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Chapter 1

OFDMA Synchronization

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Behind its strong theoretical advantages such as high spectral efficiency and simplified equalization, two important practical drawbacks are associated to the OFDM technology. The more fundamental of the two is referred to as PAPR (Peak to Average Power Ratio). Through the inverse Fourier transform operation, OFDM turns the frequency-domain transmitted data into a time-domain sequence with samples of potentially very strong and very low amplitudes. This compels both transmitter and receiver ends to be highly sensitive to large magnitudes of signals. This problem is even increased when multiple access (OFDMA) systems are considered in which multiple users enjoy different modulation codebooks or different power allocations, since then the resolution of the receiving devices from low power users need to accommodate the strong signals in destination to their neighbors. The second practical issue of OFDM systems lies in the so-called CFO (carrier frequency offset), which corresponds to a misalignment between reference frequencies at the transmitter and at the receiver. As shown in Section 1.2, even a small CFO (compared with the inter-carrier spacing) might be detrimental to the reliability of the communication. Formerly, when OFDM was only used in wired systems (such as xDSL), frequency misalignment issues were solved thanks to a single synchronization step prior to the proper communication. On the contrary, wireless communications at high mobility come along with fast varying *Doppler shifts*, which contribute, as will be shown in Section 1.2.1, to dynamically change the CFO value. A mere initial synchronization step is in this case not enough to ensure reliable data transmission. The frequency reference must be constantly tracked. This explains why, political reasons put aside, more than ten years were needed for the first mobile OFDM systems to appear.

The following chapter will detail the complete synchronization steps needed for practical OFDMA-based systems to rapidly enter the proper data exchange phase. It will be shown in Section 1.2 that system designers have to anticipate all synchronization problems to model a viable OFDMA communication scheme. It will be then shown in Section 1.3 that no theoretically optimal solution to recover synchronization has ever been proposed; all classical synchronization techniques summarize to various solutions, each of them being either more "robust" to some channel conditions, more appropriate to some configuration scheme or easier to implement than others. In Section 1.4, the particular example of synchronization for the 3GPP-LTE (Long Term Evolution) standard will be thoroughly studied in light of the discussions treated in the technical sections. In Section 1.5 the authors anticipate future challenges for the synchronization field throughout their own contributions in synchronization-related domains. Finally, the conclusions of this chapter are drawn in Section 1.6.

1.1 Introduction

Before the fundamental work of Shannon in 1948 [1] and the introduction of the channel capacity, no theoretical bound for data transmission rate had been proposed; therefore, at that time, communication-related questions were investigated without any objective comparison tool nor any performance evaluation bound. In the realm of synchronization, some sixty years later, the time has not yet come for such a unification of the field. That is, there exists no theoretical bounds on the amount of energy (or time) required for a transmitter-receiver link to synchronize their system parameters, e.g. reference frequency, timing, clock speed etc. Besides, there does not exist any theoretical derivation of the capacity of a time-limited communication taking into account the need for synchronization and channel estimation. In spite of a few recent contributions [2]-[3], the amount of bits dedicated to synchronization that is needed to maximize the transmission capacity is yet unknown; too little synchronization effort leads to numerous decoding errors while too much synchronization effort leaves little room for bits dedicated to the actual communication. In both extreme scenarios, the impact on capacity is disastrous but still no satisfying *reliability* versus *spectrum efficiency* trade-off study has yet been proposed. In fact, one might even say that the processes dedicated to synchronization should not be isolated from the effective data to be transmitted, as both synchronization parameters and useful data are equally unknown entities to the receiver; instead the whole *data plus synchronization parameters* should be encoded in such a way to achieve the optimal transmission rate in a finite time. Therefore the whole field of synchronization is not yet fully understood and the set of proposed synchronization parameter recovery processes is only based on many different solutions, which are not unified by strong theoretical bases. The latter attempt to tackle either individual parameter estimations problems or joint decoding, joint channel estimation and synchronization, etc. In the coming sections, those solutions will be divided into *rough* (also referred to as *coarse*) or *fine* estimators and *data-aided* (DA) or *non data-aided* (NDA) algorithms; however this division is merely conventional and does not reflect any theoretical foundation for synchronization, as will be discussed in Section 1.5.

If most academic studies on OFDM often consider ideal synchronization, the reader must understand that synchronization is an important task and, as such, should not be undermined. The fundamental difficulty in OFDM is to preserve the orthogonality between subcarriers when mobile terminals are in motion and thus are subject to Doppler frequency shifts. Besides, wireless networks encourage more and more *packet-switched* (e.g. IEEE 802.16 WiMax [24], 3GPP-Long Term Evolution [23]) than *connected* (e.g. Digital Video Broadcasting [25], Digital Audio Broadcasting [26]) transmission modes: the former has the strong advantage to be highly dynamic and has copped with its past latency problem. Future communication technologies will therefore rely on short data (i.e. packet) transmissions, compelling the synchronization recovery processes to operate very fast[1]. In most OFDM technologies, the synchronization phase consists first in a *power detection* process,

[1]note also that the progressive integration of mobile data transfers to the Internet requires to adapt to packet-switched communications.

meant to roughly identify a power source. The next procedure is classically an *acquisition* phase aiming to give a rough estimate of the system synchronization parameters (e.g. slot start time, reference frequency). First reliable data exchanges are in general possible at a low rate at this point. During the rest of the communication, especially in the connected mode, the synchronization processes enter the *tracking* phase to regularly update the parameter estimates. In the rest of this chapter, we concentrate on the three following main synchronization parameters,

- the Carrier Frequency Offset (CFO) which corresponds to a mismatch between the transmitter and the receiver frequency references. Even small values of CFO are detrimental to the system performance; as a consequence, frequency offsets must be efficiently corrected. In a mobile system particularly, CFO is fast varying due to Doppler shifts. It is thus very challenging for mobile OFDM designers to ensure a continuous quality of service in high mobility conditions.

- the Symbol Timing Offset (STO) which is defined as the time difference between the real and estimated beginnings of the received OFDM symbol. As is detailed in the subsequent sections, small STO are not critical for OFDM since the length of the cyclic prefix, usually slightly longer than the maximum channel delay spread, can absorb negative time offsets: this avoids inter-symbol interference; moreover, STO correction might not even be necessary, since the induced frequency rotation effect can be considered as part of the communication channel: therefore, channel estimation usually allows to conceal the STO problem. As a consequence, STO estimation for OFDM is less tackled in the literature than CFO. However, when no channel estimation is performed (so typically during the initial synchronization procedure), the phase rotations introduced by timing offsets in the received frequency-domain signal might disturb the synchronization processes.

- the Sampling Clock Offset (SCO) which is in general a negligible effect of misalignment between the local oscillators of the two communicating ends. Typically, a shift in those oscillators is due to the physical sensitivity (e.g. temperature, pressure variations) of embedded crystal oscillators. Since symbol timing shifts due to SCO are classically harmful only after the reception of hundreds or thousands OFDM symbols, SCO synchronization does not need to be performed very often.

In a mobile distributed system, those parameters need to be estimated both at the mobile devices and at the fixed base station to ensure reliable communication in both downlink and uplink. The synchronization procedure at the mobile receiver is often treated as multiple *single-parameter estimation* problems or as a *joint parameter estimation* problem. In a single-user scenario in which the communication link to the base station is exclusive to a specific user, synchronization at the base station is similar to synchronization at the mobile terminal. However, in systems based on OFDMA, all terminals transmitting in the uplink have different parameter offsets, so that the allocated user bands overlap one another. For the base station, this means the users cannot be separated in the frequency domain. One major consequence is the introduction of strong Inter-Carrier Interference (ICI), which means on a processing viewpoint that Discrete Fourier Transform (DFT) operations are better avoided for synchronization purposes at the base station. Most of the classical frequency-domain synchronization algorithms are therefore unavailable to the base station; this constitutes a fundamental difference between OFDM and OFDMA systems. Note that the parameter offsets in the uplink are in general very small when downlink and uplink communications are scheduled along a Time Division Multiplexing (TDD) strategy, for which time and frequency references are the same in uplink and downlink. For this data duplexing scheme, usually in practice, the transmitting terminals already have a good estimate of the time and frequency offsets based on the primary downlink transmissions (i.e. primary synchronization sequences) that allow for an initial rough synchronization. As a consequence, in TDD, it will be in general acceptable to consider small STO and CFO for synchronization algorithms in the uplink. On the contrary, in Frequency Division Duplex (FDD) mode, which is used more often in practice[2], can only share timing synchronization between downlink and uplink transmissions, if both links share are time-synchronous. However, frequency references being different in downlink and uplink, in FDD, one has to assume potentially large CFO.

Notation: In the following, boldface lower-case symbols represent vectors, capital boldface characters denote matrices ($\mathbf{I}_N$ is the $N \times N$ identity matrix). The transpose and Hermitian

[2]the TDD mode has the strong disadvantage to require a thorough synchronization of transmissions in the time-domain. In particular, guard periods need be taken into account that absorb the (potentially long) propagation delay. The strong advantage of TDD however lies in an easy tuning of the ratio downlink rate/uplink rate, which is fixed in single-user OFDM with FDD.

transpose are denoted $(\cdot)^{\mathsf{T}}$ and $(\cdot)^{\mathsf{H}}$ respectively. The symbol $\mathrm{E}[\cdot]$ denotes expectation.

1.2 Effects of timing and frequency shifts

1.2.1 Origins of parameter offsets

The synchronization offsets originate from various physical phenomena. Some are due to static hardware defects (e.g. SCO, CFO) or to an imperfect initial synchronization process (e.g. STO, CFO) while others are mainly due to dynamic effects that depend on the channel conditions (e.g. Doppler shifts in CFO).

Static effects

The central bandwidth frequency and the sampling frequency are always imposed by the technology standard. All communicating entities are then required to align to these frequencies. However, the precision of the hardware material is often impacted by environmental conditions. Typically, in a mobile phone device, both SCO and CFO are aligned on the embedded *crystal oscillator* frequency. Those crystals are sensible to external conditions such as pressure, temperature and aging. A mismatch between the oscillator frequencies at the transmitter and the receiver causes frequency offsets. In practice, this mismatch is the main reason that explains SCO. Since both offsets are closely related, oscillator mismatch might also be the main explanation for CFO. However, this statement only stands when the communication channel is static. Indeed, when at least one communicating side is in motion, then dynamic Doppler effects come into play and usually become the critical reason to explain CFO. In Figure 1.1, the typical response to temperature of the cheap digital crystal oscillator (DXO) and the onerous voltage controlled temperature compensated crystal oscillator (VCTCXO) is depicted. Note that the effect of temperature is rather important: if the central frequency is set to 2 GHz, then a frequency drift of ± 5 ppm corresponds to ± 10 KHz, which is of the order of the typical subcarrier spacings.

STO is of a different nature. Indeed, while SCO and CFO might be ideally null before the beginning of any communication (assuming perfect crystal oscillators at both communication ends and no motion), STO appears when the communication begins, since too little prior information is available for both communicating entities before their first handshake. To align the timing references, the beginning of each OFDM symbol must be identified. However, as will be presented in Section 1.2.2, the symbol timing parameter is not required to be finely tracked since even a rough estimate might not lead to any performance loss. By rough, we mean here that the timing error is not larger than the Cyclic Prefix (CP) duration.

