A Novel Dynamic Network Architecture Model Based on Stochastic Geometry and Game Theory

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***Abstract*—In this paper, a novel paradigm of user-provided connectivity in wireless networks is introduced using certain class of wireless terminals that can be turned temporarily into access points at any time while connected to the Internet. We show that a DNA (Dynamic Network Architecture) model improves the connectivity and capacity of ultra-dense wireless access networks without need to reconfigure the network infrastructure. The DNA operators motivate terminals to participate in this concept by providing incentives. An example is to allow terminals to transmit additional free traffic volume if they share their free bandwidth by acting as access points. The DNA operators manage the dynamic network in order to maximize their own profit by adjusting jointly their price and incentive rate. In addition, we model the joint problem of operator pricing and user resource sharing as a non-cooperative game and the resulting game admits a unique Nash equilibrium solution. Simulation results show high gains in such networks for terminals acting as access points and operators.**

Keywords. Stochastic geometry, dynamic network architecture, ultra-dense wireless networks, microeconomics, game theory.

I. Introduction

Small Cell Networks (SCN) are able to efficiently support the performance requirements of future wireless access networks such as high peak transmission rates, high spectrum utilization, and high energy efficiency [1], [2]. Unfortunately the problem of where and when to deploy these small cells still remains a challenging issue [3], [4], [5] because a pre-established scenario is expensive for the network operators. In the case of dynamic spatial traffic distribution, most of the time fixed pre-deployed network infrastructure is not used. In [6], [7], [8], and [9] new User-Provided Connectivity (UPC) paradigms for a low-cost ubiquitous connectivity are presented to solve the small cell deployment problem. Here, we further extend this concept to make the network more flexible and self-adjustable to traffic density variations.

In this paper, a new paradigm in wireless access network is considered where certain classes of wireless terminals can be turned into access points at any time while connected to the Internet. This would create a Dynamic Network Architecture (DNA) model [6] where the number and location of these access points vary in time. This ultra-dense DNA can be also thought of as a Dynamic Small Cell (DSC) network where new small cells can be temporally activated anywhere without an additional cost for reconfiguring the network infrastructure.

The main contributions of our paper are as follows:

– A new dynamic networking model is introduced as DNA to improve overall network coverage and connectivity. In this kind of network model operators motivate users to share their resources instead of installing costly hardware/software infrastructure equipment everywhere.
– New microeconomics of ultra-dense DNA wireless network is introduced where the operators motivate users to act as an access point by offering free traffic volume depending on the extent of their cooperation.
– A game-theoretic model is developed for the non-cooperative behavior of the players (access points and operators) and we show that the resulting game admits a unique unilaterally stable state (i.e. a Nash equilibrium).

The rest of the paper is organized as follows. Section II provides a related work for Dynamic Architecture Networks. In Section III, the network model is presented, while section IV provides the system performance analysis. Section V presents the numerical results and finally, Section VI concludes the paper.

II. Related Work

In the last decade, the mixture of wired and wireless connectivity has transformed the way of accessing the Internet. This heterogeneous connectivity promises to provide a high level of ubiquitous broadband access. As a result, we are witnessing the emergence of wireless hotspots characterized by the high-density deployment of WLAN access points (APs) [10], [11] and [4]. An important feature of the high-density deployment is that users can find multiple APs in its vicinity, from the same or different service providers. Due to the limited number of channels, multiple APs may operate over the same channels. Thus, the effective management of these APs to optimize the users' throughput becomes an important challenge [6]. Dynamic Multi-hop cellular networks (MCN) [12], in which the traffic of a UE is relayed to a cellular infrastructure node by means of intermediate mobile nodes, have received significant interest in recent years as a means to enhance the capacity, data rates and coverage of cellular networks. For example, architectural aspects and routing protocols were studied in [13], [14]. Recently, one promising networking model has been proposed which allow users of MVNOs to share their unused bandwidth with other users [6], [15]. When the End User(EU) becomes a spectrum provider and shares wireless opportunities based on incentive [16] in order to provide a radio communication channel available for the neighbor EUs. Incentive mechanisms are fundamental for UPN/ DNA networking models, because their operators motivate EUs to cooperate by sharing their unused resources e.g bandwidth. These new networking models depend on the EU's willingness to share their wireless connectivity, storage capabilities, and energy resources. UPN/DNA [17], [18] can enable widespread communications without depending upon traditional ISP.

