Free deconvolution for OFDM multicell SNR detection

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***Abstract*—In this paper, a new multicell OFDM blind power detection method is proposed. Relying on recent results of free deconvolution, our algorithm enables the terminal to count the number of surrounding base stations and to determine the power received from each of them, based on a limited number of snapshots. This is in sharp contrast with classical asymptotic blind techniques. A theoretical analysis is proposed to study the impact of frequency selectivity and the number of receive/transmit antennas. Simulations are provided to sustain the theoretical claims and are compared against classical techniques.**

I. Introduction

The ever increasing demand of high data rate has pushed system designers to exploit the wireless channel medium to the smallest granularity. In this respect, OFDM (Orthogonal Frequency Division Multiplexing) modulation has been chosen for the next common standard in most wireless communication systems (e.g. Wi-Max [3], 3GPP-LTE [2]). OFDM converts a frequency selective fading channel into a set of flat fading channels [12], therefore providing a high flexibility in terms of power and rate allocation. However, due to this flat fading nature, OFDM suffers from a lack of diversity and from strong interferers in multi-cell systems. This requires system level interference management (with proper multi-cell scheduling) from a network MIMO (Multiple Input Multiple Output) point of view [4].

In order to design a viable network-wide OFDM system, a key parameter that needs to be estimated is the Signal to Interference plus Noise Ratio (SINR), i.e. the power received at the terminal originating from one base station over the cumulated power of the interfering base stations plus background noise power. Ideally, one needs to access the respective Signal to Noise Ratio (SNR) of every cell in the network (i.e. the ratio between the power of the received signal received that originated from the transmitter and the noise level). In the realm of cognitive networks [16], each terminal needs to determine the uplink quality link (via the downlink channel in TDD mode) with reduced feedback from the network in order to decide autonomously how to split the different packet data. Several examples are provided in [17]-[18]. Usually, this difficult problem of *source separation* is treated with respect to the signal statistics [13], [14], [15] under the hypothesis of a high number of received snapshots. However, in practice, this hypothesis can never be met due to the high mobility of the users.

In this paper, we use recent results on free deconvolution to show that one can estimate the received power of every relevant (i.e. in detection range) base station in an OFDM network. Apart from second order statistics, no prior knowledge on the input signal constellations is required. Also large amounts of received samples (compared to the FFT size) are not demanded for our scheme to perform efficiently. Interestingly, one can count the total number of effective surrounding cells and derive every individual SNR level (seen from the terminal) of the base stations. To the authors knowledge, no previous contribution has yet considered this OFDM multiple SNR detection setting. This work makes extensive use of *free deconvolution* techniques. Those techniques were initially applied in [5] to derive the respective powers of users in a Code Division Multiple Access (CDMA) network.

The paper is structured as follows: In section II, we introduce the multicell environment model. In section III, we review classical methods used to derive SINR in OFDM systems. In section IV, we provide the algorithm to determine the per-cell SNR. A discussion on the gains and limitations of this novel method is carried out in section V, before we present the simulation results that sustain the theoretical claims. Finally, in section VII we draw our conclusions.

Notations: In the following, boldface lower case symbols represent vectors, capital boldface characters denote matrices ($\mathbf{I}_N$ is the size-N identity matrix). The spaces $\mathcal{M}(\mathcal{A}, i, j)$ and $\mathcal{M}(\mathcal{A}, i)$ are the sets of $i \times j$ and $i \times i$ matrices over the algebra $\mathcal{A}$, respectively. The Hermitian transpose is denoted $(\cdot)^H$. The operator $\mathrm{diag}(\mathbf{x})$ turns the vector $\mathbf{x}$ into a diagonal matrix. C_n^k is the (k, n) binomial coefficient. The normalized trace for matrices with N columns is denoted $\mathrm{tr}_N(\cdot)$ and $\mathrm{E}[\cdot]$ is the expectation.