Those parameters usually do not encounter practical synchronization issues. Once the reference timing and sampling rate are appropriately estimated, those parameters do not significantly change during the overall communication process. If the time for communication is rather long, e.g. long enough for the local temperature to change, then refinements on the STO and SCO are desirable but do not usually face any difficulty. In mobile multi-cell networks, when a terminal hands over a neighboring unsynchronized base stations, this initial synchronization process will be triggered anew.

Dynamic effects

By dynamic effects, we refer to the fast varying phenomena which impact the synchronization parameters of the system. In particular, in mobile OFDM systems, the relative distance $d(t)$ between the transmitting and receiving entities varies along with the time t. Consider the simple scenario of a fixed base station transmitting a sinusoidal waveform $x(t)$ of period T_0, and a mobile handset at initial distance $d_0 = d(0)$ from the base station moving at a constant speed $v \ll c$ (with c the light speed) at an angle ϕ from the base station-handset direction. This scenario is presented in Figure 1.2. Using Al-Kashi's geometrical relations, at time $t = T_0$,

$$d(T_0) = \left(d_0^2 + v^2 \cdot T_0^2 - 2v \cdot d_0 T_0 \cdot \cos(\phi)\right)^{\frac{1}{2}}$$

Therefore, the relative frequency of the sent signal, that is $f_{BS} = \frac{1}{T_0}$ from the base station's viewpoint, is different from the handset's viewpoint and equals

$$f_H = \left(T_0 \sqrt{1 + \left(\frac{v}{c}\right)^2 - 2\frac{v}{c}\cos(\phi)} \right)^{-1} \tag{1.1}$$

$$\simeq f_{BS} \left(1 - \frac{v}{c}\cos(\phi)\right)^{-1} \tag{1.2}$$

$$\simeq f_{BS} \left(1 + \frac{v}{c}\cos(\phi)\right) \tag{1.3}$$

for ϕ not too close to $\frac{\pi}{2}$ (otherwise the Taylor coefficient of second order must be taken into account).

This relative shift in frequency is referred to as *Doppler effect*. The received signal $y(t)$ then reads

$$y(t) = \rho x(t-\tau)e^{2\pi i\xi t} + w(t) \tag{1.4}$$

with $w(t)$ the additive noise process, ρ the channel attenuation, τ the propagation delay and $\xi = f_H - f_{BS}$ the Doppler shift. To understand the $e^{2\pi i\xi t}$ factor, one needs to observe that in the Fourier domain, due to the Doppler effect, the received signal originates from the transmitted signal convolved by a frequency shift, i.e. a Dirac function, of amplitude ξ; back in the original domain, this Dirac convolution turns into a complex exponential product.

The model (1.4) is generalized to practical realistic situations where not only one but numerous scatterers are present in the medium. Those scatterers are gathered into subsets of common Doppler shift and propagation delay. This yields the model

$$y(t) = \int_\tau \int_\xi \rho(\xi,\tau)x(t-\tau)e^{2\pi i\xi t}d\xi \mathrm{d}\tau \tag{1.5}$$

where $\rho(\xi,\tau)$ accounts for the mean (complex) fading of the subset of scatterers which induce a propagation delay τ and a Doppler shift ξ.

The *Doppler spectrum* $D(\xi)$, which denotes the relative signal power received at Doppler shift ξ, is computed as

$$D(\xi) = \int_\tau \mathrm{E}[|\rho(\xi,\tau)|^2]\mathrm{d}\tau \tag{1.6}$$

from which one derives the *Doppler spread* B_d, defined as the standard deviation of the random variable ξ (whose density function is given by $D(\xi)/\{\int_\xi D(\xi)\mathrm{d}\xi\}$),

$$B_d = \left(\frac{\int_\xi (\xi - \xi_0)^2 D(\xi)\mathrm{d}\xi}{\int_\xi D(\xi)\mathrm{d}\xi} \right)^{\frac{1}{2}} \tag{1.7}$$

with ξ_0 the mean value of ξ.

CFO estimation consists in tracking the value of ξ_0, so to minimize the effects of frequency offsets in the received signal. Note in particular that a large Doppler spread would be detrimental to the decoding of the received OFDM symbol. Indeed, as shall be discussed in Section 1.2.2, if much power is received outside the exact subcarrier frequency, then the decoding Bit Error Rate (BER) dramatically increases. However, large Doppler spreads typically come along with very short *channel coherence time* (i.e. the time during which the consecutive channel realizations are strongly correlated) which does not satisfy the OFDM fundamentals that require the channel realization to be constant at least during an OFDM symbol[3].

1.2.2 Performance impacts of parameter offsets

Thanks to the time-frequency duality of the OFDM modulation, the effects of STO, SCO and CFO are very similar. Basically, a constant offset in a representation domain translates into a phase rotation in the dual Fourier domain. However, for every particular offset, some fundamental differences arise that we develop in the following.

Consider an OFDM system of N subcarriers, N_{CP} cyclic prefix samples and sampling period T_s. Therefore the subcarrier spacing Δ_f equals $1/(NT_s)$. For notational ease, the entries of the discrete vectors $\mathbf{a} = [a_1, \ldots, a_N]^\mathsf{T}$ sampled from a continuous waveform $a(t)$ are denoted $a_k = a(t_0 + kT_s)$ with t_0 the beginning of the OFDM symbol. The OFDM data symbol to transmit is denoted $\mathbf{s} = [s_1, \ldots, s_N]^\mathsf{T}$; its variance $\mathrm{E}[\mathbf{s}^\mathsf{H}\mathbf{s}]$ is denoted P. The time-domain OFDM symbol vector $\mathbf{x} = [x_1, \ldots, x_N]^\mathsf{T} = \mathbf{F}^\mathsf{H}\mathbf{s}$, with $\mathbf{F}$ the Fourier matrix of size N, is sent through the channel $\mathbf{h} = [h_1, \ldots, h_L]^\mathsf{T}$ of delay spread L symbols (we assume

[3]indeed, if the channel varies during one OFDM symbol, the channel matrix in the time-domain is no longer circulant and then no longer diagonalizable in the Fourier basis.

that $L \leq N_{\text{CP}}$). The noisy time-domain received signal is called $\mathbf{y} = [y_1, \dots, y_N]^\mathsf{T}$ and its Fourier dual is denoted $\mathbf{r} = [r_1, \dots, r_N]^\mathsf{T} = \mathbf{Fy}$. The OFDM system aims to decode $\mathbf{r}$ from the original data $\mathbf{s}$ with the smallest possible BER. Under perfect synchronization, we have the classical discrete channel convolution effect, for all $n \in \{1, \dots, N\}$

$$y_n = \sum_{l=0}^{L-1} h_l x_{n-l} + w_n \tag{1.8}$$

with $\mathbf{w} = [w_1, \dots, w_N]^\mathsf{T}$ the noise process of variance $\mathrm{E}[\mathbf{w}^\mathsf{H}\mathbf{w}] = \sigma_{\mathbf{w}}^2$.

Effects of STO

Consider now that the system comprises a single transmitter and that the timing synchronization to the receiver under investigation is offset by θT_s. Equation (1.8) becomes

$$y_n = \sum_{l=0}^{L-1} h_l x_{n-l-\theta} + w_n \tag{1.9}$$

Assuming an infinitely small energy acquisition time at the receiver and a perfect square pulse shape for the time-domain signal, θ can be taken as an integer without generality restriction.

If $0 \leq \theta \leq N_{\text{CP}} - L$, then the received OFDM symbol does not suffer from the channel leakage due to previous blocks. Then the cyclic prefix property and hence the orthogonality property hold. The output of the discrete Fourier transform (DFT) block at the receiver therefore outputs, for $k \in \{1, \dots, N\}$,

$$r_k = e^{2\pi i \frac{k\theta}{N}} H_k s_k + W_k \tag{1.10}$$

where $[H_1, \dots, H_N]^\mathsf{T} = \mathbf{Fh}$ and $[W_1, \dots, W_N]^\mathsf{T} = \mathbf{Fw}$ respectively denote the Fourier transform of $\mathbf{h}$ and $\mathbf{w}$.

This results into a phase rotation of the received symbols. As shall be detailed in Section

1.3, this effect is easily corrected and might even be harmless. By incorporating the phase rotation into the channel frequency response: $e^{2\pi i \frac{k\theta}{N}} H_k$, a mere channel estimation process suffices to absorb the STO effect.

However, if $\theta \notin [0, N_{\mathrm{CP}} - L]$, then the system orthogonality collapses, with the direct consequence to introduce Inter-Symbol-Interference (ISI) from adjacent OFDM blocks. The system output is generally modelled [4] as the expected DFT weighted by an attenuation factor $\alpha(\theta)$ plus an additional interference process $I(\theta)$ of power σ_I^2 due to ISI,

$$r_k = e^{2\pi i \frac{k\theta}{N}} \alpha(\theta) H_k s_k + I(\theta) + W_k \tag{1.11}$$

The relative performance loss is classically measured through the degradation $\gamma_{STO}(\theta)$ between the Signal-to-Noise Ratio (SNR) in the synchronized ($\mathsf{SNR}_{sync} = \frac{P}{\sigma_{\mathbf{w}}^2}$) and the unsynchronized cases ($\mathsf{SNR}_{unsync} = \frac{P\alpha(\theta)^2}{\sigma_{\mathbf{w}}^2 + \sigma_I(\theta)^2}$) [4],

$$\gamma_{STO}(\theta) = \frac{\mathsf{SNR}_{sync}}{\mathsf{SNR}_{unsync}} = \frac{1}{\alpha(\theta)^2} \left(1 + \frac{\sigma_I(\theta)^2}{\sigma_{\mathbf{w}}^2} \right) \tag{1.12}$$

the behaviour of which is depicted in Figure 1.3 for an OFDM system with $N = 128$ DFT size, $N_{\mathrm{CP}} = 9$ cyclic prefix, communicating through an exponential decaying channel of length $L = 5$.

In multiple access uplink scenarios, the problem is more involved and might be very harmful to the system performances. Indeed, every user k faces a different STO θ_k so that, even when all θ_k belong to the ISI-free region (i.e. $0 \leq \theta_k \leq N_{\mathrm{CP}} - L_k$ with L_k the length of the channel seen from user k), the DFT output at the receiver introduces multiple access interference (MAI). In those situations, the performance limiting factor is linked to the largest STO gap ($\max_{k,k'} |\theta_k - \theta_{k'}|$) among all pairs of users. Therefore system performance is dictated by the ill-conditioned users; this is one of the reasons (the PAPR problem and the similar SCO, STO effects studied in the following sections are other reasons) why OFDMA is rarely used in uplink schemes in practice.

Effects of CFO

Suppose now perfect timing synchronization (i.e. $\theta = 0$) and introduce a frequency offset δ between the transmitter and the receiver. To observe the consequences of frequency offsets, the continuous frequency representation of the OFDM signals must be examined. The receive symbol of Equation (1.8) is there updated as

$$y_n = e^{2\pi i \frac{n\delta}{N}} \sum_{l=0}^{L-1} h_l x_{n-l} + w_n \tag{1.13}$$

Assuming again perfect square pulse shaping in time, after some computation, the signal at the output of the receiver DFT is [6]

$$r_k = e^{2\pi i \delta \frac{N+N_{\text{CP}}}{N}} \sum_{k'=1}^{N} H_{k'} s_{k'} \text{sinc}\left(\pi[\delta + k' - k]\right) e^{i\pi \frac{(\delta+k'-k)(N-1)}{N}} + W_k \tag{1.14}$$

in which we remark that when δ is a multiple of the subcarrier spacing Δ_f, i.e. $\delta = p \in \mathbb{Z}$, the sum in (1.14) reduces to a single non-null term which corresponds, up to a constant phase, to the data symbol intended to the p^{th} next subcarrier. Therefore, *integer* frequency offsets merely engender a phase rotation and a circular shift of all subcarriers. The adaptive decoding processes required to compensate for integer CFOs are therefore not a challenging task.