III. Network Model

We consider a global wireless network where the users' location is distributed according to a homogeneous PPP (Poisson Point Process) Φ_u with density λ_u. Every potential user is able to work in two different state modes: access point or terminal. It is assumed that $\lambda_\mathrm{Tr} = (1 - \mathrm{P_c})\lambda_\mathrm{u}$ and $\lambda_\mathrm{A} = \mathrm{P_c}\lambda_\mathrm{u}$ are the densities of terminals and access points where $\mathrm{P_c}$ is the probability that a user acts as an access point. Every user has a maximum transmission range and each terminal is served by its nearest access point. This means that the covered area of each access point forms a Voronoi tessellation [19]. According to [20], the probability that a randomly chosen access point does not have any terminal to serve is $\mathrm{P_{idle}} = (1 + \frac{\lambda_\mathrm{Tr}}{3.5\lambda_\mathrm{A}})^{-3.5} = (1 + \frac{1-\mathrm{P_c}}{3.5\mathrm{P_c}})^{-3.5}$. Therefore, the probability that an access point is active is $\mathrm{P_{ac}} = \mathrm{P_c}(1 - \mathrm{P_{idle}})$ and the density of active access points is $\lambda_\mathrm{ac} = \mathrm{P_{ac}}\lambda_\mathrm{u} = \mathrm{P_c}(1 - \mathrm{P_{idle}})\lambda_\mathrm{u}$.

In DNA networks every user operates either as a terminal consuming network resources or as an access point augmenting network resources. In the former case, the terminal must pay to the network an amount proportional to its resource consumption while in the latter case the network reimburses the user proportionally to its contribution to the overall augmentation of the network resources. This payment can be in the form of credit to freely transmit a certain amount of traffic volume. Hence, a potential user chooses its sharing rate $\alpha \in [0, 1]$ and decides on the probability P_c to work as an access point while the operator controls the process by adjusting its incentive rate θ and price ϑ.

IV. Performance Analysis

A. Network Throughput

In order to support the DNA communication paradigm a minimum SINR has to be supported by the network operator for all simultaneous active transmissions. Fig.1 presents one snapshot of the interference model on downlink transmission in a DNA network. Besides the base station the interferers are also active in downlink. In Fig.1, d is the distance between the base station and a typical access point while its distance to the nearest interfering access point is represented by a random variable X. Parameters r and D represent the distances from a typical terminal to the access point and the base station, respectively. The access point chooses its sharing rate α in order to provide a DNA access link for the typical terminal with a maximum download rate αW where W is the maximum download rate of a given access point from the base station.

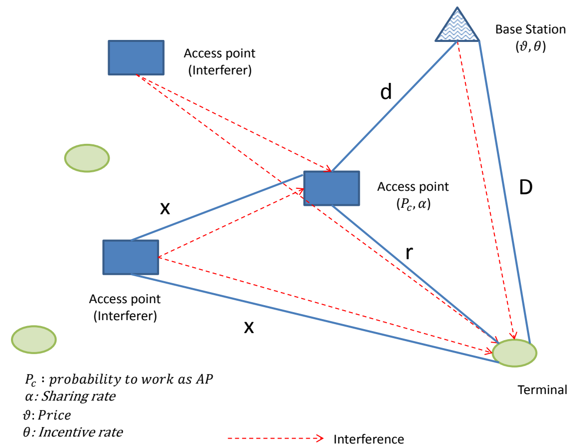

Fig. 1. Interference Model.

In a DNA network, we assume that the parameter $\Theta_2 = \frac{P_{bs} g d^{-2}}{N_0 + \sum_{i \in \Phi_{ac}} P_{ap} g |X_i|^{-2}}$ represents the signal to interference ratio of link bs-ap (base station-access point) where interferers are the activated access points, P_{ap} and P_{bs} are the transmitter powers of ap and bs, respectively, N_0 represents noise power (for interference limited scenario we discard this parameter in our analysis) and g is the channel gain.

