Stochastic modeling, mean-field and smart cities

Building and analyzing models of bike sharing systems

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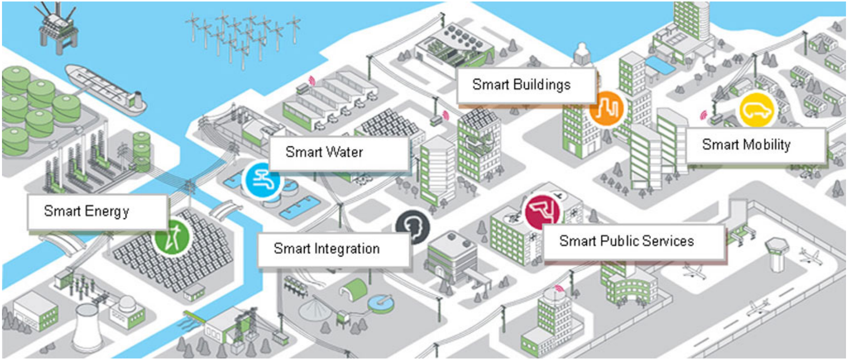
COOPERATION

What are "smart" cities?

What are “smart” cities?

According to google image:



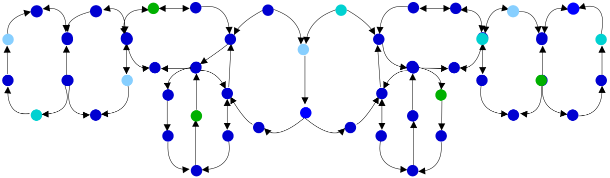
What are "smart" cities?

According to google image:

What are "smart" cities?

Smart-* = Monitor, Model, Manage

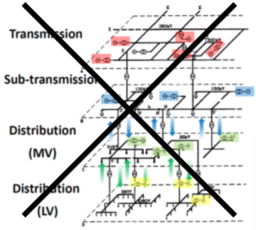
Smart cities are composed of many interacting individuals

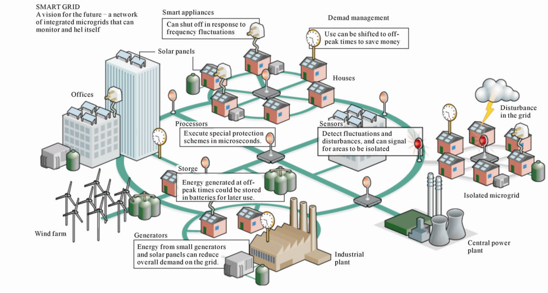


- Individual objectives lead to collective behaviour.

“Smart”-* involve more decentralized control

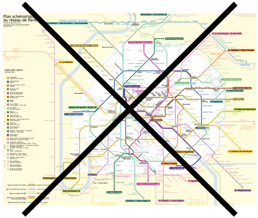
Example 1: Smart-grids





“Smart”-* involve more decentralized control

Example 2: bike-sharing systems



Research challenge

Develop tractable models for collective adaptive systems.

- Build model from systems (automatic)
- Obtain macroscopic properties in order to help system designers.

Example of questions that we want to answer

- **Smart grid** – (How) Can we use prices for distributed control?

  Gast, Le Boudec, Proutière, Tomozei – Impact of Storage on the Efficiency and Prices in Real-Time Electricity Markets. ACM e-Energy '13,
  Gast, Le Boudec, Tomozei – Impact of demand-response on the efficiency and prices in real-time electricity markets. ACM e-Energy '14,

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- **Bike-sharing** – Can we regulate the system without manually redistributing the bikes?

  Fricker Gast (2014) – Incentives and redistribution in homogeneous bike-sharing systems with stations of finite capacity. EURO Journal on Transportation.
  Waserhole, Jost (2012) – Vehicle Sharing System Pricing Regulation : A Fluid Approximation

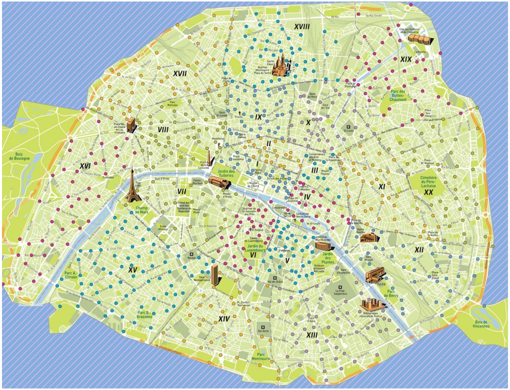
Outline

1. Bike-sharing systems: an overview
2. Mean-field approximation for performance evaluation
3. Macroscopic properties of bike-sharing systems
    - The homogeneous model
    - Adding some heterogeneity
    - Frustration of the demand
4. Conclusion

Outline

Who has already used a bike-sharing system and what was your experience?

