy models. Our results are then compared to the practical clustering of images from the MNIST database, thereby revealing an important match between theory and practice.

J18. F. Benaych-Georges, R. Couillet, "Spectral Analysis of the Gram Matrix of Mixture Models" ESAIM : Probability and Statistics, DOI http ://dx.doi.org/10.1051/ps/2016007 2016.

Abstract. This text is devoted to the asymptotic study of some spectral properties of the Gram matrix $W^{\mathsf{T}}W$ built upon a collection $w_1, \ldots, w_n \in \mathbb{R}^p$ of random vectors (the columns of W), as both the number n of observations and the dimension p of the observations tend to infinity and are of similar order of magnitude. The random vectors $w_1, \ldots, w_n$ are independent observations, each of them

belonging to one of k classes $\mathcal{C}_1, \ldots, \mathcal{C}_k$. The observations of each class $\mathcal{C}_a$ $(1 \leq a \leq k)$ are characterized by their distribution $\mathcal{N}(0, p^{-1}C_a)$, where $C_1, \ldots, C_k$ are some non negative definite $p \times p$ matrices. The cardinality n_a of class $\mathcal{C}_a$ and the dimension p of the observations are such that n_a/n $(1 \leq a \leq k)$ and p/n stay bounded away from 0 and $+\infty$. We provide deterministic equivalents to the empirical spectral distribution of $W^\mathsf{T}W$ and to the matrix entries of its resolvent (as well as of the resolvent of WW^T). These deterministic equivalents are defined thanks to the solutions of a fixed-point system. Besides, we prove that $W^\mathsf{T}W$ has asymptotically no eigenvalues outside the bulk of its spectrum, defined thanks to these deterministic equivalents. These results are directly used in the companion paper (?(?) ?17), which is devoted to the analysis of the spectral clustering algorithm in large dimensions. They also find applications in various other fields such as wireless communications where functionals of the aforementioned resolvents allow one to assess the communication performance across multi-user multi-antenna channels.

J19. R. Couillet, Estimation robuste et matrices

aléatoires, revue Traitement du Signal, vol. 33, no. 2-3, pp. 273-320, 2016.

Abstract. This article provides a technical survey of the recent advances between the fields of robust estimation of scatter and of large dimensional random matrix theory. An exposition of the theoretical results will be made which we shall apply to various contexts in the area of statistics and signal processing at large. The theoretical results essentially show that, while robust estimators of scatter are implicitly defined and thus difficult objects to manipulate, in the large dimensional random matrix regime where both the population size and the number of samples are simultaneously large, these implicit robust estimators tend to behave similar to much simpler random matrix models, amenable to analysis. This induces that many statistical properties of these estimators could be unearthed which we shall discuss. In terms of applications, these robust estimators of scatter are long-standing structural elements to handle both outliers and heavy-tailed behavior in the observed data. These impulsiveness harnessing effects will be precisely documented and shall be instrumental to develop improved robust statistics methods

for detection and estimation in antenna arrays, portfolio optimization, etc.

J20. R. Couillet, G. Wainrib, Perspectives en matrices aléatoires et grands réseaux, revue Traitement du Signal, vol. 33, no. 2-3, pp. 351-376, 2016.

Abstract. In this article, several research perspectives in random matrix theory applied to graph theory at large are discussed. Specific focus will be made on the spectrum analysis of the adjacency or Laplacian matrices of large dimensional graphs for community detection in networks, of kernel random matrices for clustering in large datasets, along with applications to neural networks.

J21. M. Sadeghi, L. Sanguinetti, R. Couillet, Y. Chau, "Large System Analysis of Power Normalization Techniques in Massive MIMO", IEEE Transactions on Vehicular Technologies, vol. 66, no. 10, pp. 9005-9017, 2017.

Abstract. Linear precoding has been widely studied in the context of Massive MIMO together with the two common power normalization techniques, namely, matrix normalization (MN) and vector normalization (VN). However, the effect of both on the system performance has not been thoroughly studied. The

aim of this paper is to address this problem using large system analysis. Considering a system model that accounts for channel estimation, pilot contamination, arbitrary pathloss, and per-user channel correlation, we compute tight approximations for the signal-to-interference-plus-noise ratio (SINR) and the rate of each user equipment (UE) in the system while employing maximum ratio transmission (MRT), zero forcing (ZF), and regularized ZF (RZF) precoding under both MN and VN techniques. Exploiting such results, we reveal the effect of power normalization on the performance of MRT and ZF, and determine how it affects noise, interference, pilot contamination, and signal powers of any arbitrary UE. We show that the power normalization can convey a notion of fairness or sum rate maximization for ZF. Numerical results are used to validate the accuracy of the asymptotic analysis and to show that in Massive MIMO, non- coherent interference and noise, rather than pilot contamination, are often the major limiting factors of the considered precoding schemes. .

J22. M. Sadeghi, L. Sanguinetti, R. Couillet, Y. Chau, "Reducing the Computational Complexity of Multicasting

in Large-Scale Antenna Systems", IEEE Transactions on Wireless Communications, vol. 16, no. 5, pp. 2963-2975, 2017.