Similarity let $\Theta_1 = \frac{P_{ap} g r^{-2}}{N_0 + \sum_{i \in \Phi_{ac}} P_{ap} g |X_i|^{-2} + P_{bs} g D^{-2}}$ denotes the signal to interference ratio of link ap-tr (access point-terminal) where the interferers are the activated access point and the base station. According to [20] and [21], the outage probability for link ap-tr with the propagation loss factor 2 is given as

$$O_1 = Pr(\Theta_1 \leq \gamma_1) = 1 - e^{(-\frac{P_s \lambda_{ac} \pi \gamma_1}{r^{-2} - \psi^{-1} D^{-2} \gamma_1})} \qquad (1)$$

where $\psi = \frac{P_{ap}}{P_{bs}}$, and P_s represents the competetive/uniform selection probability of access point by terminals. The other parameters are indicated in Fig.1. In (1), γ_1 is the minimum SINR level of link ap-tr. Likewise, the outage probability of link bs-ap is given as

$$O_2 = Pr(\Theta_2 \leq \gamma_2) = 1 - e^{(-P_s \lambda_{ac} \pi d^2 \psi \gamma_2)} \qquad (2)$$

where γ_2 is the minimum SINR level of link bs-ap. By considering the outage probability for both links, downlink throughput between the base station and a typical terminal in a DNA network is given by $C_\mathrm{DNA} = min((1 - O_1)log(1 + \gamma_1), \alpha(1 - O_2)log(1 + \gamma_2))$.

Terminals can choose the best access point from the available access point list. So, every access point can affect the terminal choices by offering a higher-capacity link according to its sharing rate α. In addition to the sharing rate α, the aggregated interference from the nearest active access points can influence the offered capacity by the access point. For this reason, every access point whose nearest interferers are further has more chance to be selected by incident terminals because it can offer more capacity to terminals. This condition provides a competitive environment for the access points trying to be selected by terminals. We assume that the sharing rates follows a normal distribution which depends on the traffic pattern of each access point. Therefore, the average competitive selection probability of a typical access point ap_i with α_i, which is inside the area A (Maximum coverage area of a given terminal) surrounding the tagged terminal is given by

$$\mathrm{S_i} = \sum_{m=1}^{\infty} \frac{\alpha_i}{(m+1)\mu_\alpha}(1 - exp(-\lambda_{ac}\pi {I_i}^2))\frac{\rho^m exp(-\rho)}{m!} \quad (3)$$

where m represents the number of active DNA access points in the area A, the term $\frac{\alpha_i}{(m+1)\mu_\alpha}$ is the relative sharing rate of ap_i in comparison to other available access points, I_i represents the expected distance from ap_i to its nearest interferer, and $\rho = \lambda_u P_c|A|$ is the access point density for the area A. In equation (3), the quantity $\frac{\rho^m exp(-\rho)}{m!}$ is a Poisson distribution with mean ρ representing the probability of having m access point inside the area A . Also, the term $1 - exp(-\lambda_{ac}\pi {I_i}^2)$ is the PDF (probability distribution function) of the distance between ap_i and its nearest interferer. Notice that when there is no competition between the access points, they can be selected uniformly by the same probability $\frac{1}{\bar{m}}$ where $\bar{m}$ represents the average number of access point inside the area A. Thus, in (1), $P_s = \frac{1}{\bar{m}}$ if terminals uniformly select access points and $P_s = \mathrm{S_i}$ if access points compete to encourage terminals to select them by increasing their sharing rates.

B. Utility Functions

In the proposed model, every user decides about its working mode and sharing rate α. On the other hand, the operator controls the DNA network by adjusting the price ϑ and the incentive rate θ. Therefore, every user tunes the parameters $(\mathrm{P_c}, \alpha)$ to optimize its individual optimum utility while the operator optimizes its own profit by adjusting (ϑ, θ). The utility of each user includes the gains of both terminal and access point modes. In the terminal mode, the user sends only its own traffic while in the access point mode the user benefits from the base station proportionally to its sharing rate. In view of all these, the utility function of a user can be modeled as

$$\begin{aligned}&\mathbf{u}_1(P_c, \alpha) = (1 - P_c)f_T(R) + P_c P_s\{f_{AP}(K_1\alpha R)- \\ &\vartheta((1-\alpha)R - K_2\theta\mathbf{C}_{\mathrm{DNA}})-(\zeta_r + \zeta_s)R\}\end{aligned} \quad (4)$$

where $f_T(.)$ and $f_{AP}(.)$ are concave functions representing the perceived utility of the user from not sharing(terminal mode) and sharing bandwidth (access point mode), K_1 and K_2 are positive constants, the parameters ζ_s and ζ_r are proportional to energy consumption rates in send and receive modes, and $R = W(1-O_2)log(1+\gamma_2)$ is data rate, where W is the available bandwidth. In (4), the first term represents the revenue in the terminal mode and the second part represents cost and gains for the access point mode. The term $K_2\theta\mathbf{C}_{\mathrm{DNA}}$ is the traffic volume delivered to terminals by the access point, $(1-\alpha)R$ as the downloaded traffic volume by the access point for itself, and $\vartheta((1-\alpha)R - K_2\theta\mathbf{C}_{\mathrm{DNA}})$ is the cost that must be paid to the operator by the access point when it uses $(1-\alpha)R$ for itself and $\mathbf{C}_{\mathrm{DNA}}$ for incident terminals. On the other hand, the revenue of operator from a typical DNA user is given as