However, if δ is fractional, every received sample r_k suffers from ICI from all subcarriers (and not only from neighboring subcarriers). Following Speth's SNR degradation measure γ_{CFO} [5], the performance loss for small values of δ is approximated by

$$\gamma_{CFO}(\delta) = \frac{\mathsf{SNR}_{sync}}{\mathsf{SNR}_{unsync}} \simeq 1 + \frac{\pi^2\delta^2}{3}\mathsf{SNR}_{sync} \tag{1.15}$$

As depicted in Figure 1.4, the performance is dramatically impacted even for small values of δ. For instance, at $\mathsf{SNR}_{sync} = 20$ dB, a CFO of $4\% \times \Delta_f$ leads to $\mathsf{SNR}_{unsync} \simeq 18$ dB, which might turn out a sufficient loss to prevent the transmission of a 64-QAM modulation for instance. Fast CFO estimators are then required to recover synchronization.

Identically to the STO increased complexity in OFDMA schemes, CFO in multiple access technologies is more involved to compensate. Those topics are discussed in the subsequent sections.

The uplink case

We dedicate a section to the OFDMA uplink, since the major difficulty in OFDMA synchronization lies in the uplink. For this reason and also because of the PAPR problem, OFDMA is not often used in the uplink of centralized mobile networks. For instance, in 3GPP-LTE, Single-Carrier Frequency Division Multiple Access (SC-FDMA) is used in the uplink in place for OFDMA. The uplink synchronization issue is twofold: (i) multiple users face different STO and CFO, turning the parameter estimation problem into a vectorial-parameter estimation problem, (ii) contrary to the single-user scenario where STO and CFO effects can be counteracted at the receiver (e.g. counter-rotation of CFO shift and clock-adjustment to STO delay), the problem of general multi-parameter offsets is only solved via maximum-likelihood NP-hard algorithms.

The model for the uplink of an OFDMA cell with M transmitting users indexed by $m \in \{1, \dots, M\}$, with respective STO θ_m and CFO δ_m, reads

$$y_n = \sum_{m=1}^{M} e^{2\pi i \delta_m \frac{n}{N}} \sum_{l=0}^{L-1} h_l^{(m)} s_{n-l-\theta_m}^{(m)} + w_n \tag{1.16}$$

where the subscript (m) indicates that the considered channel and signal belong to the m^{th} user.

In the frequency domain, assuming $\delta_m = 0$ for all m,[4] from Equation (1.14), the post-DFT receive signal reads

$$r_k = \sum_{m=1}^{M} e^{2\pi i \delta_m \frac{N+N_{\text{CP}}}{N}} \sum_{k' \in \mathcal{S}_m} H_{k'}^{(m)} s_{k'}^{(m)} \text{sinc}\left(\pi[\delta_m + k' - k]\right) e^{i\pi \frac{(\delta_m + k' - k)(N-1)}{N}} + W_k \tag{1.17}$$

[4] or, as will be seen later, assuming equivalently that δ_m is included into the channel $H^{(m)}$.

where $\mathcal{S}_m$ is the set of subcarriers allocated to user m (these sets are obviously mutually exclusive in this case).

As suggested above, while in Equation (1.14) it is clear that changing k by $k - \delta$ solves the CFO problem (i.e. by an appropriate shift of the radio interface frequency reference), it is impossible here to undo the ICI effect by a mere frequency shift at the receiver. Assuming large frequency offsets δ_m, the ICI effect on the general performance is dramatic and cannot be completed annihilated for orthogonality between users cannot be recovered.

In the following sections, we provide techniques and algorithms which allow to recover STO and CFO. The scope of these sections is restricted to the main key methods used in practice. In the literature of OFDM synchronization, and synchronization at large, there exist a large number of other methods, so that the authors do not claim gathering in the next pages the totally of the contributions in the synchronization field. Also, some recent work from the authors are presented in the last sections, which provide an information-theoretical Bayesian view to synchronization.

1.3 Synchronization recovery

Synchronization recovery is an information theoretic dilemma. The ideal transmission scheme on a given channel, whose performance is assessed thanks to its ergodic capacity [1], contains no *excess bandwidth* (i.e. no *useful* information is transmitted more than necessary). However, synchronization parameters, which need to be shared or estimated by both communication ends, are considered *non-useful* information for the data transmission purpose. As a consequence, two situations classically arise in practical OFDMA systems,

- specific pilot sequences are transmitted to allow fast synchronization at the receiver. These techniques, qualified data-aided (DA), have been used in most of the existing telecommunication systems for they are easier to implement and allow for fast synchronization. However, they imply transmitting non-useful data at the expense of a reduction in the achievable useful data rate. This statement is even more verified for systems, such as mobile communication handsets, that require to constantly track the synchronization parameters: in those scenarios (see e.g. 3GPP-LTE, Section 1.4), many pilot sequences might be used to parameter estimation purposes.

- parameter estimation is conducted by exploiting excess bandwidth inherent to the system. If the communication scheme shows good transmission rate performance relatively to the channel capacity, then little excess bandwidth is available so that those estimators are usually very slowly converging. Moreover, the excess bandwidth might turn out very impractical to exploit, contrary to pilot sequences designed for synchronization purpose. Therefore, those processes, often referred to as non-data aided (NDA), are usually complemented by DA methods. Some other schemes similarly exploit excess bandwidth due to transmitted constellations, redundancy due to excessive channel coding etc. Those are usually isolated from the NDA group into the special class of *blind techniques*. Following our excess bandwidth philosophy, we shall indifferently qualify them NDA or *blind* in what follows.

Regarding inherent redundancy, the OFDM case is particularly simple. Thanks to the subcarrier orthogonality, the *spectral efficiency* (i.e. how much of the frequency spectrum is used) achieves the theoretic Shannon's ergodic capacity. However, in the time-domain, the cyclic prefix duration is completely lost for useful communications for it consists in a mere copy of transmitted symbols which are discarded at the receiver. This cyclic prefix therefore constitutes the major part of the OFDM excess bandwidth. This is why, already fifteen years ago, the pioneering work on OFDM synchronization [7], [8] exploited symbol repetition either in dedicated pilots or in the cyclic prefix.

As previously mentioned, the classical approach to synchronization is a multi-step process: quick and rough estimators are firstly used before advanced tools perform refined estimates. We shall review in the following the main historical synchronization techniques found in the current literature.

1.3.1 STO estimation

Downlink STO estimation

A first very rough STO estimation is often handled as a first synchronization step in OFDM. Indeed, as long as the beginning of the OFDM sequence is not approximately found, pilot sequences cannot be read and in particular DFT operations cannot be performed without being impacted by a strong inter-symbol interference from consecutive OFDM blocks. The very rough STO estimator often consists in a mere correlation process with a pilot sequence designed to enjoy desirable correlation properties. This is the case in particular for the popular Zadoff-Chu (ZC) sequences [11] with properties detailed in Section 1.4. At the end of this first STO process, the OFDM symbol timing error is expected to be less than the cyclic prefix length.

When this very rough estimation is obtained, classical methods are used to perform the so-called rough, or coarse, STO acquisition. The latter consists in exploiting the time correlation properties of a repetitive structure *insensitive to CFO* so that CFO can be evaluated in a posterior phase. It is desirable to carry out the first estimates in the time domain since, as we already noticed, even small mismatches in the synchronization parameters spawn dramatic signal distortion after DFT processing. For instance, in [12], a pilot sequence $\mathbf{x}$ made of the concatenation of two identical vectors $\{x_1, \ldots, x_{N/2}\} = \{x_{N/2+1}, \ldots, x_N\}$ of size $N/2$ is designed for STO estimation. The time-domain received sequence reads

$$\begin{cases} y_n & = e^{2\pi i \delta n/N} \sum_{l=0}^{L-1} h_l x_{n-l-\theta} + w_n \\ y_{n+N/2} & = e^{2\pi i \delta n/N} e^{\pi i \delta} \sum_{l=0}^{L-1} h_l x_{n-l-\theta} + w_{n+N/2} \end{cases}, \; n < N/2 \quad (1.18)$$

Thanks to a window of size $N/2$ sliding along hypothetic values for θ, the absolute value of the cross-correlation between the first and second part of $\mathbf{y}$ is computed. This allows to remove the CFO rotation effect. The maximum value $\hat{\theta}$ of the correlations is then sought to generate the STO estimate,

$$\hat{\theta} = \arg\max_{\tilde{\theta}} \frac{\left| \sum_{n=1}^{N/2} y_{n+\tilde{\theta}} y^*_{n+N/2+\tilde{\theta}} \right|}{\sum_{n=1}^{N/2} \left| y_{n+\tilde{\theta}} \right|^2} \tag{1.19}$$

Note that this maximum is usually not unique. Indeed, as described in Section 1.2.2, when the cyclic prefix is longer than the channel delay spread, then as long as $L - N_{\mathrm{CP}} < \theta < 0$ the fundamental subcarrier orthogonality is preserved. This indicates that the solution of (1.19) is not a unique value but a continuum of size $N_{\mathrm{CP}} - L$. Note that this also allows for a rough estimation of the channel length. This is pictured in Figure 1.5 in which an exponential decaying channel of length $L = 5$ is used for an OFDM system with $N = 128$, $N_{\mathrm{CP}} = 9$, under different SNR values. Some, seeing in this *plateau* a synchronization inconvenience, proposed refined algorithms [13] that result in a smaller continuum of solutions (containing the perfect synchronization value) at the expense of a higher false alarm rate in the detection of the maximum.

The same method can be used without reference signals thanks to the OFDM inner redundancy. Indeed, if the cyclic prefix is larger than the channel length, then $N_{\mathrm{CP}} - L$ symbols are duplicated in the signal and the STO can be therefore blindly estimated by cross-correlation of the cyclic prefix symbols. However, this technique is rarely used in practice for its reliability depends on the channel conditions (e.g. N_{CP} might not be fairly larger than L and the correlation size might be very small). Note that all those techniques have the strong advantage to be independent of the channel realization, which is a feature typically sought when one does not have access to any channel estimation.

There does not exist a large literature for fine OFDM timing estimation, at least in the downlink case. Indeed, provided that the compensated STO after the estimation processes verifies $L - N_{\mathrm{CP}} \leq \theta \leq 0$, the consequence of a synchronization mismatch is a mere symbol rotation in frequency. When performing channel estimation, this rotation might be seen as part of the channel, with an increased frequency selectivity. As a consequence, as long as the channel estimation procedure can cope with the increase of the channel frequency selectivity, the performance in OFDM decoding in downlink is not impaired.

Uplink STO estimation

In uplink OFDMA, as already mentioned, the STO problem is slightly more involved due to the multiple STO values involved. A classical solution to cope with this multiple STO issue is for the whole system to align downlink and uplink timing. Indeed, from the downlink timing information, the user already has a synchronized uplink STO up to twice the propagation delay. If the system allows for a large enough cyclic prefix length (i.e. large enough to cover the channel delay spread and the double propagation delay), timing offsets coupled with channel estimation for every user's handset do not produce any harm to the system performance. However, spectral efficiency and overall throughput performance suffer from the cyclic prefix extension and therefore only short spatial coverage is tolerable in such uplink OFDMA technologies. If the cyclic prefix is limited to the maximum channel delay spread of all users, then the synchronization problem is heavily more critical and requires exhaustive multi-parameter search (e.g. joint decoding and timing acquisition) for all $\theta_k,\ k \in \{1, \ldots, K\}$.