Abstract. In this paper, we study the physical layer multicasting to multiple co-channel groups in large-scale antenna systems. The users within each group are interested in a common message and different groups have distinct messages. In particular, we aim at designing the precoding vectors solving the so-called quality of service (QoS) and weighted max-min fairness (MMF) problems, assuming that the channel state information is available at the base station (BS). To solve both problems, the baseline approach exploits the semidefinite relaxation (SDR) technique. Considering a BS with N antennas, the SDR complexity is more than $O(N^6)$, which prevents its application in large-scale antenna systems. To overcome this issue, we present two new classes of algorithms that, not only have significantly lower computational complexity than existing solutions, but also largely outperform the SDR based methods. Moreover, we present a novel duality between transformed versions of the QoS and the weighted MMF problems. The duality explicitly determines the solution to the weighted MMF

problem given the solution to the QoS problem, and vice versa. Numerical results are used to validate the effectiveness of the proposed solutions and to make comparisons with existing alternatives under different operating conditions.

J23. L. Sanguinetti, R. Couillet, M. Debbah, "Large System Analysis of Base Station Cooperation for Power Minimization" IEEE Transactions on Wireless Communications, vol. 15, no. 8, pp. 5480-5496, 2016.

Abstract. Abstract This work focuses on a large-scale multi-cell multi-user MIMO system in which L base stations (BSs) of N antennas each communicate with K single-antenna user equipments. We consider the design of the linear precoder that minimizes the total power consumption while ensuring target user rates. Three configurations with different degrees of cooperation among BSs are considered : the single cell processing scheme (no cooperation between BSs), the coordinated beamforming scheme (only channel state information is shared between BSs) and the coordinated multipoint MIMO processing technology (channel state and data cooperation). The analysis is conducted assuming that N and K grow large with a non trivial ratio K/N and

imperfect channel state information is available at the BSs. Tools of random matrix theory are used to compute, in explicit form, deterministic approximations for : (i) the parameters of the optimal precoder ; (ii) the powers needed to ensure target rates ; and (iii) the total transmit power. These results are instrumental to get further insight into the structure of the optimal precoders and also to reduce the complexity of its implementation in large-scale networks. Numerical results are used to validate the asymptotic analysis in the finite system regime and to make comparisons among the different configurations.

J24. A. Abboud, F. Iutzeler, R. Couillet, H. Siguerdidjane, M. Debbah, "Distributed Production-Sharing Optimization and Application to Power Grid Networks," IEEE Transactions on Signal and Information Processing over Networks, vol. 2, no. 1, pp. 1628, 2016.

Abstract. Based on recent works on asynchronous versions of the distributed Alternating Direction Method of Multipliers (ADMM) algorithm, we develop and prove the convergence of a distributed asynchronous method for Production-Sharing Problems over networks. The asynchronous na-

ture of the algorithm allows both for the relaxation of the synchronization constraint often inherent to distributed ADMM-based methods and distributed optimization methods at large, but also allows for random local failures to occur in fully centralized methods. These two considerations motivate the application of the method to the Direct-Current Optimal Power Flow (DC-OPF) problem in power transmission networks. Applied to the DC-OPF, this method leads to an overall network optimal production obtained through a sequence of local computations in subareas of the network (each area waking up randomly while the rest of the network is non-operational) and neighboring data exchanges. In another scenario, the DC-OPF is performed via iterations of a centralized network-wide ADMM method which may contain disconnected nodes (in general with low probability and for a short duration). In both cases, this method still converges and thus provides additional flexibility to classical DC-OPF algorithms. The proposed algorithm, inherently designed for networks of overlapping subareas, is then extended to networks of non-overlapping areas. Simulations are carried out on the IEEE-30 and IEEE-118 bus test sys-

tems which illustrate the convergence, scalability and effectiveness of the proposed algorithms.

J25. A. Kammoun, R. Couillet, F. Pascal, M.-S. Alouini, "Optimal Design of the Adaptive Normalized Matched Filter Detector using Regularized Tyler Estimator" IEEE Transactions on Aerospace and Electronic Systems, vol. 54, no. 2, pp. 755–769, 2018.

Abstract. This article addresses improvements on the design of the adaptive normalized matched filter (ANMF) for radar detection. It is well-acknowledged that the estimation of the noise-clutter covariance matrix is a fundamental step in adaptive radar detection. In this paper, we consider regularized estimation methods which force by construction the eigenvalues of the scatter estimates to be greater than a positive regularization parameter ρ. This makes them more suitable for high dimensional problems with a limited number of secondary data samples than traditional sample covariance estimates. While an increase of ρ seems to improve the conditioning of the estimate, it might however cause it to significantly deviate from the true covariance matrix. The setting of the optimal regularization parameter is a difficult ques-

tion for which no convincing answers have thus far been provided. This constitutes the major motivation behind our work. More specifically, we consider the design of the ANMF detector for two kinds of regularized estimators, namely the regularized sample covariance matrix (RSCM), appropriate when the clutter follows a Gaussian distribution and the regularized Tyler estimator (RTE) for non-Gaussian spherically invariant distributed clutters. The rationale behind this choice is that the RTE is efficient in mitigating the degradation caused by the presence of impulsive noises while inducing little loss when the noise is Gaussian. Based on recent random matrix theory results studying the asymptotic fluctuations of the statistics of the ANMF detector when the number of samples and their dimension grow together to infinity, we propose a design for the regularization parameter that maximizes the detection probability under constant false alarm rates. Simulation results which support the efficiency of the proposed method are provided in order to illustrate the gain of the proposed optimal design over conventional settings of the regularization parameter.

J26. A. Kammoun, R. Couillet,

F. Pascal, M.-S. Alouini, "Convergence and Fluctuations of Regularized Tyler Estimators" IEEE Transactions on Signal Processing, vol. 64, no. 4, pp. 1048-1060, 2016.

