Cost-Efficient Power Allocation in OFDMA Cognitive Radio Networks

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***Abstract*—Cognitive radio (CR) systems allow opportunistic, secondary users (SUs) to access portions of the spectrum that are unused by the network's licensed primary users (PUs), provided that the induced interference does not compromise the PU' performance guarantees. To account for interference constraints of this type, we consider a flexible spectrum access pricing scheme that charges SUs based on the interference that they cause to the system's PUs (individually, globally, or both), and we examine how SUs can maximize their achievable transmission rate in this setting. We show that the resulting non-cooperative game admits a unique Nash equilibrium under very mild assumptions on the pricing mechanism employed by the network operator. In addition, we derive a dynamic power allocation policy that converges to equilibrium within a few iterations (even for large numbers of users), and which relies only on local signal-to-interference-plus-noise ratio (SINR) measurements. Our theoretical analysis is complemented by extensive numerical simulations which illustrate the performance and scalability properties of the proposed pricing scheme under realistic network conditions.**

***Index Terms*—Cognitive radio; multi-carrier systems; interference temperature; pricing; exponential learning.**

I. Introduction

A key challenge faced by current mobile networks is the projected spectrum crunch: if not properly managed, the existing radio spectrum will not be able to accommodate the soaring demand for wireless broadband and the ever-growing volume of data traffic [1]. To make matters worse, studies by the US Federal Communications Commission (FCC) and the National Telecommunications and Information Administration (NTIA) have shown that this vital commodity is effectively squandered through underutilization and inefficient use: for instance, only 15% to 85% of the licensed radio spectrum is used on average, leaving ample spectral voids that could be exploited via efficient spectrum management techniques [1, 2]. Accordingly, in this often unregulated context, the emerging paradigm of cognitive radio (CR) has attracted considerable interest as a promising way out of the spectrum gridlock [3–6].

At its most basic level, cognitive radio comprises a two-level hierarchy between wireless users induced by spectrum licencing: the network's licensed, primary users (PUs) have purchased spectrum rights from the network operator (often in the form of contractual quality of service (QoS) guarantees), but they allow unlicenced secondary users (SUs) to access the spectrum provided that the induced co-channel interference (CCI) remains below a certain threshold [3, 5]. Put differently, by sensing the wireless medium, the network's cognitive SUs essentially free-ride on the PUs' licensed spectrum and they try to communicate under the constraints imposed by the PUs (though, of course, without any QoS guarantees). Thus, by opening up the unused part of the spectrum, overall utilization is increased without needing to deploy more (and more expensive) wireless interfaces [4, 7].

In CR systems, PU requirements are often treated as interference temperature (IT) [8] constraints that are coupled across the network's SUs. The theoretical analysis of the resulting system then aims to characterize the network's optimum/equilibrium states and to provide the means to converge to such states [9–13]. These constraints are then enforced indirectly via exogenous pricing mechanisms that charge SUs based on the aggregate interference that they cause to the network's PUs (and, of course, PUs are reimbursed commensurately). In this context, the authors of [9] introduced a spectrum-trading mechanism based on a market-equilibrium approach [14] and they provided an algorithm allowing SUs to estimate spectrum prices and adjust their spectrum demands accordingly.

In this paper, we focus on cost-efficient throughput maximization in multi-carrier CR networks where SUs are charged based on the interference that they cause to the network's PUs (either on an aggregate or a per-user basis). Our system model is presented in Section II where we consider a general game-theoretic formulation that is flexible enough to account for both aggregate (flat-rate), temperature-based, and per-user pricing schemes. In the case of static channels (Section III), we show that the resulting game admits a unique Nash equilibrium almost surely, provided that the SUs' pricing schemes satisfy some fairly mild requirements (for instance, that a user's transmission cost increases with his radiated power).