$$\begin{aligned}&u_2(\vartheta, \theta) = (1 - P_c)\{R(\vartheta - \bar{\vartheta})\} \\ &+ P_c P_s\{\vartheta((1-\alpha)R - K_2\theta\mathbf{C}_{\mathrm{DNA}}) + \vartheta\mathbf{C}_{\mathrm{DNA}} - \\ &\bar{\vartheta}((1-\alpha)R + \mathbf{C}_{\mathrm{DNA}})\}\end{aligned} \quad (5)$$

where $\bar{\vartheta}$ represents the basic cost for operator to provide services, the access point pays $\vartheta((1-\alpha)R - K_2\theta\mathbf{C}_{\mathrm{DNA}})$ to the operator, $\vartheta\mathbf{C}_{\mathrm{DNA}}$ is paid by the terminal that uses the access point, and $\bar{\vartheta}((1-\alpha)R + \mathbf{C}_{\mathrm{DNA}})$ is the cost to provide service for the access point and the terminal. We can identify $\lambda_{ac}u_2$ as spatial revenue of DNA network.

C. System Optimization

To analyze the interactions between the user and the operator who seek to optimize the objectives (4) and (5) respectively, we first model the scenario where both access point and operator adjust only one of their parameters as a two player game. The first player (access point) adjusts its sharing rate in accordance with the price of operator and the second player (operator) adjusts its price. Here, it is assumed that P_c and θ are constants. In the next subsection, a general game model is proposed where players can adjust all these parameters concurrently. We propose the following game-theoretic formulation:

- The set of *players* (decision-makers) of the game is $\mathcal{N} = \{1, 2\}$, with the index "1" referring to the user which can act as terminal/access point and "2" to the operator/ base station.
- The user's control variables is the bandwidth sharing rate $\alpha \in [0, 1]$. For notational convenience, we denote α by x_1 , and by $\mathcal{X}_1 \equiv [0, 1]$ the user's action space.
- The operator's control variables is the price ϑ. Again, for convenience, we replace ϑ by $x_2 = \vartheta/\vartheta_{\max}$, and use $\mathcal{X}_2 = [0, 1]$ to denote the operator's action space.
- The utility function of player 1 is given by (4) and denoted by $u_1 \equiv u_1(x_1, x_2)$; likewise, the utility function of player 2 is given by (5) and denoted by $u_2(x_1, x_2)$.

In this context, we say that the network is at Nash equilibrium (NE) when the user and operator have no incentive to change their control variables. Formally, we say that the action profile $x* = (x_1^*, x_2^*)$ is at *Nash equilibrium* when $u_k(x^*) \geq u_k(x_k; x_{-k}^*)$, for all $k \in \mathcal{N}$ and for all $x_k \in \mathcal{X}_k$, $x_{-k} \in \mathcal{X}_{-k}$ (In the above and in what follows, "$-k$" denotes the opponent of player k). With this in mind, a natural question that arises is whether the game defined above admits a Nash equilibrium x^*. Otherwise, if this is not the case, the user and operator could be constantly changing their sharing rate and price, leading the system to instability. To answer this question, note first that (4) is a concave function of α and continuous in its other arguments. Likewise, (5) is linear in ϑ and continuous in its other arguments. In particular, this implies that the game's payoff functions $u_k(x_k; x_{-k})$ are continuous and individually concave in each player's action variable x_k. Hence, given that the player's action spaces $\mathcal{X}_k$

are compact and convex, the existence of an equilibrium is guaranteed by Debreu's theorem [22].

To examine the uniqueness of the game's Nash equilibria, we focus on the players'*marginal utilities* $V_k(x) = \frac{\partial u_k}{\partial x_k}$, $k \in \{1, 2\}$. Clearly, the players' marginal utilities represent each player's individual direction of steepest payoff ascent; hence, it is reasonable to expect that tracking them may lead the system to equilibrium. To that end, we propose the following *adjustment scheme* as a learning mechanism for the game's players:

$$x_k(n+1) = x_k(n) + \delta_n x_k(n)(1 - x_k(n))V_k(x(n)), \tag{AS}$$

where:

- $n = 0, 1, \ldots,$ is the stage of the players' learning process.
- $x_k(n)$ is the action of player $k \in \{1, 2\}$ at stage n.
- δ_n is a non-increasing step-size sequence (typically of the form $\delta_n = 1/n^p$ for some $p \in [0, 1]$).