Abstract. This article studies the behavior of regularized Tyler estimators (RTEs) of scatter matrices. The key advantages of these estimators are twofold. First, they guarantee by construction a good conditioning of the estimate and second, being a derivative of robust Tyler estimators, they inherit their robustness properties, notably their resilience to the presence of outliers. Nevertheless, one major problem that poses the use of RTEs in practice is represented by the question of setting the regularization parameter ρ. While a high value of ρ is likely to push all the eigenvalues away from zero, it comes at the cost of a larger bias with respect to the population covariance matrix. A deep understanding of the statistics of RTEs is essential to come up with appropriate choices for the regularization parameter. This is not an easy task and might be out of reach, unless one considers asymptotic regimes wherein the number of observations n and/or their size N increase together. First asymptotic

results have recently been obtained under the assumption that N and n are large and commensurable. Interestingly, no results concerning the regime of n going to infinity with N fixed exist, even though the investigation of this assumption has usually predated the analysis of the most difficult N and n large case. This motivates our work. In particular, we prove in the present paper that the RTEs converge to a deterministic matrix when $n \to \infty$ with N fixed, which is expressed as a function of the theoretical covariance matrix. We also derive the fluctuations of the RTEs around this deterministic matrix and establish that these fluctuations converge in distribution to a multivariate Gaussian distribution with zero mean and a covariance depending on the population covariance and the parameter ρ.

J27. D. Morales-Jimenez, R. Couillet, M. McKay, "Large Dimensional Analysis of Robust M-Estimators of Covariance with Outliers" IEEE Transactions on Signal Processing, vol. 63, no. 21, pp. 5784-5797, 2015.

Abstract. A large dimensional characterization of robust M-estimators of covariance (or scatter) is provided under the assumption that the dataset comprises independent (essentially

Gaussian) legitimate samples as well as arbitrary deterministic samples, referred to as outliers. Building upon recent random matrix advances in the area of robust statistics, we specifically show that the so-called Maronna M-estimator of scatter asymptotically behaves similar to well-known random matrices when the population and sample sizes grow together to infinity. The introduction of outliers leads the robust estimator to behave asymptotically as the weighted sum of the sample outer products, with a constant weight for all legitimate samples and different weights for the outliers. A fine analysis of this structure reveals importantly that the propensity of the M-estimator to attenuate (or enhance) the impact of outliers is mostly dictated by the alignment of the outliers with the inverse population covariance matrix of the legitimate samples. Thus, robust M-estimators can bring substantial benefits over more simplistic estimators such as the per-sample normalized version of the sample covariance matrix, which is not capable of differentiating the outlying samples. The analysis shows that, within the class of Maronna's estimators of scatter, the Huber estimator is most favorable for rejecting outliers. On the contrary,

estimators more similar to Tyler's scale invariant estimator (often preferred in the literature) run the risk of inadvertently enhancing some outliers.

J28. L. Yang, R. Couillet, M. McKay, "A Robust Statistics Approach to Minimum Variance Portfolio Optimization" IEEE Transactions on Signal Processing, vol. 63, no. 24, pp. 6684–6697, 2015.

Abstract. We study the design of portfolios under a minimum risk criterion. The performance of the optimized portfolio relies on the accuracy of the estimated covariance matrix of the portfolio asset returns. For large portfolios, the number of available market returns is often of similar order to the number of assets, so that the sample covariance matrix performs poorly as a covariance estimator. Additionally, financial market data often contain outliers which, if not correctly handled, may further corrupt the covariance estimation. We address these shortcomings by studying the performance of a hybrid covariance matrix estimator based on Tyler's robust M-estimator and on Ledoit-Wolf's shrinkage estimator while assuming samples with heavy-tailed distribution. Employing recent results from random matrix theory, we develop a consistent esti-

mator of (a scaled version of) the realized portfolio risk, which is minimized by optimizing online the shrinkage intensity. Our portfolio optimization method is shown via simulations to outperform existing methods both for synthetic and real market data.

J29. R. Couillet, A. Kammoun, F. Pascal, "Second order statistics of robust estimators of scatter. Application to GLRT detection for elliptical signals" Elsevier Journal of Multivariate Analysis, vol. 143, pp. 249-274, 2015.

Abstract. A central limit theorem for bilinear forms of the type $a^* \hat{C}_N(\rho)^{-1} b$, where $a, b \in \mathbb{C}^N$ are unit norm deterministic vectors and $\hat{C}_N(\rho)$ a robust-shrinkage estimator of scatter parametrized by ρ and built upon n independent elliptical vector observations, is presented. The fluctuations of $a^* \hat{C}_N(\rho)^{-1} b$ are found to be of order $N^{-\frac{1}{2}}$ and to be the same as those of $a^* \hat{S}_N(\rho)^{-1} b$ for $\hat{S}_N(\rho)$ a matrix of a theoretical tractable form. This result is exploited in a classical signal detection problem to provide an improved detector which is both robust to elliptical data observations (e.g., impulsive noise) and optimized across the shrinkage parameter ρ.

J30. A. Müller, R. Couillet, E. Bjørnson, S. Wag-

ner, M. Debbah, "Interference-Aware RZF Precoding for Multi-Cell Downlink Systems" IEEE Transactions on Signal Processing, vol. 63, no. 15, pp. 3959-3973 2015.