Moreover, extending the exponential learning techniques of [15], we also derive a dynamic power allocation policy that converges to Nash equilibrium in a few iterations, even for large numbers of users and/or subcarriers per user. In particular, the

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proposed algorithm has the following desirable attributes:

1) *Distributedness:* user updates are based on local information and signal measurements.
2) *Statelessness:* users do not need to know the state (or topology) of the system.
3) *Unilateral reinforcement:* each user tends to increase his own utility; put differently, the algorithm is aligned with each user's individual objective.

Our analysis is supplemented in Section IV by extensive numerical simulations where we illustrate the performance gains of the proposed approach under realistic conditions.

II. System Model

Consider a set $\mathcal{K} = \{1, \ldots, K\}$ of (unlicensed) secondary users (SUs) that seek to connect to a common receiver over a set $\mathcal{S} = \{1, \ldots, S\}$ of non-interfering subcarriers (typically in the frequency domain if an orthogonal frequency division multiplexing (OFDM) scheme is employed). Focusing on the uplink case, the aggregate received signal y_s over the s-th subcarrier will then be:

$$y_s = \sum\nolimits_{k\in\mathcal{K}} h_{ks}x_{ks} + z_s, \tag{1}$$

where

1) $x_{ks} \in \mathbb{C}$ denotes the transmitted signal of user $k \in \mathcal{K}$ over the s-th subcarrier.
2) $h_{ks} \in \mathbb{C}$ is the corresponding transfer coefficient.
3) $z_s \in \mathbb{C}$ denotes the aggregate interference-plus-noise received from all sources not in $\mathcal{K}$.

In this context, the average transmit power of user k on subcarrier s will be

$$p_{ks} = \mathbb{E}\left[|x_{ks}|^2\right], \tag{2}$$

where the expectation is taken over the (Gaussian) codebook of user k; furthermore, each user's *total* transmit power $p_k = \mathbb{E}[\mathbf{x}_k^\dagger \mathbf{x}_k] = \sum_s p_{ks}$ will have to satisfy the power constraint

$$p_k = \sum\nolimits_{s\in\mathcal{S}} p_{ks} \leq P_k, \tag{3}$$

where $P_k > 0$ denotes the maximum transmit power of user $k \in \mathcal{K}$. In this way, the set of admissible power allocation vectors for user k is the S-dimensional polytope

$$\mathcal{X}_k = \left\{\mathbf{p}_k \in \mathbb{R}^{\mathcal{S}} : p_{ks} \geq 0 \text{ and } \sum\nolimits_{s\in\mathcal{S}} p_{ks} \leq P_k\right\}, \tag{4}$$

and the system's *state space* will be the product $\mathcal{X} = \prod_k \mathcal{X}_k$.

In this multi-carrier (MC) framework, each user's achievable transmission rate depends on his individual signal-to-interference-plus-noise ratio (SINR)

$$\mathsf{sinr}_{ks}(\mathbf{p}) = \frac{g_{ks}p_{ks}}{\sigma_s^2 + \sum_{\ell\neq k} g_{\ell s}p_{\ell s}}, \tag{5}$$

where $g_{ks} = |h_{ks}|^2$ denotes the channel gain coefficient for user k over the s-th subcarrier. Thus, in the single user decoding (SUD) regime (where interference by all other users is treated as additive noise), the maximum information transmission rate for user k (achievable with random Gaussian codes) will be:

$$R_k(\mathbf{p}) = \sum\nolimits_{s\in\mathcal{S}} \log\left(1 + \mathsf{sinr}_{ks}(\mathbf{p})\right) \tag{6}$$

where

$$w_s(\mathbf{p}) = \sum\nolimits_k g_{ks}p_{ks}, \quad s = 1, \ldots, S, \tag{7}$$

denotes the *aggregate* SU interference level per subcarrier (for convenience we will also write $\mathbf{w} = (w_1, \ldots, w_S)$ for the SUs' aggregate interference profile over all subcarriers $s \in \mathcal{S}$).