Intuitively, the adaptive scheme (AS) means that player k follows the direction indicated by his marginal utility $V_k(x)$: in particular, the player increases x_k when doing so would lead to a unilateral increase in payoff, and decreases x_k otherwise. The adjustment factor $x_k(1 - x_k)$ is then included to ensure that the process remains well-defined – i.e. that the feasibility constraints $x_k \in \mathcal{X}_k = [0, 1]$ are not violated.

In this non-cooperative game, a Nash equilibrium represents a state where each player's marginal utility vanishes or a boundary point of the game's state space $\mathcal{X} = \mathcal{X}_1 \times \mathcal{X}_2$. As a result, every stationary point x^* of (AS) also is a Nash equilibrium of the game. However, since the game may admit several equilibria, it is not clear where the learning dynamics (AS) may end up converging – if at all. On the other hand, if the game admits a unique equilibrium, it is to be hoped that the dynamics (AS) converge to it from every initialization.

To study whether the game admits a unique equilibrium, we follow the approach of [23] and consider the game's "weighted Hessian" matrix $H_{ij} = a_i \frac{\partial V_i}{\partial x_j} + a_j \frac{\partial V_j}{\partial x_i}, \quad i, j \in \{1, 2\}$, where $a_i > 0$, $i \in \{1, 2\}$, are positive constants that represent the Hessian's weighing. Then, to have a unique equilibrium, the following constraints should hold (implying that H is negative-definite):

$$\begin{aligned} \varrho_1 &= \frac{M \pm \sqrt{M^2 + 4{N_1}^2}}{2} < 0, x_1 \leq \frac{A}{B} \\ \varrho_2 &= \frac{M \pm \sqrt{M^2 + 4{N_2}^2}}{2} < 0, x_1 > \frac{A}{B} \end{aligned} \tag{6}$$

where ϱ_1 and ϱ_2 are eigenvalues of the Hessian matrix, $M = 2a_1 P_c P_s f''_{AP}(K_1 x_1 R)$, $f''_{AP}(K_1 x_1 R)$ is the second order derivative of $f_{AP}(K_1 x_1 R)$, $N_1 = P_c P_s \vartheta_{max}\{a_1 R + a_1 K_2 \theta B - a_2 R - a_2 K 2\theta B + a_2 B\}$, $N_2 = P_c P_s \vartheta_{max} R\{a_1 - a_2\}$, $A = (1 - O_1)log(1 + \gamma_1)$ and $B = (1 - O_2)log(1 + \gamma_2)$. These constraints ensure that there are two negative eigenvalues for the Hessian matrix. In (6), concavity of function $f_{AP}(K_1 x_1 R)$ grantees that there are two negative eigenvalues. This matrix measures the impact of one player's actions on the payoff of the other player, so it plays a crucial role in determining whether the game admits a unique equilibrium. In particular, we obtain the following theorem:

Theorem 1. *Let $\mathcal{G}$ denote the game between user and operator defined above. Then, every Nash equilibrium of $\mathcal{G}$ is a stationary point of the adjustment scheme* (AS). *Furthermore, if Condition (6) holds, the game admits a unique Nash equilibrium $x^* \in \mathcal{X}$ and the adjustment scheme* (AS) *converges to x^* from every initialization $x(0) \in \mathcal{X}$, provided that δ_n is chosen small enough and $\sum_{n=1}^{\infty} \delta_n^2 < \sum_{n=1}^{\infty} \delta_n = \infty$.*

Proof: For the first claim of our theorem, assume initially that x^* is an interior Nash equilibrium of $\mathcal{G}$. Then, by assumption, we have $V_k(x^*) = 0$ for all $k \in \mathcal{N}$, i.e. x^* is stationary under (AS). Otherwise, if x^* is not interior, we can only have $V_k(x^*) = 0$ if $x_k^* = 0$ or $x_k^* = 1$ (i.e. x_k^* is an extreme point of $\mathcal{X}_k$). In that case however, the update step of (AS) again yields $x_k(n+1) = x_k(n)$, so x^* is stationary under (AS), as claimed.