Abstract. Recently, the structure of the optimal linear precoder for multi-cell downlink systems has been described. Other references have used simplified versions of the precoder to obtain promising performance gains. These gains have been hypothesized to stem from providing additional degrees of freedom that allow for interference mitigation through interference relegation to orthogonal subspaces. However, no conclusive or rigorous understanding has yet been proposed. In this paper, we take an interference-aware adaption of the generally optimal precoding structure and analyze the rate performance in multi-cell scenarios. A special emphasis is placed on induced interference mitigation. For example, we will verify the intuitive expectation that the precoder structure can either completely remove induced inter-cell or intra-cell interference. We state new results from large-scale random matrix theory, that make it possible to give more intuitive and insightful explanations of the precoder behavior, also for cases involving imperfect chan-

nel state information (CSI). We remark especially that the interference-aware precoder makes use of all available information about interfering channels to improve performance. Even extremely bad CSI can be used to enhance the sum rate. Our obtained insights are then used to propose heuristic precoder parameters for arbitrary systems, whose effectiveness is shown in more involved system scenarios. Furthermore, determining these parameters does not require explicit inter base station cooperation. Using a simple heuristic version of the interference aware precoder, one finds that a sum rate performance, close to the optimally parameterized precoder one, can be achieved.

J31. R. Couillet, "Robust spiked random matrices and a robust G-MUSIC estimator" Elsevier Journal of Multivariate Analysis, vol. 140, pp. 139-161, 2015.

Abstract. A class of robust estimators of scatter applied to information-plus-impulsive noise samples is studied, where the sample information matrix is assumed of low rank ; this generalizes the study (?(?) ?34) (restricted to a noise only setting) to spiked random matrix models. It is precisely shown that, as opposed to sample covariance matrices which

may have asymptotically unbounded (eigen-)spectrum due to the sample impulsiveness, the robust estimator of scatter has bounded spectrum and may contain isolated eigenvalues which we fully characterize. We show that, if found beyond a certain detectability threshold, these eigenvalues allow one to perform statistical inference on the eigenvalues and eigenvectors of the information matrix. We use this result to derive new eigenvalue and eigenvector estimation procedures, which we apply in practice to the popular array processing problem of angle of arrival estimation. This gives birth to an improved algorithm based on the MUSIC method, which we refer to as robust G-MUSIC.

J32. R. Couillet, M. McKay, "Large Dimensional Analysis and Optimization of Robust Shrinkage Covariance Matrix Estimators" Elsevier Journal of Multivariate Analysis, vol. 131, pp. 99-120, 2014.

Abstract. This article studies two regularized robust estimators of scatter matrices proposed (and proved to be well defined) in parallel in (Chen et al., 2011) and (Pascal et al., 2013), based on Tyler's robust M-estimator (Tyler, 1987) and on Ledoit and Wolf's shrinkage covariance ma-

trix estimator (Ledoit and Wolf, 2004). These hybrid estimators have the advantage of conveying (i) robustness to outliers or impulsive samples and (ii) small sample size adequacy to the classical sample covariance matrix estimator. We consider here the case of i.i.d. elliptical zero mean samples in the regime where both sample and population sizes are large. We demonstrate that, under this setting, the estimators under study asymptotically behave similar to well-understood random matrix models. This characterization allows us to derive optimal shrinkage strategies to estimate the population scatter matrix, improving significantly upon the empirical shrinkage method proposed in (Chen et al., 2011).

J33. Y. Chitour, R. Couillet, F. Pascal "On the convergence of Maronna's M-estimators of scatter" IEEE Signal Processing Letters, vol. 22, no. 6, pp. 709-712, 2014.

Abstract. In this paper, we propose an alternative proof for the uniqueness of Maronna's M-estimator of scatter for N vector observations $y_1, \ldots, y_N \in \mathbb{R}^m$ under a mild constraint of linear independence of any subset of m of these vectors. This entails in particular almost sure uniqueness for random vectors y_i

with a density as long as $N > m$. This approach allows to establish further relations that demonstrate that a properly normalized Tyler's M-estimator of scatter can be considered as a limit of Maronna's M-estimator. More precisely, the contribution is to show that each M-estimator, verifying some mild conditions, converges towards a particular Tyler's M-estimator. These results find important implications in recent works on the large dimensional (random matrix) regime of robust M-estimation.

J34. R. Couillet, F. Pascal, J. W. Silverstein, "The Random Matrix Regime of Maronna's M-estimator with elliptically distributed samples", vol. 139, pp. 56-78, Elsevier Journal of Multivariate Analysis, 2015.

Abstract. This article demonstrates that the robust scatter matrix estimator $\hat{C}_N \in \mathbb{C}^{N\times N}$ of a multivariate elliptical population $x_1, \ldots, x_n \in \mathbb{C}^N$ originally proposed by Maronna in 1976, and defined as the solution (when existent) of an implicit equation, behaves similar to a well-known random matrix model in the limiting regime where the population N and sample n sizes grow at the same speed. We show precisely that $\hat{C}_N \in \mathbb{C}^{N\times N}$ is defined for all n large with probability one and

that, under some light hypotheses, $\|\hat{C}_N - \hat{S}_N\| \to 0$ almost surely in spectral norm, where $\hat{S}_N$ follows a classical random matrix model. As a corollary, the limiting eigenvalue distribution of $\hat{C}_N$ is derived. This analysis finds applications in the fields of statistical inference and signal processing.

J35. J. Vinogradova, R. Couillet, W. Hachem, "Estimation of Toeplitz covariance matrices in large dimensional regime with application to source detection large", IEEE Transactions on Signal Processing, vol. 63, no. 18, pp. 4903-4913, 2015.