Bike-sharing is a rather new transportation system.

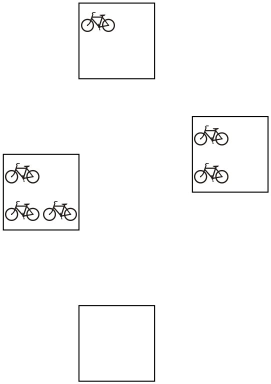


Map of Velib' stations in Paris (France).

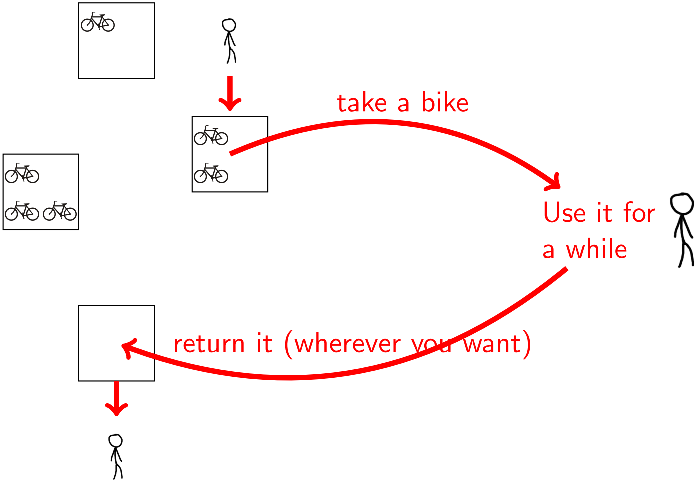
Example of Velib':

- 20000 bikes
- 2000 stations.

Bike-sharing systems



Bike-sharing systems



The main problem is the lack of resource

(a) Empty station

(b) Full station

Problematic states

The system's operator want to anticipate and avoid those states.

How to manage them?

To take good strategic decisions, one need to identify bottlenecks.

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To take good strategic decisions, one need to identify bottlenecks.

Decisions:

- Planning (number of stations, location, size)
- Long term: static pricing, number of bikes.
- Short term operating decisions: dynamic pricing, repositioning.

State of the art

Visualization of existing systems

- Traces analysis, clustering (Borgnat et al. 10, Vogel et al. 11, Nair et al. 11, Côme et al. 13...)

Short-term / mid-term prediction of availability

- (Ji Won Yoon et al. 12, Guenther et al. 12)

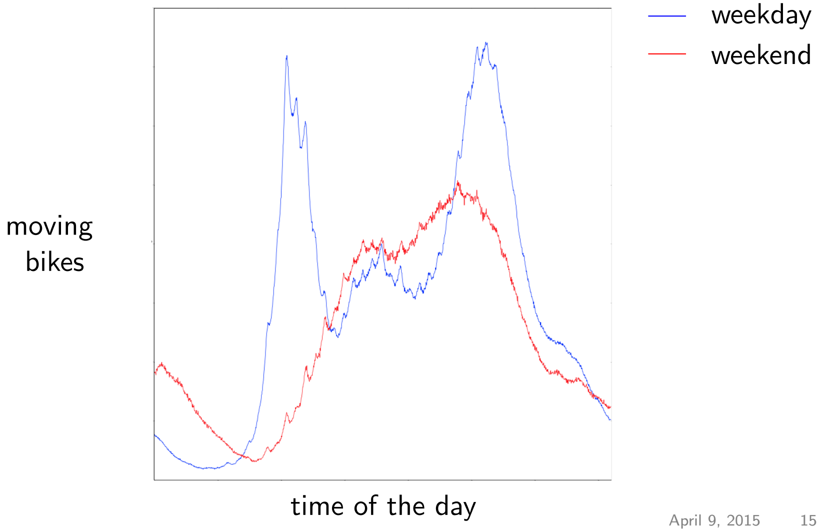
Bike re-positioning (classical RO problem)

- Redistribution based of forecast [Raviv et al. 11, Chemla et al. 13, Pfrommer 13,...]