II. Downlink Model

Let us consider, as depicted in figure 1, a set of N_B base stations and one UE (User Equipment) with $N_r = 1$ receiving antennae. The network uses OFDM modulation with a size N FFT. Let us also denote by M the *expected maximum* number of base stations (ideally $N_B \leq M$). In the following, we shall only deal with M and no longer use N_B, considering

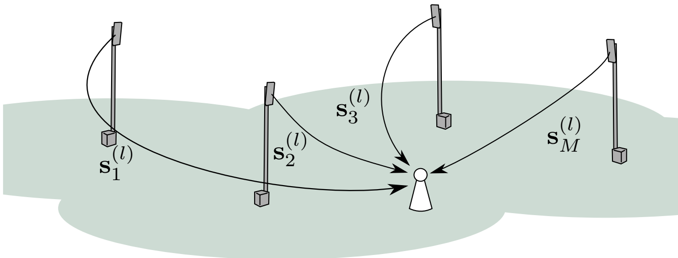

Fig. 1. System Model

L	Estimated $\tilde{\mathbf{P}}$ [our algorithm]	$\Vert\mathbf{P}-\tilde{\mathbf{P}}\Vert_2$
	$N=256$, $\mathbf{P}=\{P_1,P_2,P_3\}=\{4,2,1\}$	
512	$\{5.82, 2.90, -1.7\}$ [$\{3.80, 2.16, 1.00\}$]	11.42
1024	$\{4.26, 3.60, -0.8\}$ [$\{3.62, 2.37, 0.94\}$]	5.87
2048	$\{4.52, 2.69, -0.2\}$ [$\{4.22, 1.55, 1.12\}$]	1.77
4096	$\{4.20, 2.65, 0.18\}$ [$\{4.09, 2.06, 0.78\}$]	1.41
8192	$\{4.10, 2.28, 0.58\}$ [$\{4.05, 1.89, 0.92\}$]	0.27
16384	$\{3.97, 2.42, 0.89\}$ [$\{3.95, 2.24, 0.99\}$]	0.19
32768	$\{4.07, 1.95, 0.99\}$ [$\{4.03, 1.95, 0.98\}$]	0.01

TABLE I
CLASSICAL MOMENT-BASED METHOD

then a network of M base stations, of which some could be of null power. The link between the UE and the base station k is a fast-fading complex Gaussian channel $\mathbf{h}_k \in \mathbb{C}^N$, coupled with a slow-fading path loss $L_k = 1/P_k$ with P_k the mean received power originating from base station k. The UE also suffers Additive White Gaussian Noise (AWGN) of power σ^2. The base station k sends at time instant l the frequency-domain OFDM symbol $\mathbf{s}_k^{(l)} = (s_{1k}^{(l)}, \ldots, s_{Nk}^{(l)})^T$ that we first assume standard Gaussian (i.e. with zero mean and unit variance). The noise $\sigma\mathbf{n}^{(l)}$ is added at the reception, with $\mathbf{n}^{(l)} = (n_1^{(l)} \ldots, n_N^{(l)})^T$ also standard Gaussian. Therefore, the received signal vector $\mathbf{y}^{(l)} = (y_1^{(l)}, \ldots, y_N^{(l)})^T$ at time instant l reads

$$\mathbf{y}^{(l)} = \sum_{k=0}^{M-1} P_k^{\frac{1}{2}} \mathbf{D}_k \mathbf{s}_k^{(l)} + \sigma\mathbf{n}^{(l)} \tag{1}$$

with $\mathbf{D}_k = \mathrm{diag}(\mathbf{h}_k) = \mathrm{diag}([h_{k1} \ldots h_{kN}])$.

This summation over the M cells can be rewritten

$$\mathbf{y}^{(l)} \quad = \mathbf{H}\mathbf{P}^{\frac{1}{2}}\boldsymbol{\theta}^{(l)} + \sigma\mathbf{n}^{(l)} \tag{2}$$

with $\boldsymbol{\theta}^{(l)} = (\mathbf{s}_1^{(l)}, \ldots, \mathbf{s}_M^{(l)})^T$.