Abstract. In this article, we derive concentration inequalities for the spectral norm of two classical sample estimators of large dimensional Toeplitz covariance matrices, demonstrating in particular their asymptotic almost sure consistence. The consistency is then extended to the case where the aggregated matrix of time samples is corrupted by a rank one (or more generally, low rank) matrix. As an application of the latter, the problem of source detection in the context of large dimensional sensor networks within a temporally correlated noise environment is studied. As opposed to standard procedures, this application is performed online, i.e., without

the need to possess a learning set of pure noise samples.

J36. R. Couillet, W. Hachem, "Analysis of the limiting spectral measure of large random matrices of the separable covariance type", Random Matrix Theory and Applications, vol. 3, pp. 1-23, 2014.

Abstract. Consider the random matrix $\Sigma = D^{1/2} X \widetilde{D}^{1/2}$ where D and $\widetilde{D}$ are deterministic Hermitian nonnegative matrices with respective dimensions $N \times N$ and $n \times n$, and where X is a random matrix with independent and identically distributed centered elements with variance $1/n$. Assume that the dimensions N and n grow to infinity at the same pace, and that the spectral measures of D and $\widetilde{D}$ converge as $N, n \to \infty$ towards two probability measures. Then it is known that the spectral measure of $\Sigma\Sigma^*$ converges towards a probability measure μ characterized by its Stieltjes Transform. In this paper, it is shown that μ has a density away from zero, this density is analytical wherever it is positive, and it behaves in most cases as $\sqrt{|x-a|}$ near an edge a of its support. In addition, a complete characterization of the support of μ is provided. Aside from its mathematical interest, the analysis underlying these results

finds important applications in a certain class of statistical estimation problems.

J37. J. Hoydis, R. Couillet, P. Piantanida, "The Second-Order Coding Rate of the MIMO Rayleigh Block-Fading Channel," IEEE Transactions on Information Theory, vol. 61, no. 12, pp. 6591-6622, 2015.

Abstract. The second-order coding rate of the multiple-input multiple-output (MIMO) quasi-static Rayleigh fading channel is studied. We tackle this problem via an information-spectrum approach and statistical bounds based on recent random matrix theory techniques. We precisely derive a central limit theorem (CLT) to analyze the information density in the regime where the block-length n and the number of transmit and receive antennas K and N, respectively, grow simultaneously large. This result leads to the characterization of closed-form upper and lower bounds on the optimal average error probability when the coding rate is within $O(1/\sqrt{nK})$ of the asymptotic capacity.

J38. J. Vinogradova, R. Couillet, W. Hachem, "Statistical Inference in Large Antenna Arrays under Unknown Noise Pattern," IEEE Transactions on Signal Processing, vol. 61, no. 22, pp. 5633-5645, 2013.

Abstract. In this article, a general information-plus-noise transmission model is assumed, the receiver end of which is composed of a large number of sensors and is unaware of the noise pattern. For this model, and under reasonable assumptions, a set of results is provided for the receiver to perform statistical eigen-inference on the information part. In particular, we introduce new methods for the detection, counting, and the power and subspace estimation of multiple sources composing the information part of the transmission. The theoretical performance of some of these techniques is also discussed. An exemplary application of these methods to array processing is then studied in greater detail, leading in particular to a novel MUSIC-like algorithm assuming unknown noise covariance.

J39. F. Chapon, R. Couillet, W. Hachem, X. Mestre, "The outliers among the singular values of large rectangular random matrices with additive fixed rank deformation," Markov Processes and Related Fields, vol. 20, pp. 183-228, 2014.

Abstract. Consider the matrix $\Sigma_n = n^{-1/2} X_n D_n^{1/2} + P_n$ where the matrix $X_n \in \mathbb{C}^{N \times n}$ has Gaussian standard independent elements, D_n is a deterministic diagonal non-

negative matrix, and P_n is a deterministic matrix with fixed rank. Under some known conditions, the spectral measures of $\Sigma_n\Sigma_n^*$ and $n^{-1}X_nD_nX_n^*$ both converge towards a compactly supported probability measure μ as $N, n \to \infty$ with $N/n \to c$. In this paper, it is proved that finitely many eigenvalues of $\Sigma_n\Sigma_n^*$ may stay away from the support of μ in the large dimensional regime. The existence and locations of these outliers in any connected component of $\mathbb{R}\backslash\text{supp}(\mu)$ are studied. The fluctuations of the largest outliers of $\Sigma_n\Sigma_n^*$ are also analyzed. The results find applications in the fields of signal processing and radio communications.

J40. R. Couillet, F. Pascal, J. W. Silverstein, "Robust Estimates of Covariance Matrices in the Large Dimensional Regime," IEEE Transactions on Information Theory, vol. 60, no. 11, 2014.

Abstract. This article studies the limiting behavior of a class of robust population covariance matrix estimators, originally due to Maronna in 1976, in the regime where both the number of available samples and the population size grow large. Using tools from random matrix theory, we prove that, for sample vectors made of independent entries having

some moment conditions, the difference between the sample covariance matrix and (a scaled version of) such robust estimator tends to zero in spectral norm, almost surely. This result can be applied to various statistical methods arising from random matrix theory that can be made robust without altering their first order behavior.