Another classical solution for uplink synchronization, which has benefits both in the time and frequency domains, is to allocate sets of contiguous subcarriers to every uplink user. To avoid frequency overlap due to additional CFO problems, frequency guard bands, i.e. non-allocated subcarriers, are placed between these sets. This allows the receiver to individually treat each user by filtering out the other users, with a minimal impact of ICI due to hypothetical CFO problems. Then the STO of every user can be estimated independently of the other users. The same technique as in the single-user (SU) case can then be used. Equation (1.19) is still valid on a per-user basis, but here the noise term w_n in Equation (1.18) also contains interference contribution from residual ICI. However, using contiguous blocks for all users reduces the available frequency selectivity for every user, especially when a large number of users is present in the OFDMA cell. Indeed, such a subcarrier allocation makes every user very sensitive to deep channel fades. In practice, a simple workaround consists in using high-level user scheduling, such as frequency-hopping techniques [22]. If for some reason, such as short packet transmissions[5], interleaved carrier allocation is demanded, then practical computationally cheap STO estimations are yet unknown.

In order to cope with short-time transmissions issues in OFDMA when users are allocated sets of contiguous subcarriers, the authors propose in [29] an alternative solution to the OFDM modulation, referred to as α-OFDM, which provides additional frequency diversity at a minimal implementation cost. This novel modulation scheme allows users to dynamically exploit side frequency bands by sacrificing a few subcarriers on the edge of the total bandwidth. α-OFDM brings in particular significant outage capacity gain when users are allocated very small frequency bands, compared to the total bandwidth.

1.3.2 CFO estimation

Rough CFO estimation

Acquisition and tracking of the frequency offsets are the most critical synchronization tasks. The first reason was studied in Section 1.2.2: a small mismatch between local oscillators entails dramatic system performance losses. CFO estimation is also made difficult by the Doppler effect, introduced in Section 1.2.1; in short coherence time channels, every new data transmission is subject to a different frequency shift, which demands fast CFO tracking.

Similarly to the STO case, it is common to perform a very rough CFO estimation prior to any accurate CFO estimation, so to align the DC-equivalent frequencies from the base station and the terminals up to more or less one subcarrier spacing. This can be handled, like in the STO case, by correlating a training sequence with different frequency-shifted copies of this sequence. Since the channel is not known at this early step and that this estimate can be impaired by different sources of interference, the process is not very reliable. Therefore, the estimation range sought for the CFO at this stage is typically of the order of the subcarrier spacing. From this point on, rough STO estimation is performed and then proper CFO

[5] short packet transmissions lead to moreover consider performance in terms of outage capacity, instead of the long-term ergodic capacity.

evaluation can be processed.

Historically, Moose [7] was the first to provide a DA technique for CFO estimation, which is independent of the channel realization. Similarly to the STO estimation, Moose proposes a pilot OFDM symbol $\mathbf{x}$ composed of two identical vectors of size $N/2$. Assuming a prior STO estimation, the CFO effect in time (see Section 1.2.2) is a phase rotation of the transmitted symbols by an angle proportional to the time index. Therefore, the correlation of the first and second half of received data symbol results, for all $n \in \{1, \ldots, N\}$, in

$$y_n y_{n+N/2}^* = \left(e^{2\pi i \delta n/N} \sum_{l=0}^{L-1} h_l x_{n-l} + w_n \right) \left(e^{2\pi i \delta n/N} e^{\pi i \delta} \sum_{l=0}^{L-1} h_l x_{n-l} + w_{n+N/2} \right)^* \quad (1.20)$$

$$= e^{-\pi i \delta} \left| \sum_{l=0}^{L-1} h_l x_{n-l} \right|^2 + \tilde{w}_n \quad (1.21)$$

where $\tilde{w}_n$ includes the double products and the noise correlation, of null average.

Summing up coherently the $N/2$ correlations leads to the estimate $\hat{\delta}$ of δ,

$$\hat{\delta} = \frac{1}{\pi} \tan^{-1} \left(\frac{\sum_{n=1}^{N/2} \Im(y_n y_{n+N/2}^*)}{\sum_{n=1}^{N/2} \Re(y_n y_{n+N/2}^*)} \right) \quad (1.22)$$

The two main limitations in this approach are (i) the effective acquisition range that is limited to $\delta \in [-\pi, \pi]$ (or equivalently to the length of the subcarrier spacing), and (ii) in the low SNR region, the noise $\tilde{w}_n$ is very strong since it contains components originating from cross-correlation to the pilot. As a consequence of (i), only the decimal part of the frequency offsets can be identified through this method. Moose proposes [7] solutions to enlarge the acquisition range at the expense of a reduction in the estimation resolution. Many schemes based on the latter were then successively proposed to enhance the performance trade-off between acquisition range and resolution, the most popular of those being the Schmidl & Cox [12] and the Morelli & Mengali algorithms [14]. All those schemes are particularly adapted to circuit-switched communications or low speed mobile systems and show high accuracy in the CFO acquisition, especially for high SNR regimes. Indeed, they require specific pilot sequences that should not be made available at many time symbols (otherwise having a strong impact on the system spectral efficiency). Those are therefore not suitable for fast varying channels or short data transmissions.

To cope with this double issue, Van de Beek [8] considers a blind CFO estimator based on the cyclic prefix redundancy. The principle is essentially identical to Moose's estimator but here the correlation is obtained between the last samples of each OFDM symbol and its cyclic prefix copy. Unfortunately, this method ideally works in additive white Gaussian noise (AWGN) channels which are mono-path channels. The ISI leakage due to multipath environments is detrimental to the performance of Van de Beek's method. To correct this deficiency, [15] suggests a reduction in the correlation window to result in an ISI-free estimate. Typical performances of these methods are depicted in Figure 1.6 for $N = 128$, $N_{\mathrm{CP}} = 9$ and different channel configurations and SNR. These NDA estimators have the strong advantage not to require any dedicated sequence, but face the main issue to be slowly converging, especially for high channel delay spread and short cyclic prefix length.

Fine CFO estimation

When channel estimation can be performed, the previous CFO estimation problem is less involved and advanced accurate algorithms can be performed, which can take into account all the available information (received data, pilots, source coding structure etc.). While the previous algorithms were designed to acquire a rough CFO estimation, this other set of algorithms is meant to perform CFO tracking (i.e. constant refinement of the CFO estimation). In the acquisition phase the objective is to find a rough estimate $\hat{\delta}$ for the CFO δ, whose estimation variance was depicted for instance in Figure 1.6; in the tracking phase, this estimation is usually refined in a closed-loop operation to significantly reduce the error variance and to adapt to the hypothetical Doppler shift. Those closed-loop systems, which originate from Wiener feedback loops back in 1948 [9], are a useful tool to come up with parameter estimates in a system whose behaviour is rather complex to model. In this specific case, the complexity lies in the anticipation of the Doppler shift dynamics.

The historical feedback loop example for CFO estimation was proposed by Daffara and Adami [10] and was followed by various derived contributions whose working structures are essentially the same. The typical Daffara and Adami's block-diagram is depicted in Figure 1.7. In this loop, every symbol is first fed to the "error function f_e" block which evaluates some error due to frequency offsets. Most enhancements of Daffara and Adami's solution are based on alternative f_e functions. The next step consists in evaluating the residual term $\hat{\delta} - \delta$. This is performed through the estimation of the CFO for all symbol indexes $n \in \mathbb{N}$. This allows to update $\hat{\delta}$ by successive refinement of the angle of symbol rotation due to CFO for every new incoming symbol. This symbol rotation update is denoted by the function $\phi(n)$ in Figure 1.7. The loop is then closed by feeding back the reconstructed (i.e. counter-rotated) symbols $y_n,\ n \in \mathbb{N}$.

CFO estimation in uplink

In uplink OFDMA, the frequency synchronization model stumbles on the same multi-parameter issue as its STO counterpart. But here the problem is heavily more involved. Indeed, frequency offsets engender ICI that corrupts the data in destination to the base station such that only complex techniques would help decode the overlapping data streams. For this reason, system-wide solutions are usually exploited. Consider the situation of *localized subcarrier allocation* (i.e. every user is allocated a frequency subband of contiguous subcarriers). As mentioned previously for the STO case, a common approach is to insert frequency guard bands between adjacent users, so that the individual data can be easily filtered and the ICI minimized (since only remote subcarriers would leak on the individual user's data). The same techniques as in downlink can then be applied to individually estimate all the frequency offsets. Therefore, Equation (1.22) is still valid, on a per-user basis, in which again ICI is added to the noise w_n. When many users share the bandwidth, the number of available subcarriers per user (especially for NDA techniques) can however be so small that the CFO estimation performance is heavily impacted in the frequency domain; more computationally demanding time-domain processes are more desirable in such situations.

When the subcarrier allocation is *distributed*, as opposed to a localized allocation, it is practically impossible to separate users' frequency subbands and the ICI effect is even more detrimental. Some authors recently tackled the problem of CFO estimation in these scenarios; among those, a joint STO and CFO maximum-likelihood solution was proposed by Morelli [16], which comes along with a high complexity since then exhaustive search on a two-dimensional grid is demanded. However, Morelli uses the major assumption that at most one user in the OFDMA network is imperfectly aligned in time and frequency. For this particular user, who is assigned the set $\mathcal{S}$ of L non-consecutive subcarriers $\mathcal{S} = \{k_1, \ldots, k_L\}$, $L < N$, whose channel fades are assumed approximately known to the base station and whose STO and CFO are respectively denoted θ and δ, the data-aided ML joint estimate, i.e. when pilot sequences are used, is given by

$$(\hat{\theta}, \hat{\delta}) = \arg\min_{(\theta,\delta)} \|\mathbf{r}^{\mathcal{S}} - \mathbf{z}(\delta, \theta)\|^2 \tag{1.23}$$

where $\mathbf{r}^{\mathcal{S}}$ is the restriction of $\mathbf{r}$ to the set of subcarriers in $\mathcal{S}$ and $\mathbf{z}(\delta, \theta)$ models the noiseless received pilot plus ICI due to STO and CFO, when all other users are perfectly aligned in time and frequency. In general the joint STO-CFO ML solution requires a search over a 2-D grid spanning over possible values for θ and δ. In practice, the hypothesis that the base station knows the user's channel is not true and then the ML problem actually encompasses also the search over the channel $\mathbf{h}$ but Morelli manages still to turn the problem into a 2-D search, independent of the channel realization. Practical solutions with lesser complexity, e.g. with decoupled 1-D searches over δ and θ, are also proposed in [16], under the assumption of small CFO, which is often met in practice.

Most of the previously detailed algorithms make use of several *ad-hoc* methods that do not minimize a given performance metric. The reason for *ad-hoc* methods to be the major techniques used in practice is twofold: (i) they are usually simple to implement and very low computationally demanding, which is very important for synchronization processes that might be used very often in mobile networks, (ii) they cope with the absence of knowledge of major system parameters such as information about the communication channel. In the following, we shortly evoke advanced solutions that rely on optimal orthodox or Bayesian approaches, in the sense that they achieve Cramer-Rao bounds or use maximum entropy-based methods to deal with limited knowledge.