In the absence of other considerations, the unilateral objective of each SU would be the maximization of his individual transmission rate $R_k(\mathbf{p})$ subject to the total power constraint (3). In our CR setting however, the network operator needs to ensure that the system's PUs meet the QoS guarantees that they have already paid for – typically in the form of minimum rate requirements or maximum interference tolerance per subcarrier. Thus, to achieve this, we will consider a general spectrum access pricing scheme whereby SUs are charged according to the individual and aggregate interference that they induce.

Formally, this can be captured by the general cost model:

$$C_k(\mathbf{p}) = \pi_0(\mathbf{w}(\mathbf{p})) + \pi_k(\mathbf{p}_k), \tag{8}$$

where:

1) $\pi_0 \colon \mathbb{R}_+^{\mathcal{S}} \to \mathbb{R}_+$ is a *flat spectrum access price* that is calculated in terms of the aggregate SU interference level w_s per subcarrier $s \in \mathcal{S}$.
2) $\pi_k \colon \mathcal{X}_k \to \mathbb{R}_+$ is a *user-specific price* which is charged to user $k \in \mathcal{K}$ based on his *individual* radiated power profile $\mathbf{p}_k \in \mathcal{X}_k$.

In tune with standard economic considerations on diminishing returns [14], the only assumptions that we will make for the price functions π_0 and π_k are that:

(A1) Each π is non-decreasing in each of its arguments.
(A2) Each π is Lipschitz continuous and convex.

For concreteness, we provide below some typical examples of pricing models which we explore further in Section IV:

Model 1. Let $I_s^{\max}$ denote the PUs' interference tolerance on subcarrier s. Then, in the spirit of [11], we define the linear pricing (LP) flat-rate model as:

$$\pi_0^{\mathrm{LP}}(\mathbf{w}) = \lambda_0 \sum\nolimits_{s\in\mathcal{S}} w_s / I_s^{\max}, \tag{LP}$$

where the pricing parameter λ_0 represents the price paid by the network's SUs when saturating the PUs' interference tolerance.

Model 2. With notation as above, the violation pricing (VP) flat-rate model is defined as:

$$\pi_0^{\mathrm{VP}}(\mathbf{w}) = \lambda_0 \sum\nolimits_{s\in\mathcal{S}} \left[w_s / I_s^{\max} - 1\right]_+ \tag{VP}$$

where $\lambda_0 > 0$ is a sensitivity parameter and $[x]_+ \equiv \max\{x, 0\}$. In this model, SUs are only charged when the PUs' interference tolerance is actually violated, and the steepness of the sanction is controlled by the pricing parameter λ_0.

In light of all this, the *utility* of user k is defined as:

$$u_k(\mathbf{p}) = R_k(\mathbf{p}) - C_k(\mathbf{p}), \tag{9}$$

i.e., $u_k(\mathbf{p})$ is simply the user's achieved transmission rate minus the cost reimbursed to the network operator in order to achieve it. In turn, this leads to the *cost-efficient throughput maximization game* $\mathcal{G} \equiv \mathcal{G}(\mathcal{K}, \mathcal{X}, u)$, defined as follows:

1) The game's *players* are the system's secondary users $k \in \mathcal{K} = \{1, \dots, K\}$.
2) The *action set* of each player/user is the set of feasible power allocation profiles $\mathcal{X}_k = \{\mathbf{p}_k \in \mathbb{R}^{\mathcal{S}} : p_{ks} \geq 0 \text{ and } \sum_{s\in\mathcal{S}} p_{ks} \leq P_k\}$.
3) Each player's *utility function* $u_k \colon \mathcal{X} \equiv \prod_k \mathcal{X}_k \to \mathbb{R}$ is given by (9).