$\mathbf{H} \in \mathcal{M}(\mathbb{C}, N, MN)$ is the concatenated matrix of the matrices $\mathbf{D}_k$, $k \in [1, M]$

$$\mathbf{H} = \begin{bmatrix} h_{11} & \cdots & 0 & \cdots & h_{M1} & \cdots & 0 \\ \vdots & \ddots & \vdots & \ldots & \vdots & \ddots & \vdots \\ 0 & \cdots & h_{2N} & \cdots & 0 & \cdots & h_{MN} \end{bmatrix} \tag{3}$$

and $\mathbf{P} \in \mathcal{M}(\mathbb{R}, N)$ the diagonal matrix

$$\mathbf{P} \quad = \mathrm{diag}([P_1,\ P_2,\ \ldots,\ P_M]) \otimes \mathbf{I}_N \tag{4}$$

where the symbol $\otimes$ denotes the Kronecker product.

Let us now assume that the M channels are slowly varying, so that we can concatenate L samples $\mathbf{y}^{(l)}$ $(l = 1, \ldots, L)$ into matrix $\mathbf{Y} = [\mathbf{y}^{(1)} \cdots \mathbf{y}^{(L)}] \in \mathcal{M}(\mathbb{C}, N, L)$ and have the more general matrix product

$$\mathbf{Y} = \mathbf{H}\mathbf{P}^{\frac{1}{2}}\boldsymbol{\Theta} + \sigma\mathbf{N} \tag{5}$$

where $\boldsymbol{\Theta} \in \mathcal{M}(\mathbb{C}, MN, L)$ and $\mathbf{N} \in \mathcal{M}(\mathbb{C}, N, L)$ are concatenation matrices of the L vectors $\boldsymbol{\theta}^{(l)}$ and $\mathbf{n}^{(l)}$ respectively. The entries of $\mathbf{H}$ are fixed over the L channel uses.

This then imposes the minimum of all M channel coherence times to be greater than LT_s with T_s the OFDM symbol period.

III. Classical Power Detection

Usual power detectors consider the second as well as higher order statistics of the received signals. This requires to estimate the following *empirical moments*: $\left\{(\frac{1}{L}\mathbf{Y}\mathbf{Y}^H)^k,\ k \geq 1\right\}$. These techniques work well when L goes to infinity while N is finite, or at least when the ratio N/L tends to zero when both N and L go to infinity. Indeed, with growing L, $(\frac{1}{L}\mathbf{Y}\mathbf{Y}^H - \sigma^2\mathbf{I}_N)^k$ tends to $\mathrm{E}[(\mathbf{H}\mathbf{P}\mathbf{H}^H)^k]$, which gives access to matrix $\mathbf{P}$, as shown in IV.

As a consequence, one can retrieve the values P_k (as will be proved in section IV) directly from the normalized traces (also called *moments*) of $\{\frac{1}{L}(\mathbf{Y}\mathbf{Y}^H - \sigma^2\mathbf{I}_N)^k\}$ when L is large compared to N. However, in practice, this case is rarely met due to mobility of the UE. In fact, we wish N to be fairly large (such that the ratio between the system bandwidth over the coherence bandwidth is small) while L is limited by the *channel coherence time*. Therefore, even if N and L are large, the problem falls in a situation where the ratio N/L is not close to zero so that expectation-based methods are far from accurate. In this context, the previous classical method does not work since the expectation taken for large L is not valid when N grows along with L. This is shown in table I which provides the results and quadratic errors of estimates of $\mathbf{P}$ under different values of L using the same algorithm as described in section IV-A based on the moments of $\frac{1}{L}(\mathbf{Y}\mathbf{Y}^H - \sigma^2\mathbf{I}_N)$ (instead of the moments of $\frac{1}{L}\mathbf{H}\mathbf{P}\mathbf{H}^H$ for which comparative results, based on the algorithm of section IV, are presented in brackets). This confirms that large L/N ratios are required for this classical scheme to be valid.