Assume now that (6) also holds and x^* is a Nash equilibrium of $\mathcal{G}$. The fact that x^* is the unique Nash equilibrium of $\mathcal{G}$ follows from Theorems 2 and 6 of [23]. As for the convergence of (AS) to x^*, we proceed in two steps: first, we consider a dynamical system in continuous time that represents the "mean field" of (AS) and we show that this dynamical system converges to x^*; then, we show that the trajectories of (AS) are asymptotically close to these "mean trajectories", so they also converge to x^*.

To make all this precise, consider first the mean dynamics

$$\dot{x}_k = x_k(1 - x_k)V_k(x). \tag{7}$$

We then claim that the function

$$V(x) = \sum_{k=1,2} a_k \left[x_k^* \log \frac{x_k^*}{x_k} + (1 - x_k^*) \log \frac{1 - x_k^*}{1 - x_k} \right] \tag{8}$$

is a strict Lyapunov function for (7), i.e. $V(x(t))$ is strictly decreasing for every solution trajectory of (7) unless $x(t) = x^*$ for all $t \geq 0$. Indeed, by differentiating with respect to t, we obtain:

$$\begin{aligned} \dot{V}(x) &= \sum_{k=1,2} a_k \left[-x_k^* \frac{\dot{x}_k}{x_k} + (1 - x_k^*) \frac{\dot{x}_k}{1 - x_k} \right] \\ &= - \sum_{k=1,2} a_k (x_k - x_k^*) V_k(x). \end{aligned} \tag{9}$$

By condition (6) (cf. Theorem 6 in [23]), this last expression is strictly negative unless $x(t) = x^*$. It follows that $V(x(t))$ is strictly decreasing for every non-stationary solution orbit of (7), so $V(x(t)) \to \min_{x \in \mathcal{X}} V(x) = 0$. Since the global minimizer of $V(x)$ is x^*, we conclude that $\lim_{t \to \infty} x(t) = x^*$, as claimed.

For the convergence of the actual adaptive scheme (AS), note first that the iterates $x(n)$ of (AS) comprise an asymptotic pseudo-trajectory (APT) of (7) in the sense of [24]. To proceed, let

$$\begin{aligned} D_n &= V(x(n)) \\ &= \sum_{k=1,2} a_k \left[x_k^* \log \frac{x_k^*}{x_k(n)} + (1 - x_k^*) \log \frac{1 - x_k^*}{1 - x_k(n)} \right] \end{aligned} \tag{10}$$

A Taylor expansion of V then yields:

$$D_{n+1} \leq D_n + \delta_n \sum_{k=1,2} a_k(x_k(n) - x_k^*)V_k(x(n)) + \frac{1}{2}\delta_n^2 C \sum_{k=1,2} \|V_k(x(n))\|^2 \quad (11)$$

where we have used the last line of (9) in the derivation of the second term and $C > 0$ is a positive constant. Assume now that $x(n)$ always stays a bounded distance away from x^* so $\sum_{k=1,2} a_k(x_k(n) - x_k^*)V_k(x(n)) \leq -q$ for some $q > 0$ and for all n by Eq. (6). Thus, telescoping (10), we obtain $D_{n+1} \leq D_0 - q\sum_{j=1}^n \delta_j + \frac{1}{2}CV^2 \sum_{j=1}^n \delta_j^2$, where $V = \max_{k=1,2} \max_{x\in\mathcal{X}} V_k(x)$. Since $\sum_{j=1}^n \delta_j^2 < \sum_{j=1}^n \delta_j = \infty$, we have $D_n < 0$ for sufficiently large n, a contradiction.

From the above, we conclude that $x(n)$ must visit every neighborhood of x^* infinitely often. Since $x(n)$ is an asymptotic pseudo-trajectory (APT) of the mean dynamics (7) and the latter are globally attracted to x^*, Theorem 6.10 in [24] shows that $x(n) \to x^*$, as claimed. ■

D. Additional user and operator control variables

In the game-theoretic formulation above, the probability P_c that the user acts as an access point has been treated as a parameter of the game under study and likewise for the operator's incentive rate θ. In a setting where the user-operator interactions span a longer time horizon, these parameters could also be adjusted during communication, so they can be considered as additional control variables, to be optimized separately.