In this context, we will say that a power allocation profile $p^* \in \mathcal{X}$ is at *Nash equilibrium (NE)* when

$$u_k(\mathbf{p}_k^*; \mathbf{p}_{-k}^*) \geq u_k(\mathbf{p}_k; \mathbf{p}_{-k}^*) \quad \text{for all } \mathbf{p}_k \in \mathcal{X}_k \text{ and for all } k \in \mathcal{K}, \tag{NE}$$

i.e., when *each user's chosen power profile $p_k^* \in \mathcal{X}_k$ is individually cost-efficient given the power profile of his opponents* (so no user has a unilateral incentive to deviate). Accordingly, our goal in the rest of the paper will be to characterize the Nash equilibria of $\mathcal{G}$ and to provide distributed optimization methods allowing selfish (and myopic) SUs to converge to equilibrium in the absence of centralized medium access control mechanisms.

III. Learning

In this section, we focus on how players can attain an equilibrium state by means of a simple, adaptive learning process. Our proposed algorithm will rely on the users' *marginal utilities*:

$$\mathbf{v}_k(\mathbf{p}) = \nabla_k u_k(\mathbf{p}) \tag{10}$$

where ∇_k denotes differentiation with respect to the power profile $\mathbf{p}_k$ of user k. In particular, writing $\mathbf{v}_k = (v_{k,1}, \dots, v_{k,S})$, some easy algebra yields the component-wise expression

$$v_{ks}(\mathbf{p}) = \frac{\partial u_{ks}}{\partial p_{ks}} = g_{ks}\left(\frac{1}{\sigma_s^2 + w_s} - \frac{\partial \pi_0}{\partial w_s}\right) - \frac{\partial \pi_k}{\partial p_{ks}}, \tag{11}$$

which shows that $v_{ks}(\mathbf{p})$ can be calculated by each individual user knowing only their SINR per subcarrier (which is measured locally) and the functional form of the price functions π_0 and π_k (which are agreed upon by the network's SUs and the PU and are thus also known locally). Indeed, Eq. (5) shows that the aggregate interference level on subcarrier s can be calculated by user k as:

$$w_s(\mathbf{p}) = \sum\nolimits_k g_{ks} p_{ks} = g_{ks} p_{ks} \frac{1 + \mathsf{sinr}_{ks}(\mathbf{p})}{\mathsf{sinr}_{ks}(\mathbf{p})}, \tag{12}$$

i.e., requiring only local SINR measurements and the knowledge of the user's channel (which can in turn be obtained through the exchange of pilot signals). As a result, the marginal utility vectors v_k can be calculated in a completely distributed fashion with locally available information.

By definition, the users' marginal utility vectors define the direction of unilaterally steepest utility ascent, i.e., the best direction that a user could follow in order to increase his utility. As such, a natural learning process would be for each user to track this steepest ascent direction with the hopes of converging to a Nash equilibrium; however, given the problem's power and positivity constraints, this method may quickly lead to inadmissible power profiles that do not lie in $\mathcal{X}$ – in which case convergence is also out of the question.

To account for these constraints, we will employ an interior point method which increases power on subcarriers that seem to be performing well, without ever shutting off a particular channel completely. Formally, consider the *exponential regularization map* $\mathbf{G} \colon \mathbb{R}^{\mathcal{S}} \to \mathbb{R}_+^{\mathcal{S}}$ given by

$$\mathbf{G}(\mathbf{v}) = \frac{1}{1 + \sum_s \exp(v_s)} \left(\exp(v_1), \dots, \exp(v_S)\right). \tag{13}$$

This map has the property that it assigns positive weight (power) to all subcarriers and exponentially more weight to the subcarriers $s \in \mathcal{S}$ with the highest marginal utilities v_s. Furthermore, if all marginal utilities are relatively low (indicating high transmission costs), all assigned weights will also be low in order to decrease the user's cost. With this in mind, our proposed exponential learning algorithm for cost-efficient rate maximization is as follows:

Algorithm 1 Exponential Learning for Cost-Efficient Rate Maximization

```
Parameter: step size γ_n.
Initialize: n ← 0; scores y_ks ← 0 for all k ∈ K, s ∈ S.
Repeat
    n ← n + 1;
    foreach user k ∈ K do
        foreach subcarrier s ∈ S do
            set transmit power p_ks ← P_k exp(y_ks) / (1 + Σ_r exp(y_kr));
            measure sinr_ks;
            update marginal utilities: v_ks ← (1/p_ks) · sinr_ks/(1 + sinr_ks) − ∂C_k/∂p_ks;
            update scores: y_ks ← y_ks + γ_n v_ks;
until termination criterion is reached.
```

We then obtain:

Theorem 1. *Let γ_n be a variable step-size sequence such that $\sum_n \gamma_n = \infty$ and $\sum_{j=1}^n \gamma_j^2 / \sum_{j=1}^n \gamma_j \to 0$. Then, Algorithm 1 converges to Nash equilibrium in the cost-efficient rate maximization game $\mathcal{G}$.*

Proof: Omitted due to space limitations. ∎

IV. Numerical Results

To evaluate the performance of the proposed cost-efficient power allocation framework for throughput maximization in cognitive radio networks, we have performed extensive numerical simulations over a wide range of system parameters. In what follows, we provide a selection of the most representative cases.

Throughout this section, and unless explicitly mentioned otherwise, we consider a population of $K = 10$ SUs uniformly distributed over a square area and $S = 10$ non-interfering subcarriers with channel gain coefficients g_{ks} drawn according to the path-loss model for Jakes fading proposed in [16]; the other relevant simulation parameters are summarized in Table I. For simplicity, we also assume that σ_s and P_k are equal for

TABLE I
Simulation Setting

Parameter	Value
Carrier frequency	$f_c = 2.4\,\mathrm{GHz}$
Channel bandwidth	$B = 10.93\,\mathrm{KHz}$
Noise spectral density	$\sigma_s = -173\,\mathrm{dBm/Hz}$
Maximum transmitting power of SUs	$P_k = 21.03\,\mathrm{dBm}$
Edge of the simulated square area	$L = 200\,\mathrm{m}$
Transmitting power of the PU	$P^{\mathrm{PU}} = 30\,\mathrm{dBm}$
Distance of the PU from the receiver	$d = 50\,\mathrm{m}$

all $s \in \mathcal{S}$ and all $k \in \mathcal{K}$; finally, we will assume that PUs have the same interference tolerance level $I_s^{\max}$ over all subcarriers $s \in \mathcal{S}$.

To begin with, Figs. 1(a)–1(c) compare the performance of the proposed power allocation scheme to the benchmark case of uniform power allocation – i.e., when SUs transmit at full power and allocate their power uniformly over the available subcarriers, irrespective of the PU's requirements. For some values of λ_0, the SUs' sum-rate under uniform power allocation is higher than the one achieved by the proposed approach, but this comes at the expense of violating the PU's minimum QoS requirements (which constitutes a contractual breach from the PU's perspective); on the contrary, our approach always respects the PU's contractual requirements (since the λ_0 pricing parameter is negotiated with the PU), while guaranteeing high throughput to the SUs. This is seen in Fig. 1(b): the PU's throughput exceeds the throughput achieved when SUs employ a uniform power allocation policy, except when the PU has no significant QoS requirements ($I^{\max} \to \infty$), in which case the SUs exploit all the available spectrum and the PU's rate is reduced. Furthermore, in Fig. 1(c) we illustrate the normalized revenue of the proposed approach w.r.t. the revenues generated by uniform power allocation policies. Note that the income generated by the proposed approach is up to 3× higher than the income generated by SUs that are not cost-/energy-aware and transmit naïvely at full power, using a uniform power allocation policy. Thus, by fine-tuning his pricing scheme, the PU not only achieves his QoS requirements, but also increases his monetary revenue against cost-aware SUs.