For a deeper analysis of those classical techniques, refer to [19]. In this contribution, we shall also use a technique based on moments, relying on recent work on *Random Matrix Theory* (RMT) [6] and *Free Deconvolution* [5], that are briefly introduced in the following section.

IV. Application of Free Deconvolution to Multiple SNR detection

In order to recover the P_i powers, one needs to access the entries of $\mathbf{H}\mathbf{P}\mathbf{H}^H$ and more specifically, as shall be shown later, the eigenvalue distribution of $\mathbf{H}\mathbf{P}\mathbf{H}^H$. In RMT, this distribution is called the *empirical distribution* of the matrix $\mathbf{H}\mathbf{P}\mathbf{H}^H$ and is denoted $\mu_{\mathbf{H}\mathbf{P}\mathbf{H}^H}$. For those distributions, we associate *free moments* M_k of order k defined as $M_k = \mathrm{E}[\mathrm{tr}_N(\mathbf{HPH}^H)^k]$. In particular, a random matrix $\mathbf{A} \in \mathcal{M}(\mathbb{C}, N)$ is called *standard Wishart* if it can be written $\mathbf{A} = \frac{1}{L}\mathbf{XX}^H$, with $\mathbf{X} \in \mathcal{M}(\mathbb{C}, N, L)$ a standard Gaussian random matrix (i.e. a matrix with standard i.i.d. Gaussian entries). The empirical distribution of $\mathbf{A}$ is the Marchenko-Pastur law [6] that we denote μ_{η_c}, with $c = N/L$. Those Wishart matrices have a generalized version in which the column entries of $\mathbf{X}$ are correlated through a covariance matrix $\Sigma_{\mathbf{X}}$. Recent work on *Free Probability* [7] and RMT [5], [9] provide several tools to derive the empirical distributions of the sum and product of random matrices. In particular, when the matrix at hand is of the *information plus noise* type (those random matrices are deeply studied in [5]), then it is possible to access the empirical distribution of the information signal given the empirical distribution of the received noisy signal. This is the main result that we use in this work, which is part of the general framework of *free deconvolution*. In the following, we shall use the symbols $\boxplus$, $\boxminus$, $\boxtimes$ and $\boxslash$ respectively to retrieve the empirical distribution of the sum, difference, product and inverse of two random matrices respectively. For instance

$$\mu_{\mathbf{A}+\mathbf{B}} = \mu_{\mathbf{A}} \boxplus \mu_{\mathbf{B}} \tag{6}$$

$$\mu_{\mathbf{C}} = \mu_{\mathbf{A}} \boxslash \mu_{\mathbf{B}} \tag{7}$$

with $\mathbf{C}$ such that $\mathbf{A} = \mathbf{CB}$.

In our problem, described in the form of model (5), it turns out that the $N \times N$ matrix $\frac{1}{L}\mathbf{YY}^H$ is an *information plus noise* matrix with $\mathbf{N}$ a Gaussian random matrix (hence $\frac{1}{L}\mathbf{NN}^H$ is a Wishart matrix). Therefore, for large N and L, one can derive the empirical distribution of $\frac{1}{L}\mathbf{HP}^{\frac{1}{2}}\mathbf{\Theta\Theta}^H\mathbf{P}^{\frac{1}{2}}\mathbf{H}^H$ (i.e. $\mu_{\frac{1}{L}\mathbf{HP}^{\frac{1}{2}}\mathbf{\Theta\Theta}^H\mathbf{P}^{\frac{1}{2}}\mathbf{H}^H}$) from $\mu_{\frac{1}{L}\mathbf{YY}^H}$. This requires knowledge of the noise power σ^2 and reads [5]

$$\mu_{\frac{1}{L}\mathbf{HP}^{\frac{1}{2}}\mathbf{\Theta\Theta}^H\mathbf{P}^{\frac{1}{2}}\mathbf{H}^H} = \left((\mu_{\frac{1}{L}\mathbf{YY}^H} \boxslash \mu_{\eta_c}) \boxminus \delta_{\sigma^2} \right) \boxtimes \mu_{\eta_c} \tag{8}$$

where $c = N/L$ since the noise matrix is $N \times L$.