1.3.3 Advanced methods

Few complete studies and optimal methods (with respect to some given performance metric) are found in the literature of OFDMA synchronization for the mathematical derivations are usually not tractable; this is mainly due to the difficulty to model systems from which one does not know much *a priori* (i.e. when trying to estimate rough STO, CFO, channel state information is usually not known). Lately, the recursive expectation-maximization (EM) algorithm has grown into a handy solution to tackle such incomplete data problems and particularly suits multiple parameter estimation problems. Indeed, EM is a recursive technique which allows to turn an *a priori* difficult problem with some missing parameters (from an *incomplete parameter set*) into a simple problem in which those parameters are known (forming then the *completed parameter set*). Under some adequate conditions [17], this method converges to a solution whose complete parameter set contains consistent parameter values. In our synchronization framework, such problems as channel estimation, decoding etc. are simple problems when all system parameters are known, while the marginal problems when some parameters are not known *a priori* are often more involved. This gives birth to joint EM estimation techniques such as joint channel estimation and parameter offset estimation [18], joint decoding and CFO estimation [21] etc. Other joint estimation studies are considered in [19], [20] which give hints on the achievable performance to be expected in orthodox probability-based approaches. In particular, theoretical limits in terms of Cramér-Rao bound of the joint SCO and channel estimation are given in [20]. Unfortunately these orthodox techniques do not take into consideration any prior knowledge on the unknown parameters and might lead to incongruous solutions, especially if the problem to maximize is not convex as a function of the unknown parameters. Optimal Bayesian maximum entropy approaches have been proposed by the authors [30]-[32] in place for EM-like solutions. This is further discussed in Section 1.5.

In Tables 1.1 and 1.2, the main synchronization algorithms for coarse and fine STO and CFO estimation are recalled.

1.4 A case study: 3GPP-LTE

Due to the synchronization problems discussed in Section 1.2.2 and to the major PAPR problem in the uplink, 3GPP decided against an OFDMA uplink setup in the first releases of the Long Term Evolution (3GPP-LTE) standard. Therefore, only downlink OFDMA is considered in the following. Few synchronization sequences are utilized in LTE to minimize the system overhead. Therefore, at the receiver, synchronization is only performed,

- either on the primary synchronization sequence, secondary synchronization sequence and/or any pilot sequence present in the LTE frame.
- either blindly thanks to such methods as the NDA techniques described in Section 1.3.

Note that, contrary to the uplink scenario, low power consuming methods are demanded at the receiving interface to minimize battery usage. The standard is then demanded to provide simple synchronization sequences while minimizing the system overhead. A typical LTE synchronization phase unfolds as follows,

1. when the user equipment is switched on, the first physical layer operation consists in detecting a power source along the licensed LTE bandwidth. This is referred to as Public Land Mobile Network (PLMN) search. This is typically handled by a mere mean power measure on the receive antenna array. A threshold on this receive power is set to decide on the presence or absence of the OFDM source.
2. when a source of power is detected, it then has to be identified. This operation is undergone thanks to a set of three orthogonal time-domain ZC sequences of length 63 which enjoy the following properties
   - two ZC sequences of different indexes show very small cross-correlation.
   - the cross-correlation of a ZC sequence with itself shifted by an integer number of samples is very small.
   - the frequency response of a ZC sequence is flat.

- third order statistics of ZC sequences are small to mitigate non-linearities in the analog front-end (e.g. analog amplifiers).

These three ZC sequences allow to map the different transmitting base stations into three groups (those groups are organized such that, in the hexagonal cell planning, two cells of a given group are never adjacent). Those sequences are called Primary Synchronization Sequences (PSS) and are found every 5 ms on the central frequency band of size 1.4 MHz. Through the ZC sequence detection, a first rough STO estimation is performed, since the beginning of the ZC sequence is then identified. Depending on the detection technique used (e.g. the classical technique is a point to point correlation with the three ZC sequences on different hypothetical central frequencies), a first rough CFO estimation is also performed. Note however that a very low sampling frequency is used at cell detection step to match the 1.4 MHz central band. Therefore the STO estimates cannot be very accurate if the effective signal bandwidth is as large as 20 MHz (i.e. the maximum usable bandwidth), since then one symbol sampled at 1.4 MHz corresponds to a set of 16 symbols at sampling rate 20 MHz. It is also important to note that 5 ms of PSS detection over different frequency references represents an considerable amount of processing. Therefore, time and frequency acquisition cannot be made using a thin time-frequency grid. To cope with the constraint of large frequency grid steps, which does not allow for a good CFO estimate, the DA but pilot-independent technique proposed in [32] and presented briefly in Section 1.5 is of particular appeal.

3. when the PSS sequence is discovered, the cell identification is completed thanks to the Secondary Synchronization Sequence (SSS) in the frequency domain. The latter is scrambled by one of three possible codes given by the index of the identified ZC sequence. This sequence uniquely identifies the selected cell (in reality, as many as 504 identifiers are available, so that, with a correct cell planning, two cells with same identifier should never interfere). Those PSS sequences are based on two interleaved binary *maximum length sequences* of size 32 whose main property is to have good cyclic cross-correlation properties. In terms of CFO estimation, they allow for a coarse evaluation of δ reducing the search range to the subcarrier spacing Δ_f.

STO and CFO acquisitions can be performed thanks to the pilot sequences introduced in the LTE standard. Note nonetheless that PSS and SSS do not have the repetitive structure advised by Moose [7] due to the structure of ZC and maximum length sequences respectively; alternative schemes must then be produced to adapt the standard. In particular, the time-domain PSS sequence $\mathbf{x}$ is *close to* a two-half mirrored sequence: $x_{N-k+2} = x_k$, $k \in [2, N/2]$. This symmetrical structure might allow to design specific CFO and STO recovery techniques. This structure is however less adequate since the treatment of channel leakage on mirrored signals is more involved than for Moose's double-half sequences. In the next section, the authors provide a CFO estimation technique whose performance is independent of the pilot sequence (as long as all entries of the time-domain sequence have the same amplitude); this method particularly targets the current LTE standard.

As for the tracking phase, the problem is even more difficult for no dedicated sequence allows for fine parameter estimation. PSS and SSS are length-64 sequences that pop up only every 5 ms. *Reference Symbols* (RS) dedicated to channel estimation purposes are scattered along the whole frequency bandwidth and spaced every 6 subcarriers; as they were designed with no synchronization consideration, only advanced (and often computationally demanding) STO or CFO estimation techniques can be performed. As a consequence, NDA techniques are appreciable to achieve accurate parameter estimates. Joint estimations based on the EM algorithm, despite their apparent complexity, turn out an interesting compromise since, practically speaking, they do not require additional software treatment (e.g. turbo decoders, channel estimators etc. are already part of the terminal software) and they actually perform reusable tasks (e.g. soft decoding and channel estimation can be reused). Those advanced solutions are especially demanded for communications over the large 64-QAM constellation whose tolerable SNR (for a channel coding rate 1/3) is of order 30 dB in SISO channels and of order 25 dB in MIMO 2×2 channels. Simulations show that this constrains the CFO estimation to be constantly of order $\delta \simeq 200$ Hz for a subcarrier spacing $\Delta_f = 15$ KHz.

From formula (1.3), on a working LTE frequency around 3 GHz and under vehicular speed of 120 Km/h, the typical Doppler shift is around 300 Hz, which is more than the maximum tolerable CFO. Therefore the CFO estimation task is highly critical and must be performed accurately in order to ensure a satisfying working SNR under high mobility conditions.

1.5 Discussion

1.5.1 Bayesian framework

We already mentioned that the synchronization field does not rely on strong information theoretic grounds. The reason for that is mainly because synchronization is usually isolated from the rest of the communication chain and, as such, is considered an independent field. The authors suggest to model synchronization parameters as simply *a priori* unknown parameters in the same way as the transmitted data $\mathbf{s}$.

The objective of a communication scheme is to optimize the useful data decoding process given some information at the receiver. This information often summarizes as the exact knowledge of the transmit codebook, the noisy receive sequences, the exact synchronization pilots etc. The general soft decoding decision for a transmitted vector $\mathbf{x}$ of a known codebook $\mathcal{X}$ and received vector $\mathbf{y}$ is based on the Bayesian probabilities,

$$p(\mathbf{x}|\mathbf{y}, I) = \int p(\mathbf{x}|\mathbf{y}, \delta, \theta, I)p(\delta, \theta|I)d\delta d\theta \tag{1.24}$$

where I stands for the prior information on the system.

When CFO and STO estimation is handled, the classical approach is to simplify (1.24) by (erroneously) setting $p(\delta, \theta|I) = \delta_{\delta}^{\hat{\delta}} \cdot \delta_{\theta}^{\hat{\theta}}$, which differs from the updated probability $p(\mathbf{x}|\mathbf{y}, \hat{\theta}, \hat{\delta}, I)$ that should now be considered. When the parameter estimators are very poor (which can happen in many situations, such as short packet transmission, low SNR, scarce synchronization resources...), soft decisions on the estimates will prove of key importance, instead of wrong hard decisions on $\hat{\theta}, \hat{\delta}$. Of course, it is usually difficult to perform the integration (1.24). Still, the latter leads to a few relevant considerations,

- as shown in Section 1.3, parameter estimations often come from the minimization of some error measure. The choice of this measure is often directed by computational simplicity or common usage (e.g. minimizing the quadratic error). Instead, the estimation error should be minimized in accordance with the operations performed in the data decoding step, i.e. so to end up with a satisfying approximation of $p(\mathbf{x}|\mathbf{y}, I)$.

- as a first refinement of the hard decisions of the synchronization parameters, $p(\mathbf{x}|\mathbf{y}, I)$ can be better approximated by

$$p(\mathbf{x}|\mathbf{y}, I) \simeq \sum_{(\delta,\theta)\in\mathcal{D}} p(\mathbf{x}|\mathbf{y}, \delta, \theta, I)p(\delta, \theta|I) \tag{1.25}$$

where $\mathcal{D}$ is some discrete space of high joint probability for the couple (δ, θ). The diameter of this space is chosen to meet the best computation load/decoding quality compromise. In [30], [31], the authors show that this approach shows significant results in the sense that, even parameter estimates with low probability can be *resurrected* by a high joint data-parameter probability $p(\mathbf{x}|\mathbf{y}, \delta, \theta, I)$. This observation is often difficult to anticipate, mainly because of the complexity hidden in the probability $p(\mathbf{x}|\mathbf{y}, \delta, \theta, I)$.

In the light of those considerations, already envisioned in schemes such as joint decoding-synchronization, joint channel estimation-synchronization etc., one realizes that, with the ever-growing computation performance of embedded hardware, much progress can be achieved in the synchronization field. Throughout the recent appeal for cognitive radio systems, novel adaptive synchronization schemes could appear and replace the already too old classical algorithms. A simple example of an optimal Bayesian CFO estimator is detailed in the following.