In Figs. 2 and 3, we investigate the length of the system's off-equilibrium phase and the convergence rate of the proposed distributed learning scheme (Algorithm 1). By Theorem 1, the iterations of Algorithm 1 converge to Nash equilibrium when using a step-size sequence γ_n such that $\sum_{j=1}^{n} \gamma_j^2 / \sum_{j=1}^{n} \gamma_j \to 0$ as $n \to \infty$. As discussed in [17], a rapidly decreasing step-size sequence slows down the algorithm, so we examine here the usage of a fixed step size to accelerate convergence. This choice makes the algorithm run faster; on the other hand, a fixed step-size may lead to unwanted oscillations around the equilibrium point, thus interfering with the algorithm's end-state. To account for this, we employ a very aggressive schedule during the first non-oscillating iterations which becomes more conservative (thus guaranteeing convergence) once oscillations are noticed.

To assess the method's efficiency, we plotted the system's equilibration level (EQL) [15]; by definition, an EQL value of 1 means that the system is at Nash equilibrium. Accordingly, in Fig. 2, we show the evolution of the EQL and the system's sum-rate at each iteration for different step-size rules and interference pricing models. As expected, a conservative step-size of the form $\gamma_n = n^{-\beta}$, $1/2 < \beta < 1$, leads to relatively slow convergence (of the order of several tens of iterations or worse). On the other hand, the use of search-then-converge (STC) and fixed-step methods greatly accelerates the users' learning rate: after only a few STC iterations the system's EQL exceeds 90%, and the algorithm's convergence is accelerated even further by increasing the constant step-size in the "exploration" phase of the STC method.

Finally, to investigate the scalability of the proposed learning scheme, we also examine the algorithm's convergence speed for different numbers of SUs. In Fig. 3 we show the number of iterations needed to reach an EQL of 95%: importantly, by increasing the value of the algorithm's step-size, it is possible to reduce the system's transient phase to a few iterations, even for large numbers of users. Moreover, we also note that the algorithm's convergence speed in the LP model depends on the pricing parameter λ_0 (it decreases with λ_0), whereas this is no longer the case under the VP model. The reason for this is again that the VP model acts as a "barrier" which is only activated when the PUs' interference tolerance is violated.

V. Conclusions

In this paper, we considered a game-theoretic formulation of the problem of cost-efficient throughput maximization in multi-carrier CR networks where SUs are charged based on the interference that they cause to the system's PUs. We showed that the resulting game admits a unique Nash equilibrium under fairly mild conditions (and for both static and ergodic channels), and we derived a fully distributed learning algorithm that converges to equilibrium using only local SINR and channel measurements (and, again, under both static and fast-fading channel conditions). Our analysis shows that the choice of the exact pricing scheme has a strong impact on the network's achievable performance (for both licensed and unlicensed users): in the "soft-pricing" regime, the PUs' requirements are violated in exchange for monetary reimbursement; by contrast, higher prices safeguard the PUs' requirements, but (somewhat surprisingly) generate no revenue to the PUs. Moreover, thanks to the fast convergence of the proposed algorithm, the system's transient (off-equilibrium) phase is minimized, so SUs avoid being unduly uncharged for relatively low throughput levels.