Also, the random entries of $\mathbf{\Theta}$ in equation (5) are standard Gaussian and independent. Therefore $\frac{1}{L}\mathbf{P}^{\frac{1}{2}}\mathbf{H}^H\mathbf{HP}^{\frac{1}{2}}\mathbf{\Theta\Theta}^H$ is a *generalized Wishart matrix* with covariance $\mathbf{P}^{\frac{1}{2}}\mathbf{H}^H\mathbf{HP}^{\frac{1}{2}}$.

As such, $\mu_{\mathbf{P}^{\frac{1}{2}}\mathbf{H}^H\mathbf{HP}^{\frac{1}{2}}}$ can be recovered from $\mu_{\frac{1}{L}\mathbf{P}^{\frac{1}{2}}\mathbf{H}^H\mathbf{HP}^{\frac{1}{2}}\mathbf{\Theta\Theta}^H}$ when the couple (N, L) is large with a constant ratio $c' = MN/L$ (M is constant) [5]

$$\mu_{\mathbf{P}^{\frac{1}{2}}\mathbf{H}^H\mathbf{HP}^{\frac{1}{2}}} = \mu_{\frac{1}{L}\mathbf{P}^{\frac{1}{2}}\mathbf{H}^H\mathbf{HP}^{\frac{1}{2}}\mathbf{\Theta\Theta}^H} \boxslash \mu_{\eta_{c'}} \tag{9}$$

The left expression of equation (8) is slightly different from the desired expression in the right part of equation (9). Still, thanks to the trace property, we have the link [6]

$$\mu_{\frac{1}{L}\mathbf{P}^{\frac{1}{2}}\mathbf{H}^H\mathbf{HP}^{\frac{1}{2}}\mathbf{\Theta\Theta}^H} = \frac{1}{M}\mu_{\frac{1}{L}\mathbf{HP}^{\frac{1}{2}}\mathbf{\Theta\Theta}^H\mathbf{P}^{\frac{1}{2}}\mathbf{H}^H} + \left(1 - \frac{1}{M}\right)\delta_0 \tag{10}$$

Finally, we similarly connect the left part of equation (9) to $\mu_{\mathbf{HPH}^H}$ through

$$\mu_{\mathbf{P}^{\frac{1}{2}}\mathbf{H}^H\mathbf{HP}^{\frac{1}{2}}} = \frac{1}{M}\mu_{\mathbf{HPH}^H} + \left(1 - \frac{1}{M}\right)\delta_0 \tag{11}$$

The empirical distribution of $\mathbf{HPH}^H$ was then derived from the empirical distribution of $\frac{1}{L}\mathbf{YY}^H$. As a consequence, the free moments $d_k = \mathrm{E}[\mathrm{tr}_N(\mathbf{HPH}^H)^k]$ can be retrieved from the free moments $m_k = \mathrm{E}[\mathrm{tr}_N(\frac{1}{L}\mathbf{YY}^H)^k]$. Surprisingly, it is shown in [8] that for any of the free classical operations (additive and multiplicative [de]convolution), the set of the first k moments of the [de]convolution results can be exactly recovered from the set of the k first moments of the operands (and vice-versa). This substancially reduces the computational effort.

The details of how to recover the moments d_k from the moments m_k as well as fundamentals of Random Matrix Theory and Free Deconvolution are provided in [19].