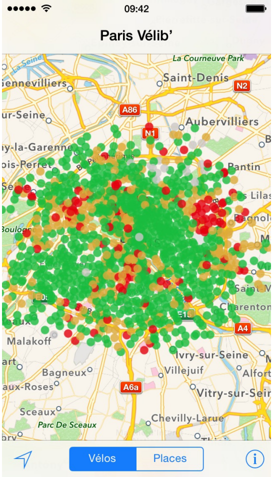
Planing using macroscopic data

Visualizing the data: usage varies (data from paris, 2014)

Example : temporal variation



Example: spatial variation



Source: http://www.bicyclette-app.com/fr/

Visualizing the data: usage varies (data from paris, 2014)

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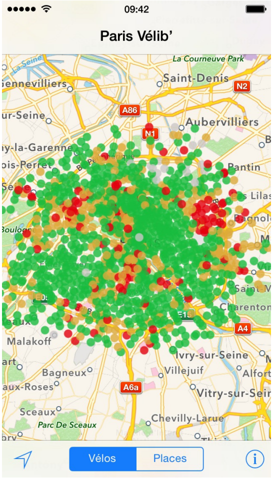
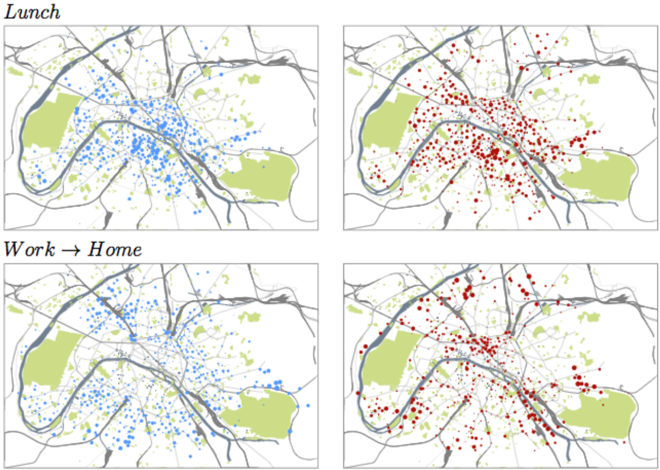


→ Solution: clustering?

Source: http://www.bicyclette-app.com/fr/

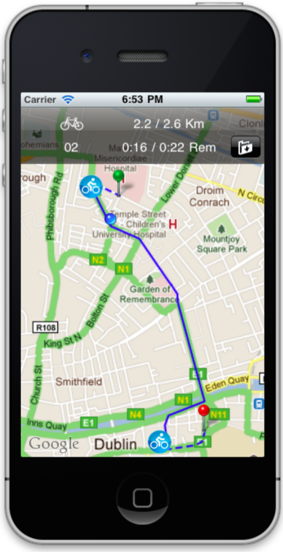
Visualizing the data: usage varies (data from paris, 2014)

Example: spatio-temporal variation



Côme et al (2013) – Spatio-temporal analysis of Dynamic Origin-Destination data using Latent Dirichlet Allocation. Application to the Vélib' Bike Sharing System of Paris

Prediction is for trip planning, multi-modal transportation

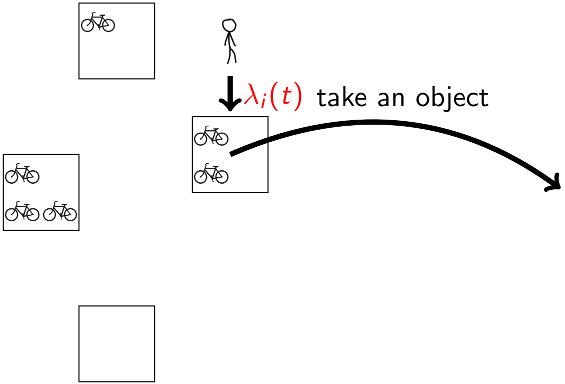


Cityride: a predictive bike sharing journey advisor
Ji Won Yoon, Fabio Pinelli, and Francesco Calabrese, 2012

Our objective

We want to understand the emergent behavior of the model and to build a rigorous mathematical model that can be analyzed quickly and fed by data.