J41. G. Geraci, R. Couillet, J. Yuan, M. Debbah, I. B. Collings, "Large System Analysis of Linear Precoding in MISO Broadcast Channels with Confidential Messages," IEEE Journal on Selected Area in Communications, vol. 31, no. 9, pp. 1660-1671, 2013. **Second prize of the 2012-2013 IEEE Australia Council Student Paper Contest.**

Abstract. In this paper, we study the performance of regularized channel inversion (RCI) precoding in large MISO broadcast channels with confidential messages (BCC). We obtain a deterministic approximation for the achievable secrecy sum-rate which is almost surely exact as the number of transmit antennas M and the number of users K grow to infinity in a fixed ratio $\beta = K/M$. We derive the optimal regularization parameter ξ and the optimal network load β that maximize the per-antenna

secrecy sum-rate. We then propose a linear precoder based on RCI and power reduction (RCI-PR) that significantly increases the high-SNR secrecy sum-rate for $1 < \beta < 2$. Our proposed precoder achieves a per-user secrecy rate which has the same high-SNR scaling factor as both the following upper bounds : (i) the rate of the optimum RCI precoder without secrecy requirements, and (ii) the secrecy capacity of a single-user system without interference. Furthermore, we obtain a deterministic approximation for the secrecy sum-rate achievable by RCI precoding in the presence of channel state information (CSI) error. We also analyze the performance of our proposed RCI-PR precoder with CSI error, and we determine how the error must scale with the SNR in order to maintain a given rate gap to the case with perfect CSI.

J42. J. Hoydis, R. Couillet, M. Debbah, "Iterative Deterministic Equivalents for the Capacity Analysis of Communication Systems," Technical Report.

Abstract. In this report, we introduce the notion of nested deterministic equivalents of functional of random matrices. Nested deterministic equivalents extend classical deterministic equivalents in or-

der to study stochastic problems with multiple independent random variables. In particular, we discuss their applications to wireless communications, and especially to the capacity analysis of doubly-scattering multiple input multiple output (MIMO) channels and of multi-hop relay channels for which we derive novel expressions.

J43. R. Couillet, S. Medina Perlaza, H. Tembine, M. Debbah, "Electrical Vehicles in the Smart Grid: A Mean Field Game Analysis," IEEE Journal on Selected Areas in Communications : Smart Grid Communications Series, vol. 30, no. 6, pp. 1086-1096, 2012.

Abstract. In this article, we investigate the competitive interaction between electrical vehicles or hybrid oil-electricity vehicles in a Cournot market consisting of electricity transactions to or from an underlying electricity distribution network. We provide a mean field game formulation for this competition, and introduce the set of fundamental differential equations ruling the behavior of the vehicles at the feedback Nash equilibrium, referred here to as the mean field equilibrium. This framework allows for a consistent analysis of the evolution of the price of electricity as well as of the ins-

tantaneous electricity demand in the power grid. Simulations precisely quantify those parameters and suggest that significant reduction of the daily electricity peak demand can be achieved by appropriate electricity pricing.

J44. J. Yao, R. Couillet, J. Najim, M. Debbah, "Fluctuations of an Improved Population Eigenvalue Estimator in Sample Covariance Matrix Models," IEEE Transactions on Information Theory, vol. 59, no. 2, pp. 1149-1163, 2013.

Abstract. In this article, the joint fluctuations of the extreme eigenvalues and eigenvectors of a large dimensional sample covariance matrix are analyzed when the associated population covariance matrix is a finite-rank perturbation of the identity matrix, corresponding to the so-called spiked model in random matrix theory. The asymptotic fluctuations, as the matrix size grows large, are shown to be intimately linked with matrices from the Gaussian unitary ensemble (GUE). When the spiked population eigenvalues have unit multiplicity, the fluctuations follow a central limit theorem. This result is used to develop an original framework for the detection and diagnosis of local failures in large sensor networks,

for known or unknown failure magnitude.

J45. R. Couillet, M. Debbah, "Signal Processing in Large Systems: a New Paradigm," IEEE Signal Processing Magazine, vol. 30, no. 1, pp. 24-39, 2013.

Abstract. For a long time, detection and parameter estimation methods for signal processing have relied on asymptotic statistics as the number n of observations of a population grows large comparatively to the population size N, i.e., $n/N \to \infty$. Modern technological and societal advances now demand the study of sometimes extremely large populations and simultaneously require fast signal processing due to accelerated system dynamics. This results in not-so-large practical ratios n/N, sometimes even smaller than one. A disruptive change in classical signal processing methods has therefore been initiated in the past ten years, mostly spurred by the field of large dimensional random matrix theory. The early works in random matrix theory for signal processing applications are however scarce and highly technical. This tutorial provides an accessible methodological introduction to the modern tools of random matrix theory and to the signal processing methods derived from them, with an em-

phasis on simple illustrative examples.

J46. R. Couillet, W. Hachem, "Fluctuations of spiked random matrix models and failure diagnosis in sensor networks," IEEE Transactions on Information Theory, vol. 59, no. 1, pp. 509-525, 2013.

Abstract. In this article, the joint fluctuations of the extreme eigenvalues and eigenvectors of a large dimensional sample covariance matrix are analyzed when the associated population covariance matrix is a finite-rank perturbation of the identity matrix, corresponding to the so-called spiked model in random matrix theory. The asymptotic fluctuations, as the matrix size grows large, are shown to be intimately linked with matrices from the Gaussian unitary ensemble (GUE). When the spiked population eigenvalues have unit multiplicity, the fluctuations follow a central limit theorem. This result is used to develop an original framework for the detection and diagnosis of local failures in large sensor networks, for known or unknown failure magnitude.

J47. A. Kammoun, R. Couillet, J. Najim, M. Debbah, "Performance of capacity inference methods under colored interference," IEEE Transactions on Information Theory, vol. 59, no. 2, pp. 1129-1148, 2013.