Our interest though is to find the diagonal entries of $\mathbf{P}$. Remarkably, it turns out that the matrix $\mathbf{HPH}^H$ is diagonal. Hence, for large (N, L) couples, we can easily derive all the theoretical *free moments* d_k of the distribution $\mu_{\mathbf{HPH}^H}$ [6] since all the $(\mathbf{HPH}^H)^p$ are diagonal matrices with entry

$$\left\{(\mathbf{HPH}^H)^p\right\}_{ij} = \left(\sum_{k=1}^{M} P_k |h_{ki}|^2\right)^p \delta_i^j \tag{12}$$

and then the p^{th} order moment $d_p = \mathrm{E}[\mathrm{tr}_N(\mathbf{HPH}^H)^p]$ of $\mathbf{HPH}^H$ can then be approximated for large N by

$$d_p = \frac{1}{N}\sum_{j=1}^{N}\left(\sum_{k=1}^{M} P_k |h_{kj}|^2\right)^p \tag{13}$$

At this point, expression (13) contains too many unknowns since, in addition to the P_k, also the h_{ij} are unknown. In fact those unknowns h_{ij} are the key that allows for the multiple SNR recovery provided that the channel *coherence bandwidth* is small compared to the system bandwidth. The reason lies in the diversity engendered by high channel selectivity. This is thoroughly discussed in section V. In the following we therefore discuss the frequency selective scenario under the hypothesis that the ratio *coherence bandwidth* over *system bandwidth* is small.

A. How to find the P_k

To deduce the cell power values from the moments d_p, we need to derive independent equations of the d_p in the variables $\{P_k, k \in \{1, \ldots, M\}\}$. Again, as will be discussed in section V, the M channels' frequency diversity is the key to provide those equations. Let us start by deriving, for large L, the moments d_p as in (13), which, for $p \in \{1, \ldots, M\}$, form a system of M equations for the M unknowns $\{P_k, k \in \{1, \ldots, M\}\}$.

In this case, since N is large, (13) can be approximated by

$$d_p = \mathbb{E}_h \left(\sum_{k=1}^{M} P_k |h_{kj}|^2\right)^p \tag{14}$$

where $\mathbb{E}_h$ denotes the expectation over the variables h_{ij}.

If N is larger than the typical coherence bandwidth size, the approximation of (14) is accurate. We then have a high confidence that the channel correlations do not have a strong impact in the final results. Based on the classical moments of the Rayleigh-distributed variables $|h_{kj}|$, we can tehrefore derive d_p as

$$d_p = \frac{p!}{2^{2p}} \sum_{\substack{k_1,\ldots,k_M \\ \sum_i k_i = p}} \prod_{i=1}^{M} \left\{ \sum_{k=0}^{k_i} \frac{(2k)!(2[k_i-k])!}{(k!)^2([k_i-k]!)^2} \right\} P_i^{k_i} \quad (15)$$

The proof of formula (15) and further details about the "large N" hyptothesis are provided in [19].

Therefore the system of equations formed by (15) for $p \in \{1,\ldots,M\}$ consists of multivariate polynomials in $P_1,\ldots,P_M$. This *homogenous symmetric multivariate polynomial* system can be rewritten

$$\sum_{k=1}^{M} P_k^p = Q_p(d_1,\ldots,d_p) \quad (16)$$

in which polynomial functions $Q_k \in R[d_1,\ldots,d_k]$ are theoretically determined in [19].

System (16) is then easier to solve. Its solution, the vector of powers $(P_1,\ldots,P_M)$, is unique and corresponds to the M roots (counted with their multiplicities) of the polynomial in X of degree M

$$X^M - \Pi_1 X^{M-1} + \Pi_2 X^{M-2} - \ldots + (-1)^M \Pi_M \quad (17)$$

where the elements Π_k are related to the $\sum_j P_j^i$ by the *Newton-Girard formula* [10]

$$(-1)^k k \Pi_k + \sum_{i=1}^{k} (-1)^{k+i} \left(\sum_{j=1}^{M} P_j^i \right) \Pi_{k-i} = 0 \quad (18)$$

A thorough and clear study of the particular $M = 3$ case as well as the complete derivations that lead to the Q_k polynomials are derived in [19].