To account for the above considerations, the game-theoretic formulation of the previous section should be modified as follows:

- The possible actions of the user are of the form $x_1 = (\alpha, P_c)$ with $\alpha \in [0,1]$ denotes the user's bandwidth sharing rate and $P_c \in [0,1]$ the probability of acting as an access point. Accordingly, the action space of Player 1 is $\mathcal{X}_1 = [0,1] \times [0,1]$.
- The possible actions of the operator are of the form $x_2 = (\vartheta/\vartheta_{\max}, \theta)$ with $\vartheta \in [0, \vartheta_{\max}]$ denotes the price set by the operator and $\theta \in [0,1]$ denotes the associated incentive rate. As such, the action space of Player 2 is $\mathcal{X}_2 = [0,1] \times [0,1]$.
- Each player's utility function $u_k : \mathcal{X}_1 \times \mathcal{X}_2 \to \mathbb{R}$ is given by Eqs. (4) and (5) above.

A key difference between this extended formulation and that of the previous section is that the players' utility functions in general are no longer individually concave – note for instance the very complicated dependence of u_1 on the probability P_c. As a result, the existence (resp. uniqueness) of a NE cannot be established by Debreu's (resp. Rosen's) theorem as in the previous section.

Nevertheless, we provide below an adaptive learning scheme which directly extends (AS) and which has the property of unilaterally increasing each player's utility:

$$\dot{x}_{ks} = x_{ks}(1 - x_{ks})V_{ks}(x), \quad k \in \{1,2\}, s \in \{1,2\}, \quad (12)$$

where the index tuple (k,s) marks the s-th component of the action $x_k \in \mathcal{X}$ of the k-th player and $V_{ks}(x) = \frac{\partial u_k}{\partial x_{ks}}$ denotes the player's marginal utility with respect to said component.

The payoff-increasing properties of (12) can be understood by noting that

$$\frac{d}{dt}u_k(x_k(t); x_{-k}) = \sum_{s=1}^{2} \dot{x}_{ks}V_{ks} = \sum_{s=1}^{2} x_{ks}(1-x_{ks})V_{ks}^2 \geq 0, \quad (13)$$

i.e. the payoff of player k increases under (AS) for every *fixed* action x_{-k} of his opponent. Of course, when both players' action profiles evolve under (AS), there is no guarantee that either player's payoff increase because of the impact of each player's actions on the payoff of his opponent. However, if (AS) converges, the above reasoning shows that this limit point must also be a Nash equilibrium of the game.

V. Numerical results

In this section, the performance of the network is evaluated by numerical analysis using MATLAB. We assume that in typical ultra-dense network terminals and access points are randomly placed in an area of $1Km^2$. DNA network is simulated to find how attractive for wireless operators is to establish DNA networks. We also show that participation in a DNA network can provide more benefit for wireless users too. Table I includes parameters used in the simulations.

TABLE I
PARAMETER VALUE USED IN THE NUMERICAL ANALYSIS.

Parameter	Value
α	$0 \leq \alpha \leq 1$
μ	$0 \leq \mu \leq 1$
γ_1	1
γ_2	1
θ	$0 \leq \theta \leq 1$
λ_u	10^{-3} node/m^2
P_c	[0.00005-0.0005]
ζ_s	10
ζ_r	1
r	$[1 - 20]$
I_i	[1-10]
K_1,K_2	1
ϑ	1
W	1MHz

Fig.2 shows the spatial throughput given as $SP_i = S_i\lambda_{ac}(1-O_1)log(1+\gamma_1)$ for a DNA access point when considering only the impact of the nearest interferer. As expected, the closer interferer is it degrades more the performance of link quality and the selection probability of access points. Therefore, terminals would select more likely those access points which are far from other interfering access points and meanwhile they offer the higher bandwidth links. The graph shows that there is a significant fall in spatial throughput when the average distances of interferers (I_i) are reduced. In this simulation (Fig.2), $\alpha_i = 0.5$ and $\mu_\alpha = 0.2$. While these results were expected Fig.2 quantifies the impacts of I_i on the system performance.

In Fig.3, we plot the spatial selection probability of a given access point versus the average sharing rates of competing APs. In other words, in a DNA network, APs have to compete

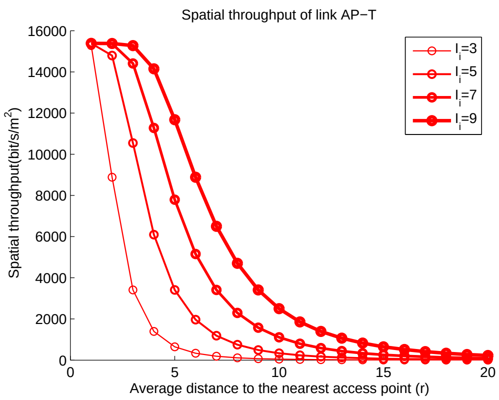

Fig. 2. Spatial throughput ($SP_i = S_i\lambda_{ac}(1 - O_1)log(1 + \gamma_1)$) of ap_i versus the average distance (r) to the nearest access point, I_i is the average distance of ap_i to its nearest interferer.