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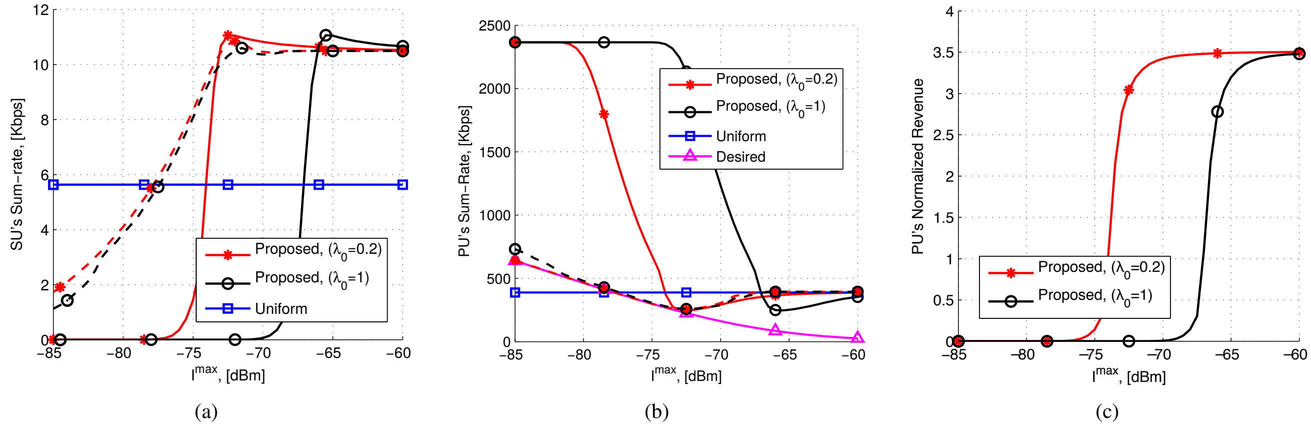

Fig. 1. Comparison between the proposed and uniform power allocation approaches: a) Sum-rate of SUs; b) Sum-rate of the PU; c) Normalized revenue of the proposed approach w.r.t. the uniform power allocation policy (LP: solid lines with star and circle markers; VP: dashed lines with star and circle markers).

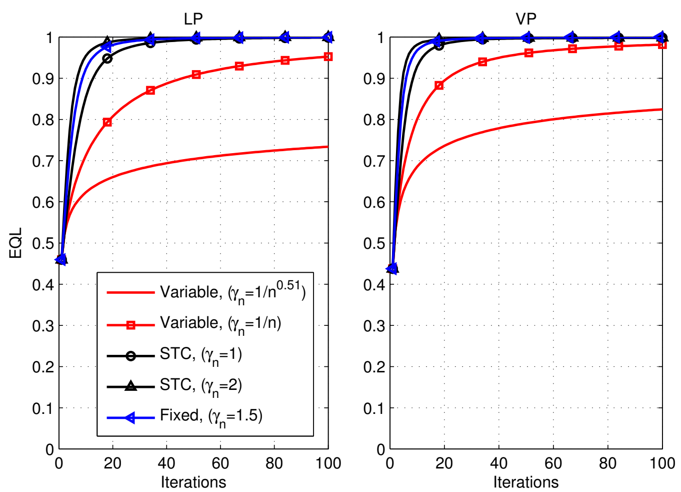

Fig. 2. Equilibration level, EQL(n), for different step-size rules under and flat-rate interference pricing models.

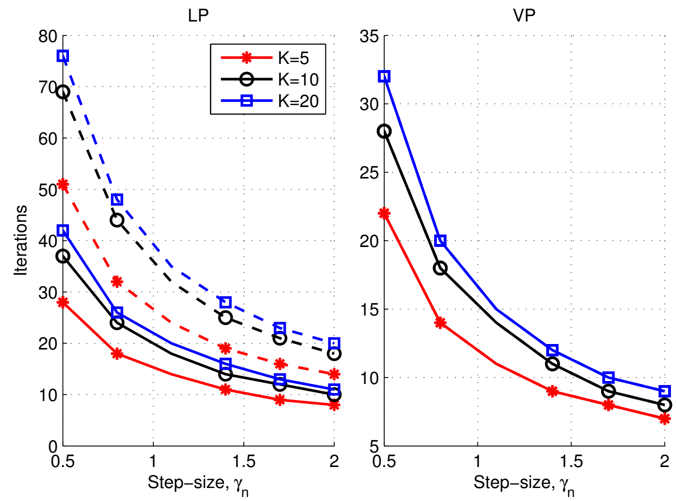

Fig. 3. Scalability of the proposed learning scheme as a function of the step-size γ for different values of the number K of SUs and pricing schemes (λ_0 = 0.1: solid lines; λ_0 = 0.5 dashed lines).

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