V. Discussion

First, as already mentioned in the previous sections, our algorithm simplifies to a mere power detector when the number of sampling periods L is larger than the FFT size N. This would be valid when L is fairly larger than N but this imposes very long accumulations, which is no longer valid for the typical coherence time encountered in OFDM. Equivalently, N could be limited to very few elements. But then most of the provided information is discarded, which will heavily degrade the performance.

The channel conditions are also of primary importance. Indeed, if only one simulation shot is run (with sufficiently large N and L), and if the channel is very small, then the channel frequency response will be rather flat over the whole bandwidth.

This implies that all the moments of $\mathbf{HPH}^H$ will form a correlated system of equations and (16) cannot be derived since equation (15) is no longer valid. At best we can retrive the approximated total power received from all cells from such a situation. This is why a short coherence bandwidth (with respect to the total bandwidth) is desired.

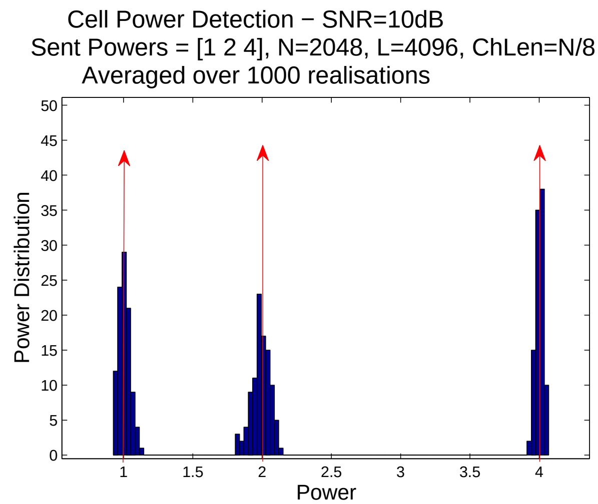

Fig. 2. Cell power detection, $N = 512$, $L = 1024$, Averaged over 1000 trials

If this small coherence bandwidth is not provided, then the scheme can be extended to use multiple antennae to introduce independent channel realizations. Thus, instead of using an $N \times L$ matrix $\mathbf{Y}$ at the reception, we can easily extend the scheme to use an $NN_r \times L$ data matrix.

VI. Simulation and Results

In the following, we use the results that were previously derived in the case of a three-cell network (i.e. $N_B = M = 3$) that the UE wishes to track. The set of cells studied in this part are of relative powers $P_1 = 4$, $P_2 = 2$, $P_3 = 1$.

For increased performance of our simulations, we shall average the estimated d_p values over 1000 channel realizations which we model as exponential decaying. Their delay spreads vary from 1 to $N/4$ samples.

In figure 2, we took matrices of size $N = 512$, $L = 1024$ and a Rayleigh channel of length $N/8$ samples. A hundred realizations of this process are run. The SNR is 10dB. Figure 2 shows that $(P_1,\ P_2,\ P_3)$ are well recovered.

The next experiment aims to estimate the noise influence on the cell power recovery. This is obtained by comparing the $SNR = 30$dB case to the $SNR = -10$dB scenario. Figure 4 provides the results (with 1000 accumulations over 100 trials) and shows that, surprisingly, whatever the noise level (even if it actually perfectly matches one specific cell power), the cell power recovery is substancially the same if $N,\ L$ are large enough.