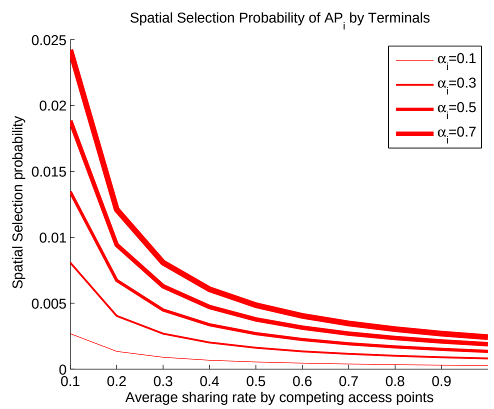

Fig. 3. Spatial Selection probability (equation (3)) of ap_i by terminals versus the average sharing rates of competing access points.

to become the first choice of terminals for downloading data. Access points may manage this process only by adjusting their sharing rates because they can not change position of the nearest interferer. Fig.3 show that access points can compete to win by adjusting their sharing rates. If they are far from other interferers, they more likely win (be selected by terminals) by offering a better download link for terminals.

In the next two figures, we present the results of our game-theoretic model. In Fig.4, spatial revenue of a DNA operator is presented versus cooperation rate of users (P_c). One can see that the operator can achieve more profit by offering the higher incentive rate (θ). As mentioned before, high participation rate of users as APs can bring more revenue to the DNA network operators. Fig.4 shows that the revenue has a upper bound and after that bound operator can not achieve to more revenue even with increase of its incentive rate. In Fig.4, the upper bound of spatial revenue is around 0.35. In Fig.5, revenue of one DNA operator is presented for different value of user density. DNA network model offers more benefit in very dense network because there are more users that like to act as access point without extra cost for infrastructure.

VI. Conclusion

This paper provided a framework for the analysis of the DNA model for ultra-dense networks. In this model, every user can work as an access point and extend the coverage of the

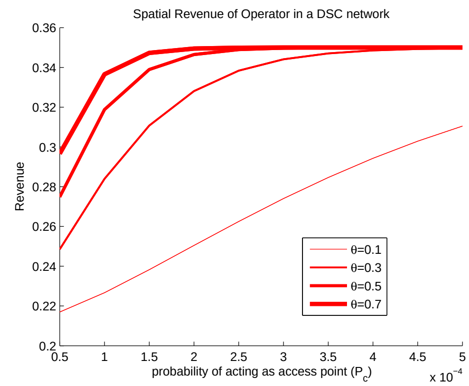

Fig. 4. The Spatial Revenue ($\lambda_{ac}u_2$) of operator for different incentive rates θ versus P_c.

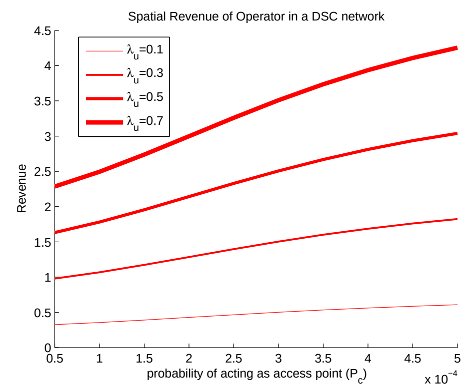

Fig. 5. The Spatial Revenue ($\lambda_{ac}u_2$) of operator for different node density λ_u versus P_c.

main network without need to reconfigure hardware/software equipments. A key contribution of our paper is a game theoretic model for DNA including terminals, access point and operators. For such a model we have developed an optimization algorithm and provided the prove of its convergency. Overall, the results of this paper indicate that DNA is an attractive network model for users and access points to extend dynamically current networks without any extra cost. As possible extensions of this work we are planning to consider terminals equipped with multiple antennas and develop beam forming schemes to select the most appropriate APs that satisfy the QoS requirements and maximize throughput. We expect that beam-forming will reduce the interference and so further improve the efficiency of these network

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