Abstract. The problem of fast point-to-point MIMO channel mutual information estimation is addressed, in the situation where the receiver undergoes unknown colored interference, whereas the channel with the transmitter is perfectly known. The considered scenario assumes that the estimation is based on a few channel use observations during a short sensing period. Using large dimensional random matrix theory, an estimator referred to as *G-estimator* is derived. This estimator is proved to be consistent as the number of antennas and observations grow large and its asymptotic performance is analyzed. In particular, the G-estimator satisfies a central limit theorem with asymptotic Gaussian fluctuations. Simulations are provided which strongly support the theoretical results, even for small system dimensions.

J48. R. Couillet, J. Hoydis, M. Debbah, "Random beamforming over quasi-static and fading channels: A deterministic equivalent approach," IEEE Transactions on Information Theory, vol. 58, no. 10, pp. 6392-6425, 2012.

Abstract. In this work, we study the performance of random isometric precoders over quasi-static and correlated fading channels.

We derive deterministic approximations of the mutual information and the signal-to-interference-plus-noise ratio (SINR) at the output of the minimum-mean-square-error (MMSE) receiver and provide simple provably converging fixed-point algorithms for their computation. Although these approximations are only proven exact in the asymptotic regime with infinitely many antennas at the transmitters and receivers, simulations suggest that they closely match the performance of small-dimensional systems. We exemplarily apply our results to the performance analysis of multi-cellular communication systems, multiple-input multiple-output multiple-access channels (MIMO-MAC), and MIMO interference channels. The mathematical analysis is based on the Stieltjes transform method. This enables the derivation of deterministic equivalents of functionals of large-dimensional random matrices. In contrast to previous works, our analysis does not rely on arguments from free probability theory which enables the consideration of random matrix models for which asymptotic freeness does not hold. Thus, the results of this work are also a novel contribution to the field of random matrix theory and applicable to a wide spectrum of practical sys-

tems.

J49. S. Wagner, R. Couillet, M. Debbah, D. T. M. Slock, "Large System Analysis of Linear Precoding in MISO Broadcast Channels with Limited Feedback", IEEE Transactions on Information Theory, vol. 58, no. 7, pp. 4509-4537, 2012.

Abstract. In this paper, we study the sum rate performance of zero-forcing (ZF) and regularized ZF (RZF) precoding in large MISO broadcast systems under the assumptions of imperfect channel state information at the transmitter and per-user channel transmit correlation. Our analysis assumes that the number of transmit antennas M and the number of single-antenna users K are large while their ratio remains bounded. We derive deterministic approximations of the empirical signal-to-interference plus noise ratio (SINR) at the receivers, which are tight as $M, K \to \infty$. In the course of this derivation, the per-user channel correlation model requires the development of a novel deterministic equivalent of the empirical Stieltjes transform of large dimensional random matrices with generalized variance profile. The deterministic SINR approximations enable us to solve various practical optimization problems. Under sum rate

maximization, we derive (i) for RZF the optimal regularization parameter, (ii) for ZF the optimal number of users, (iii) for ZF and RZF the optimal power allocation scheme and (iv) the optimal amount of feedback in large FDD/TDD multi-user systems. Numerical simulations suggest that the deterministic approximations are accurate even for small M, K.

J50. R. Couillet, J. W. Silverstein, Z. Bai, M. Debbah, "Eigen-Inference for Energy Estimation of Multiple Sources", IEEE Transactions on Information Theory, vol. 57, no. 4, pp. 2420-2439, 2011.

Abstract. In this paper, a new method is introduced to blindly estimate the transmit power of multiple signal sources in multi-antenna fading channels, when the number of sensing devices and the number of available samples are sufficiently large compared to the number of sources. Recent advances in the field of large dimensional random matrix theory are used that result in a simple and computationally efficient consistent estimator of the power of each source. A criterion to determine the minimum number of sensors and the minimum number of samples required to achieve source separation is then introduced. Simulations

are performed that corroborate the theoretical claims and show that the proposed power estimator largely outperforms alternative power inference techniques.

J51. R. Couillet, M. Debbah, J. W. Silverstein, "A Deterministic Equivalent for the Analysis of Correlated MIMO Multiple Access Channels", IEEE Transactions on Information Theory, vol. 57, no. 6, pp. 3493-3514, 2011.

Abstract. In this article, novel deterministic equivalents for the Stieltjes transform and the Shannon transform of a class of large dimensional random matrices are provided. These results are used to characterise the ergodic rate region of multiple antenna multiple access channels, when each point-to-point propagation channel is modelled according to the Kronecker model. Specifically, an approximation of all rates achieved within the ergodic rate region is derived and an approximation of the linear precoders that achieve the boundary of the rate region as well as an iterative water-filling algorithm to obtain these precoders are provided. An original feature of this work is that the proposed deterministic equivalents are proved valid even for strong correlation patterns at both communication sides.

The above results are validated by Monte Carlo simulations.

J52. R. Couillet, M. Debbah, "A Bayesian Framework for Collaborative Multi-Source Signal Sensing", IEEE Transactions on Signal Processing, vol. 58, no. 10, pp. 5186-5195, 2010.
Abstract. This paper introduces a Bayesian framework to detect multiple signals embedded in noisy observations from a sensor array. For various states of knowledge on the communication channel and the noise at the receiving sensors, a marginalization procedure based on recent tools of finite random matrix theory, in conjunction with the maximum entropy principle, is used to compute the hypothesis selection criterion. Quite remarkably, explicit expressions for the Bayesian detector are derived which enable to decide on the presence of signal sources in a noisy wireless environment. The proposed Bayesian detector is shown to outperform the classical power detector when the noise power is known and provides very good performance for limited knowledge on the noise power. Simulations corroborate the theoretical results and quantify the gain achieved using the proposed Bayesian framework.