1.5.2 Case study: Bayesian CFO estimation

In this section, the authors propose a generalized study of optimal Bayesian parameter estimation from which important conclusions shall be drawn. For a more complete study, the reader is invited to refer to [32]. Say one wants to perform CFO estimation using all the provided *a priori* system information I. As recalled in Section 1.2.2, the time-domain effect of a CFO is a mere symbol phase rotation, proportional to the time index. From the transmitted data $\mathbf{x}$, received with perfect timing synchronization as $\mathbf{y}$, one has

$$\mathbf{y} = \mathbf{D}_\delta \mathbf{H}\mathbf{x} + \mathbf{w} \tag{1.26}$$

where $\mathbf{H}$ is the circulant matrix originating from the channel vector $\mathbf{h} = [h_0, \ldots, h_{L-1}]^\mathsf{T}$ of length L, $\mathbf{w}$ the noise process with entries of power $1/\mathsf{SNR}$ and $\mathbf{D}_\delta$ the diagonal matrix of main diagonal $\{1, e^{i\delta}, \ldots, e^{(N-1)i\delta}\}$. For simplicity in the upcoming derivations, let us rewrite this model

$$\mathbf{y} = \mathbf{D}_\delta \mathbf{X}\mathbf{h} + \mathbf{w} \tag{1.27}$$

with $\mathbf{X}$ the pseudo-circulant matrix

$$\mathbf{X} = \begin{pmatrix} x_0 & x_{N-1} & \cdots & x_{N-L-1} \\ x_1 & x_0 & \cdots & x_{N-L-2} \\ \vdots & \vdots & \vdots & \vdots \\ x_{L-2} & x_{L-3} & \cdots & x_{N-1} \\ x_{L-1} & x_{L-2} & \cdots & x_0 \\ \vdots & \vdots & \vdots & \vdots \\ x_{N-1} & x_{N-2} & \cdots & x_{N-L} \end{pmatrix} \tag{1.28}$$

The estimation problem consists in evaluating, for all $\delta \in \mathbb{R}$,

$$p(\delta|\mathbf{y}, I) = p(\mathbf{y}|\delta, I)\frac{p(\delta|I)}{p(\mathbf{y})} \tag{1.29}$$

The term $p(\mathbf{y}|\delta, I)$ can be further developed

$$p(\mathbf{y}|\delta, I) = \int p(\mathbf{y}|\delta, \mathbf{h}, \mathbf{x}, I)p(\mathbf{h}, \mathbf{x}|I)d\mathbf{h}d\mathbf{x} \tag{1.30}$$

in which $p(\mathbf{y}|\delta, \mathbf{h}, \mathbf{x}, I)$ is easily evaluated from the linear model (1.26). Equation (1.30) does not consider at all the selected synchronization method (i.e. DA or NDA). This property is actually hidden in the expression $(\mathbf{x}|I)$. Two cases can arise,

- either the information on $\mathbf{x}$ is completely included in the prior information I of the receiver. This makes of $\mathbf{x}$ a pilot sequence. In this scenario, Equation (1.30) simply becomes

$$p(\mathbf{y}|\delta, I) = \int p(\mathbf{y}|\delta, \mathbf{h}, I)p(\mathbf{h}|I)d\mathbf{h} \tag{1.31}$$

- either the information on $\mathbf{x}$ contained in I is limited to some statistical properties (e.g. mean, variance), constellation knowledge etc. This leads to a semi-blind or fully blind analysis, which solution can be found in the works on joint decoding-parameter estimation.

The *a priori* distribution $p(\mathbf{h}|I)$ can be evaluated through the available prior information contained in I; if only the average power and the typical length L of the channel, i.e. the expected channel delay spread, are known, then thanks to the *maximum entropy principle* [27], the most appropriate distribution to represent $\mathbf{h}$ is a multivariate Gaussian i.i.d. density function [28]. Ordinarily, L is not supposed known, but let assume perfect knowledge on L in the following (for deeper analysis when L is unknown, refer to [31]). After further development, in the DA case (i.e. $\mathbf{x}$ is *a priori* known), one can show that,

$$p(\delta|\mathbf{y}, I) = \alpha(\mathbf{y}) \cdot p(\delta|I)e^{-C(\mathbf{y},\delta)} \tag{1.32}$$

for some function $\alpha(\mathbf{y})$ independent of δ and

$$C(\mathbf{y}, \delta) = \mathbf{y}^{\mathsf{H}} \left[\mathbf{I}_{\mathsf{N}} + \mathbf{D}_{\delta}^{\mathsf{H}}\mathbf{X} \left(\mathbf{X}^{\mathsf{H}}\mathbf{X} + \frac{1}{\mathsf{SNR}}\mathbf{I}_{\mathsf{L}} \right)^{-1} \cdot \mathsf{SNR} \cdot \mathbf{X}^{\mathsf{H}}\mathbf{D}_{\delta} \right] \mathbf{y} \tag{1.33}$$

If no prior information is known about δ apart from its belonging to a finite range $\mathcal{D}$, $p(\delta|I)$ can be considered uniform over $\mathcal{D}$ [27] and a simple CFO estimator consists in the value $\delta \in \mathcal{D}$ for which C is minimized[6]. Unfortunately $C(\mathbf{y}, \delta)$ is not convex in δ in general, which makes the solution difficult to grasp and requires exhaustive search methods. However, from exhaustive simulations, it appears that $C(\mathbf{y}, \delta)$ is convex on $[-\pi, \pi]$, i.e. on the range of one subcarrier spacing. Therefore, iterative algorithms can be produced based on steepest descent

[6]the choice of this estimator is purely subjective since no mention is made here of any ultimate objective apart CFO estimation

methods to find the maximum likelihood estimate $\hat{\delta}$. Figure 1.8 provides a comparison between Moose's correlation method discussed in Section 1.3 and the proposed Bayesian method, whose performance plot is produced from the iterative algorithm provided in [32]. The results show an increase performance in the CFO estimate. We also provided simulation results when the prior for $\mathbf{h}$ is initially wrong, i.e. the assumed channel length L_{as} is not the true channel length L; for not-too-low SNR, the solution is close to optimal. Note moreover that this optimal solution in the Bayesian sense is better than Moose's solution while being applicable to any pilot sequence $\mathbf{x}$. Also, the complexity of the algorithm, which is obviously more than Moose's correlation method, can be dynamically controlled by the number of required iteration steps. In the low SNR regime in particular, an important synchronization time is gained, at the expense of a small increase in processing complexity.

Those Bayesian considerations lead to a new approach regarding problems of parameter estimation: the authors propose here an information theoretic solution based on the state of knowledge of the receiver (which provides upper bounds on estimation performance in this information theoretic framework) and envision simplifications of the optimal solution to better suit the computational complexity requirements. This allows to keep a constant control on the performance.

Similar maximum entropy Bayesian studies are carried out in thorough details for blind MIMO signal detection [30] and pilot-based channel estimation for OFDM [31]. In the latter, we particularly observe that, even when the channel length L is unknown to the receiver, channel estimation can be equally performed as if L were known (with almost as good performance) since the missing information, carried by the incoming signal, can be automatically recovered. The resulting algorithms are shown to be more complex than classical methods, making simplification algorithms only a matter of mathematical complexity reduction, and not a matter of finding an adequate *ad-hoc* alternative.

1.6 Conclusion

To ensure reliable communication in an OFDMA system, a first timing and frequency synchronization step is necessary, for local oscillators in both communication ends generally mismatch. In addition, mobility in recent OFDM-based technologies engenders Doppler effects, which dynamically contribute to the frequency mismatch (CFO). For those reasons, STO and CFO need be estimated at device initialization and then tracked during the proper communication phase. We showed that STO estimation is not in general a critical task, while CFO can lead to dramatic performance impairment. Synchronization mainly consists in a multi-parameter estimation problem. No optimal solution has ever been proposed since there exists no strong theoretical foundation for synchronization which aims at optimizing the useful data transmission capacity. As a consequence, STO and CFO recoveries in the literature consist in a multitude of various solutions, which aim at different objectives. From those solutions, we selected the historical and most used algorithms, either based on dedicated pilot sequences, designed to synchronization purposes, or based on the redundancy found in the system overhead, and in particular in the cyclic prefix. However, we showed that in a concrete application such as 3GPP-LTE, most of those schemes are not adequate. This has led to recent proposals using information theoretic grounds on synchronization issues to conclude that DA and NDA methods are just particular cases of a more general Bayesian parameter estimation approach. In the near future, with the availability of high embedded computation rates, the synchronization field is expected to enroll into the current trend for cognitive radios and move from low complex solutions to more involved but information theoretically optimal processes.

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STO estimation	
Data-Aided (DA)	*Non-Data Aided (NDA)*
Rough correlation to pilot Schmidl's double-half sequence [12] Serpedin's finer sequence [13]	CP-based correlation

CFO estimation	
Data-Aided (DA)	Non-Data Aided (NDA)
Rough correlation to pilot Moose's double-half sequence [7] Moose's redundant sequence [7] Schmidl's refinement of Moose's sequence [12] Morelli's algorithm [14]	Van de Beek's CP-based algorithm [8] Ai's ISI-free CP-based method [15]

Table 1.1: Main OFDMA coarse synchronization techniques

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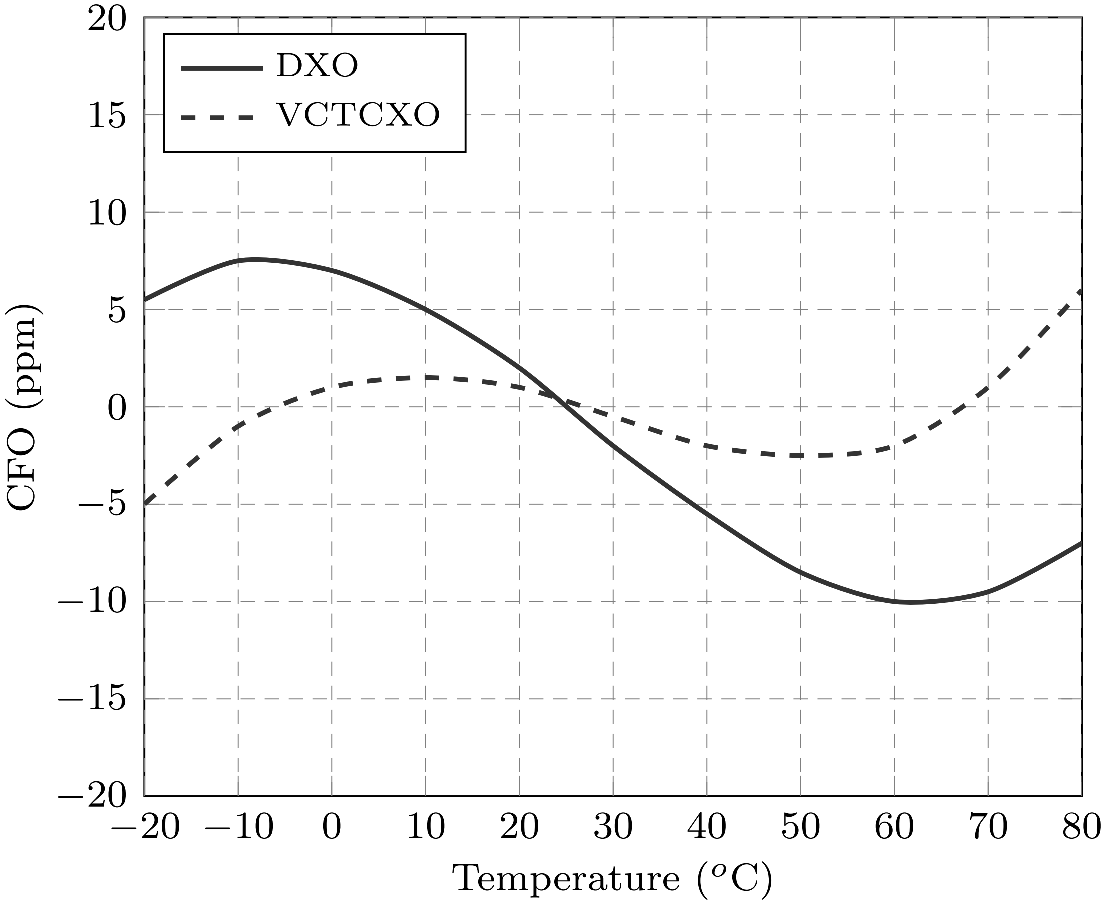