We consider a markovian model



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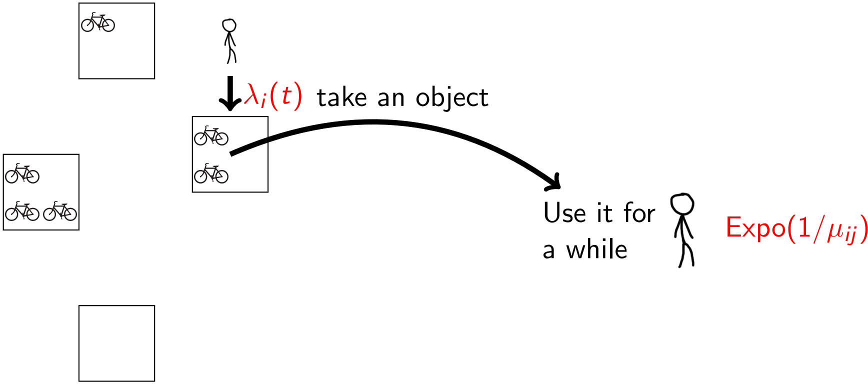

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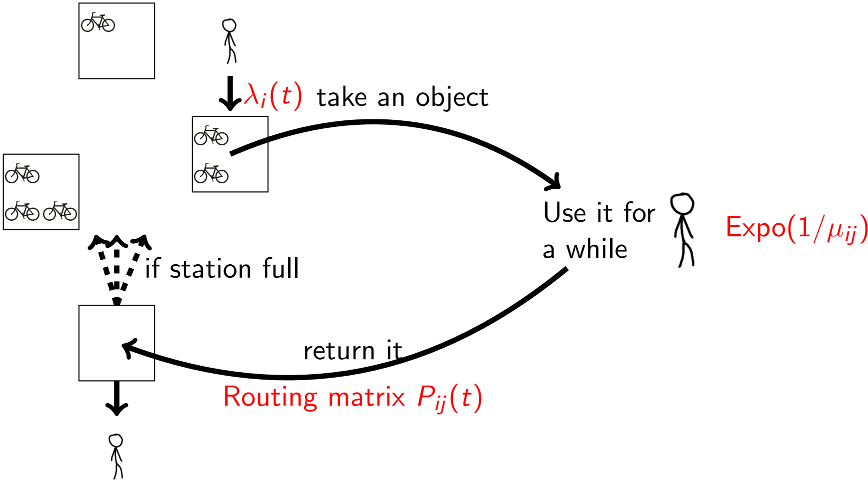


Outline

QUANTICOL Research Vision

The QUANTICOL's objective is to develop an innovative formal design framework consisting of:

- an unambiguous way of describing the behaviour;
- a logic
- model checking

Stochastic process algebras

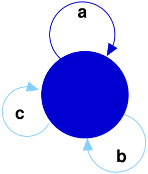
- Models consists of agents which engage in actions at some rate.

$$(\alpha, r).P$$

action type or name

activity rate (parameter of an exponential distribution)

component/ derivative



Stochastic process algebras

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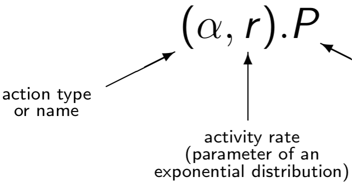
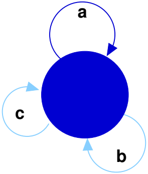
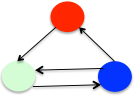


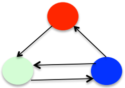


- The language is used to generate a Continuous Time Markov Chain (CTMC) for performance modelling.

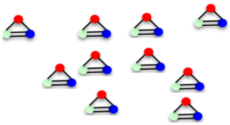


CTMC model

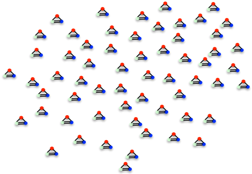
Problem: the state space grows exponentially with the number of objects.