J53. R. Couillet, A. An-

cora, M. Debbah, "Bayesian Foundations of Channel Estimation for Cognitive Radios", Advances in Electronics and Telecommunications, vol. 1, no. 1, pp. 41-49, 2010.

Abstract. In this paper, we revisit the philosophical foundations of the field of channel estimation. Our main intention is to come up with a partial answer to the question : "given some available sensed signals, how should cognitive radios ideally perform channel estimation ?". We specifically introduce a general framework to provide optimal channel estimates under any prior knowledge at the sensing device. Our discussion is articulated as a top-down approach, introducing successively (i) a discussion on the philosophical foundations of channel estimation as a simplification means for the general problem of wireless detection, (ii) an information theoretically optimal approach to channel detection assuming the sensing device has infinite memory, and (iii) a derived optimal approach when limited memory size is accounted for. The key mathematical tools used in this discussion emerge from Bayesian probability theory and are known as the maximum entropy principle and the minimum update principle. Derivations are carried out for the

particular case of channel estimation in orthogonal frequency division multiplexing (OFDM) systems. While some theoretical results will be proven to match already known techniques, such as Kalman filters, another set of novel results will be shown by simulations to perform better than known channel estimation schemes.

J54. R. Couillet, M. Debbah, "Le téléphone du futur : plus intelligent pour une exploitation optimale des fréquences" Revue de l'Electricité et de l'Electronique, no. 6, pp. 71-83, 2010.

Résumé. Au jour où les communications mobiles demandent de plus en plus de vitesse de transfert de larges volumes de données à destination d'utilisateurs de plus en plus nombreux, il apparaît que les limites physiques des protocoles de communication sont bientôt atteintes. Une révolution technologique est ainsi nécessaire et sur le point d'éclore : celle-ci passe par la mise en place de systèmes de communications opportunistes, coopératifs, autonomes et idéalement suffisamment intelligents pour servir au mieux les requêtes de l'utilisateur. Ces différents aspects, certains d'ores et déjà d'actualité, d'autres à l'état embryonnaire sont discutés successivement dans cette étude.

J55. R. Couillet, M. Debbah, "Mathematical foundations of cognitive radios", Journal of Telecommunications and Information Technologies, no. 4, 2009.

Abstract. Recently, much interest has been directed towards software defined radios and embedded intelligence in telecommunication devices. However, no theoretical framework for cognitive radios has ever been proposed. In this paper, we introduce an information theoretic point of view on cognitive radios. Specifically, our motivation in this work is to embed human-like intelligence in mobile wireless devices, following the three century-old work on Bayesian probability theory, the maximum entropy principle and minimal probability update. This allows us to partially answer such questions as "what are the signal detection capabilities of a wireless device ?" or "when facing a situation in which most parameters are missing, how to react ?". As an introduction, we will present two examples from the same authors using the cognitive framework namely multi-antenna channel modelling and signal sensing.

J56. R. Couillet, M. Debbah, "Outage performance of flexible OFDM

schemes in packet-switched transmissions", Eurasip Journal on Advances on Signal Processing, Volume 2009, Article ID 698417, 2009.

Abstract. In this paper, α-OFDM, a generalization of the OFDM modulation, is proposed. This new modulation enhances the outage capacity performance of bursty communications. The α-OFDM scheme is easily implementable as it only requires an additional time symbol rotation after the IDFT stage and a subsequent phase rotation of the cyclic prefix. The physical effect of the induced rotation is to slide the DFT window over the frequency spectrum. When successively used with different angles α at the symbol rate, α-OFDM provides frequency diversity in block fading channels. Interestingly, simulation results show a substantial gain in terms of outage capacity and outage BER in comparison with classical OFDM modulation schemes. The framework is extended to multi-antenna and multi-cellular OFDM based standards. Practical simulations, in the context of 3GPP-LTE, called hereafter α-LTE, sustain our theoretical claims.

- R. Couillet, M. Debbah, **Mathematical Foundations for Signal Processing, Communications**

and Networking, CRC Press, Taylor & Francis Group, 2011 [chapitre de livre]

Abstract. From basic transforms to Monte Carlo simulation to linear programming, the text covers a broad range of mathematical techniques essential to understanding the concepts and results in signal processing, telecommunications, and networking. Along with discussing mathematical theory, each self-contained chapter presents examples that illustrate the use of various mathematical concepts to solve different applications. Each chapter also includes a set of homework exercises and readings for additional study.

- R. Couillet, M. Debbah, **Orthogonal Frequency Division Multiple Access Fundamentals and Applications**, Auerbach Publications, CRC Press, Taylor & Francis Group, 2010 [chapitre de livre]

  Abstract. Supported by the expert-level advice of pioneering researchers, Orthogonal Frequency Division Multiple Access Fundamentals and Applications provides a comprehensive and accessible introduction to the foundations and applications of one of the most promising access technologies for current and future wireless networks. It includes authoritative coverage of

the history, fundamental principles, key techniques, and critical design issues of OFDM systems.

- R. Couillet, M. Debbah, **No. 08368023.1** "Method for short-time OFDM transmission and apparatus for performing flexible OFDM modulation"

- R. Couillet, S. Wagner, **No. 09368025.4** "Precoding process for a transmitter of a MU-MIMO communication system"

- R. Couillet, **No. 09368030.4** "Process for estimating the channel in an OFDM communication system, and receiver for doing the same"

II1. R. Couillet, **Idée Innovante** "User Subspace Clustering"