Figure 1.1: CFO effect on crystal oscillators due to temperature

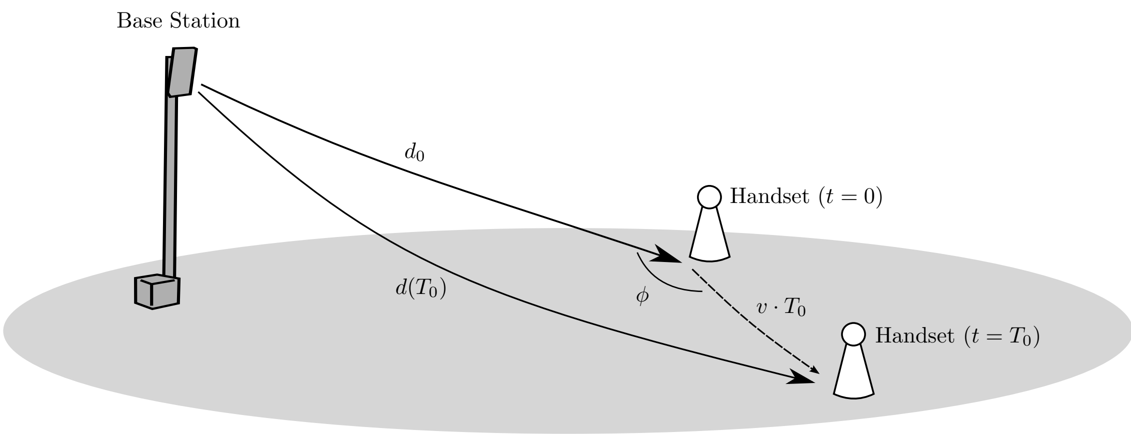

Figure 1.2: Doppler frequency shift effect

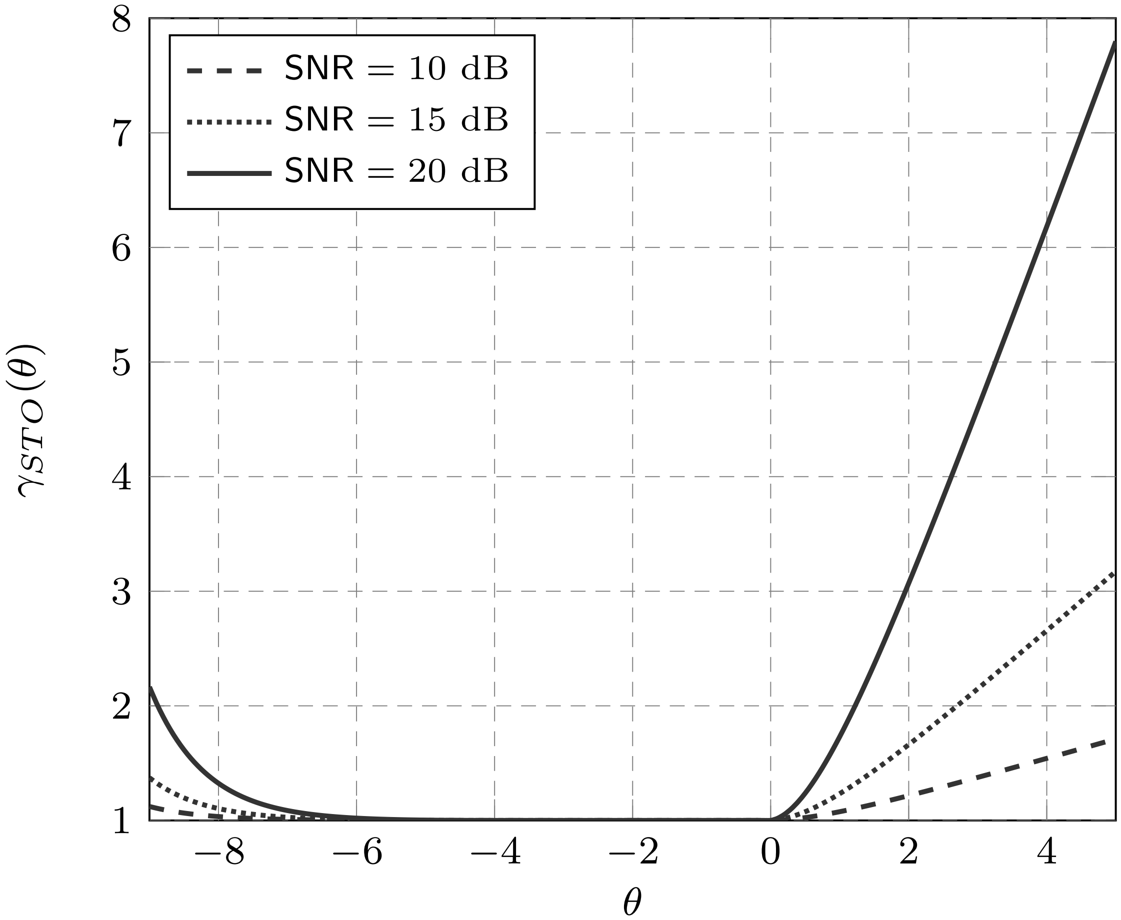

Figure 1.3: Performance decay due to STO - $N = 128$, $N_{\mathrm{CP}} = 9$, Exponential decaying channel with $L = 8$

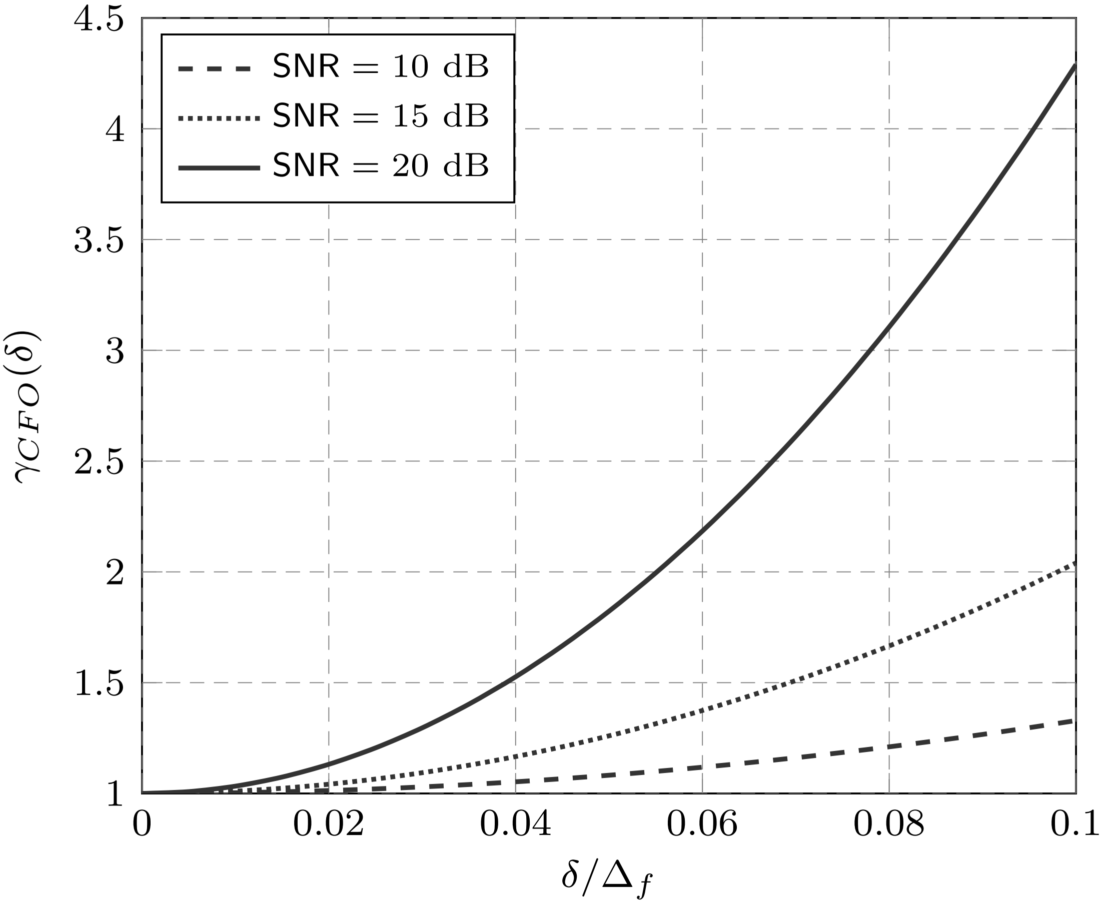

Figure 1.4: Performance decay due to CFO

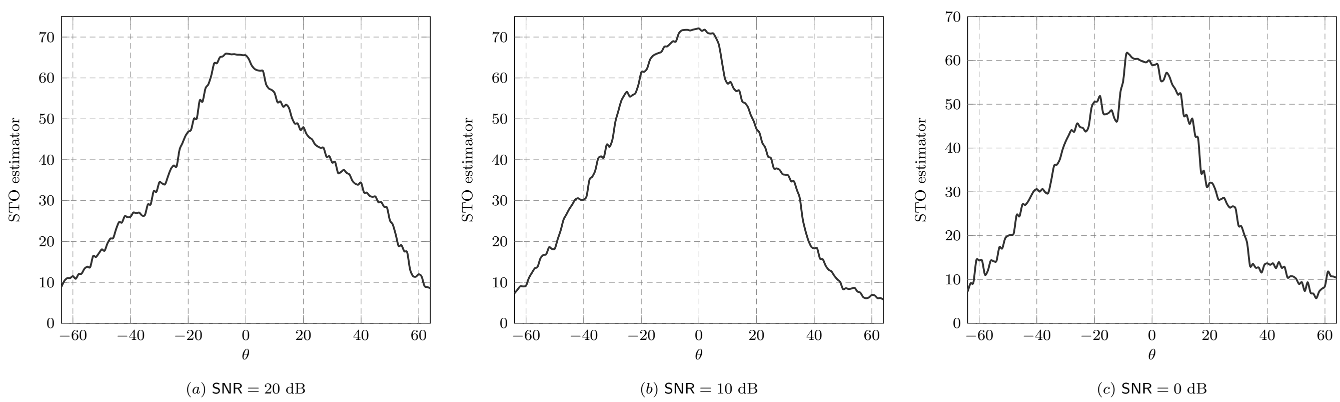

Figure 1.5: STO estimation for different SNR values, $N = 128$, $N_{\mathrm{CP}} = 9$, exponential decaying channel with $L = 5$