Also, we need to test the robustness of our algorithm against practical channels and not only "ideal" i.i.d or theoretical exponential decaying channels with high delay spread. This is shown in figure 3 that proposes a comparison between the ideal long channel situation and the 3GPP-LTE [2] standardized Extended Vehicular A (EVA) and Extended Typical Urban (ETU) channels with characteristics

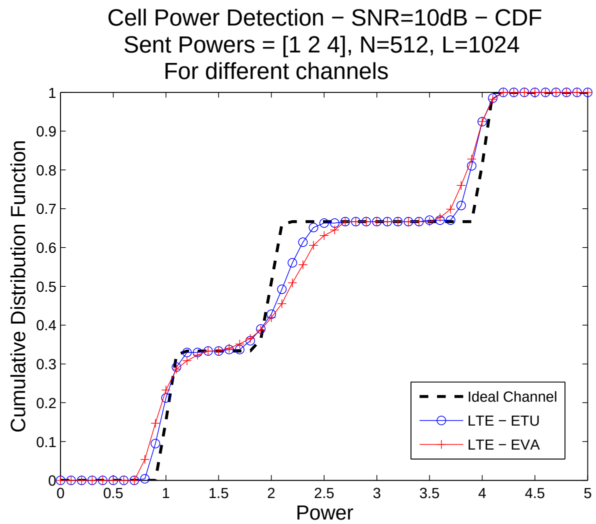

Fig. 3. Cell power detection of LTE short Channels, $L = 1024$, Averaged over 1000 trials

Channel Type	RMS Delay Spread	Channel Length
EVA	357ns	$N/27$
ETU	991ns	$N/13$

Here we considered a mobile handset with 2 antennae $N_r = 2$, working under a size $N = 256$-FFT, with $SNR = 10$dB. L equals 1024 and the results are averaged over a thousand trials, to assure fair comparison with the previous results. The cumulated distribution function of the detected power distribution for those channels is provided in figure 3. The latter shows a rather good behaviour in both ETU and EVA channels. Nonetheless their short delay spreads lead to less performant results. Note also that a certain bias in the mean power estimates is introduced in this case.

Surprisingly, it turns out that the chosen zero mean unit variance distribution of the input signals $\mathrm{s}^{(l)}$ does not matter: the results show the same performance. This is a general observation in free deconvolution which has not been proven yet. Therefore in our simulations, QPSK modulations showed the exact same behaviour as purely Gaussian distributed input signals. Also, we carried out some simulations in which we purposely took $N_B > M$ (e.g. $B_S = 4$ base stations emitting while only $M = 3$ are assumed). This has fairly bad consequences since the characteristic polynomial (17) often has non-real solutions. As a consequence, the number of base stations should always be upper bounded.

The uplink scenario in which a base station wants to determine the powers of multiple UE's in its cell can be equally derived by changing M into the number of potential users in the cell and N_r into the number of antennae at the base station.

VII. Conclusion

In this contribution, we demonstrated a practical way to blindly detect neighboring cells in a distributed OFDM network. Assuming constant transmission in those cells on a

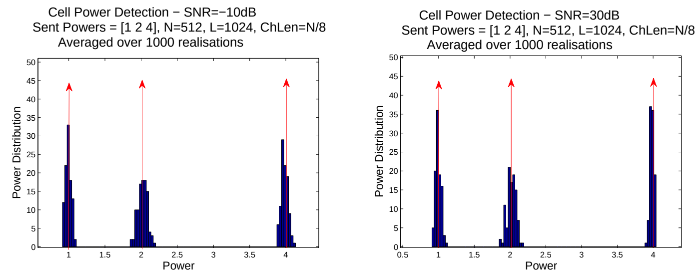

Fig. 4. Cell power detection at $SNR = -10dB$ (left) and $SNR = 30dB$ (right), $N = 512$, $L = 1024$, Averaged over 1000 trials

fairly large bandwidth (large enough to ensure that the channel coherence bandwidth is small in comparison), we showed that one can blindly determine the individual SNR of every cell. This is particularly suitable for next generation OFDM systems which aim to reduce the amount of synchronization sequences to keep track of the neighboring cells.

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