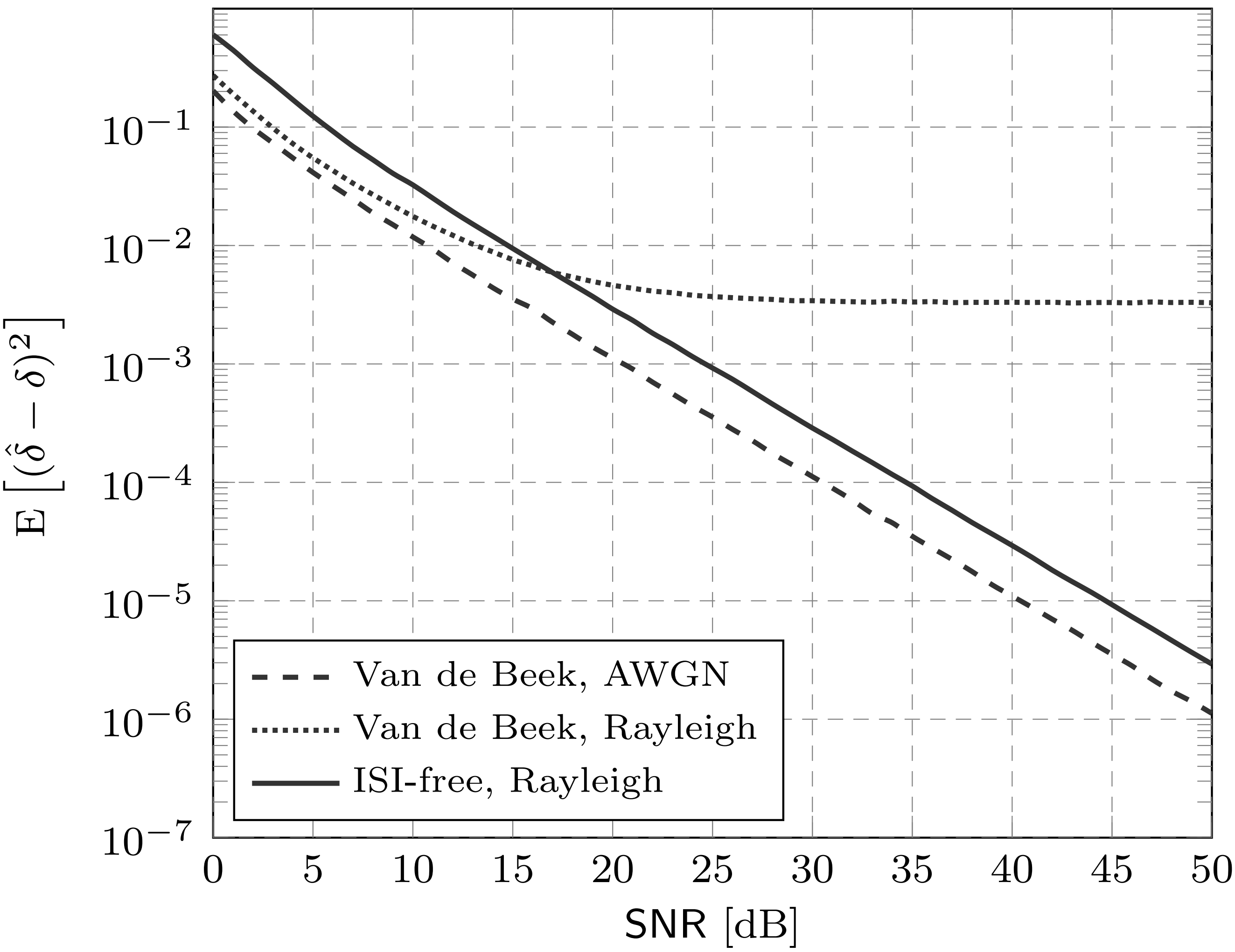

Figure 1.6: Rough CFO estimation versus SNR values, $N = 128$, $N_{\mathrm{CP}} = 9$, AWGN and Rayleigh channel with $L = 5$

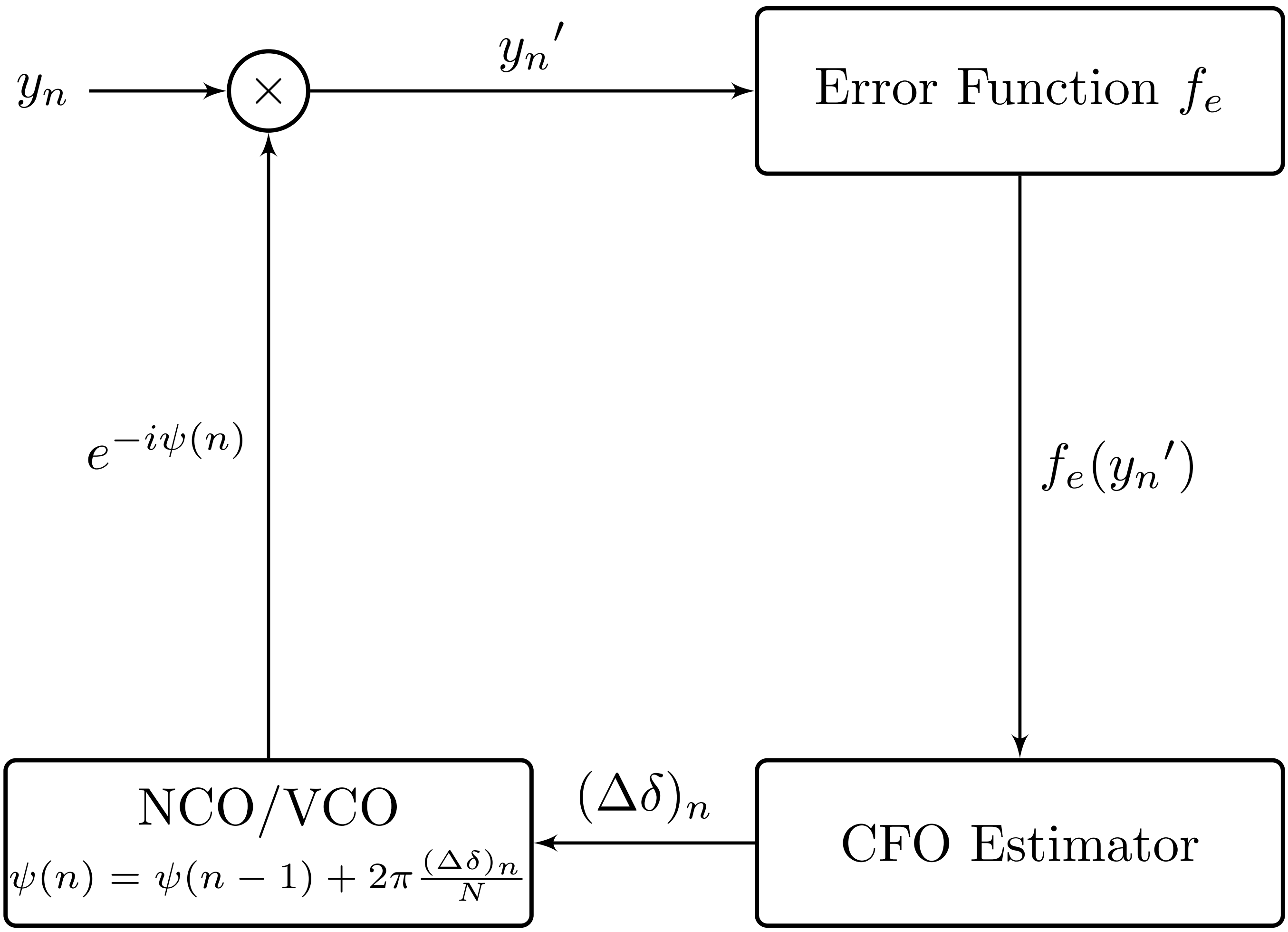

Figure 1.7: Closed-loop CFO tracking

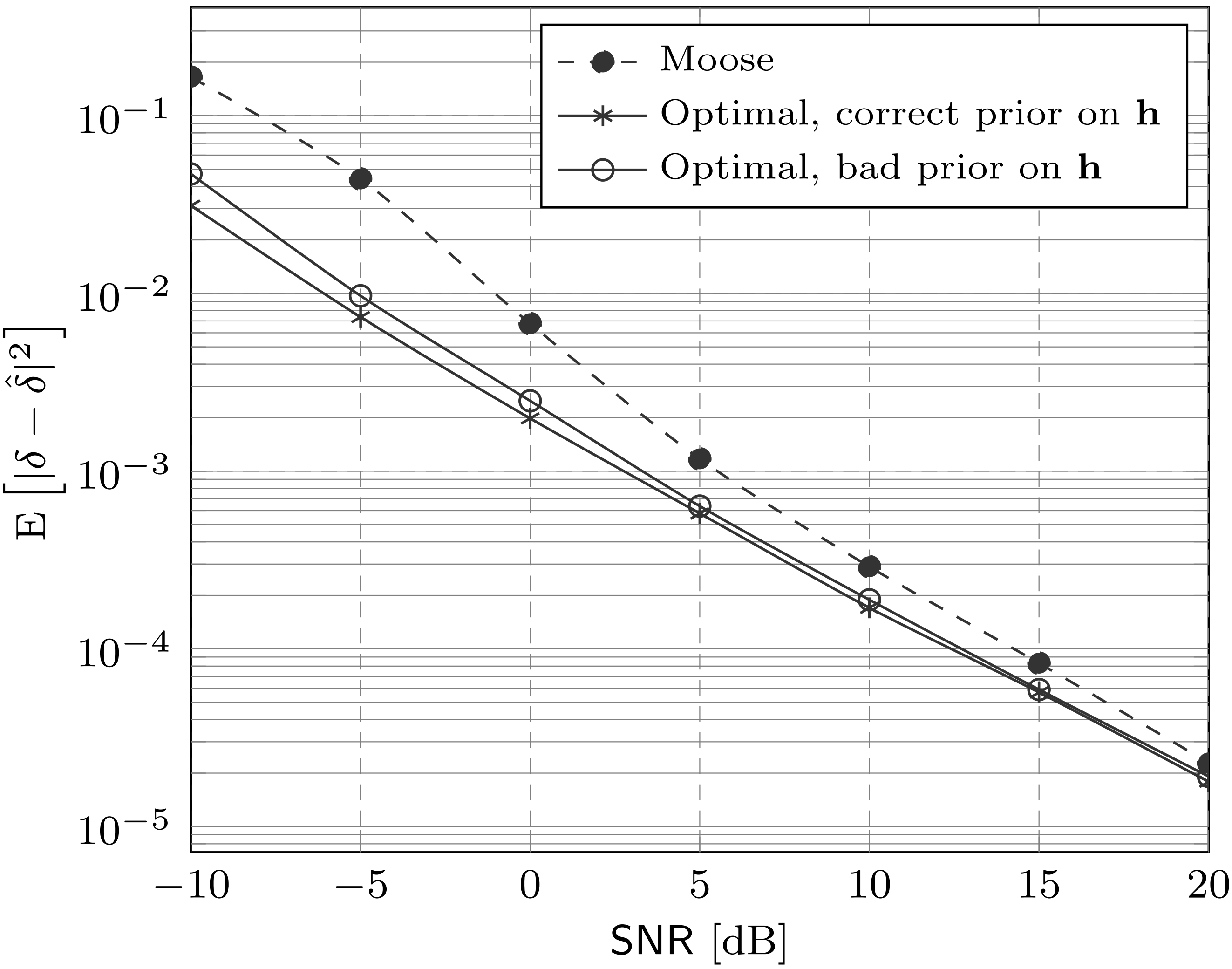

Figure 1.8: Optimal Bayesian CFO estimation - Comparison between Moose's methods [7] and Bayesian method. *Conditions:* $N = 128$, *random double-half OFDM symbol,* $\mathbf{h}$ *is i.i.d. Gaussian with length* $L = 3$, *prior on* $\mathbf{h}$ *either perfect:* $L_{as} = L$ *or imperfect* $L_{as} = 9$

STO and CFO estimation
Daffara's CFO closed-loop [10]
Joint ML CFO-STO [16]
EM-based algorithms [21]
Decision directed algorithms [19]
Maximum entropy-based algorithms [32]

Table 1.2: Main OFDMA fine synchronization techniques