1 object
3 states



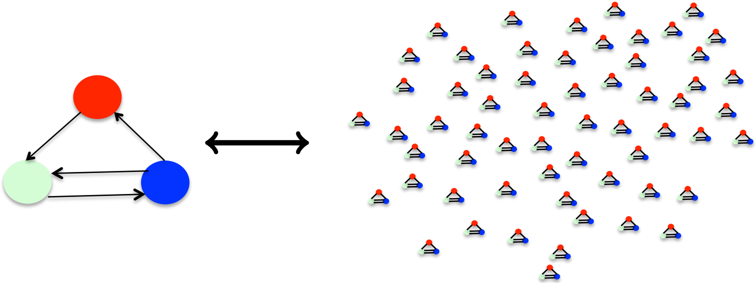
10 objects
$3^{10} \approx 10^5$ states



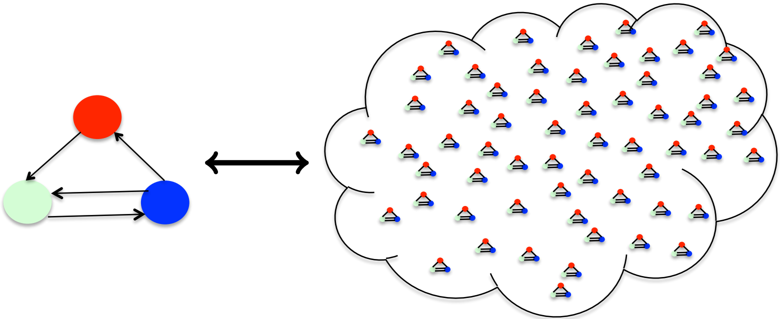
60 objects
$3^{60} \approx 10^{28}$ states

- Only simulation?

Mean Field Approximation

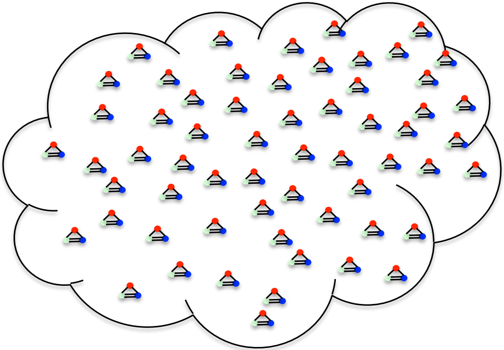


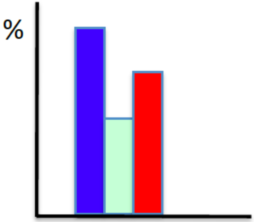
Mean Field Approximation



We view the population of objects more abstractly, assuming that individuals are indistinguishable.

Mean Field Approximation





An occupancy measure records the proportion of agents that are currently exhibiting each possible state.

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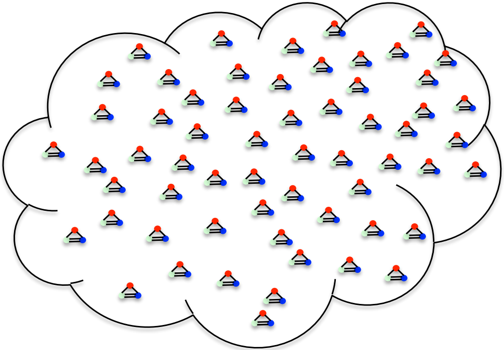

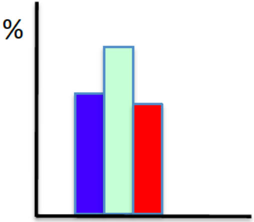

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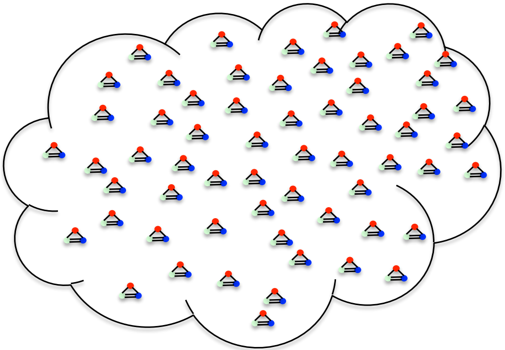

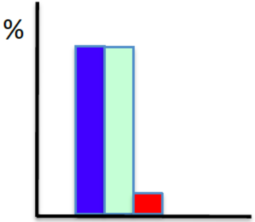

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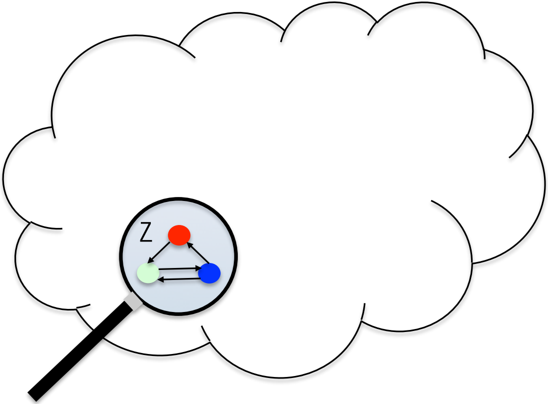
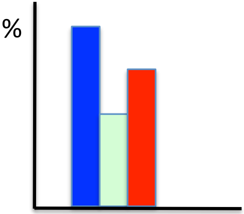
Example: Fluid Model Checking

L.Bortolussi and J.HIllston, Fluid Model Checking, CONCUR 2012





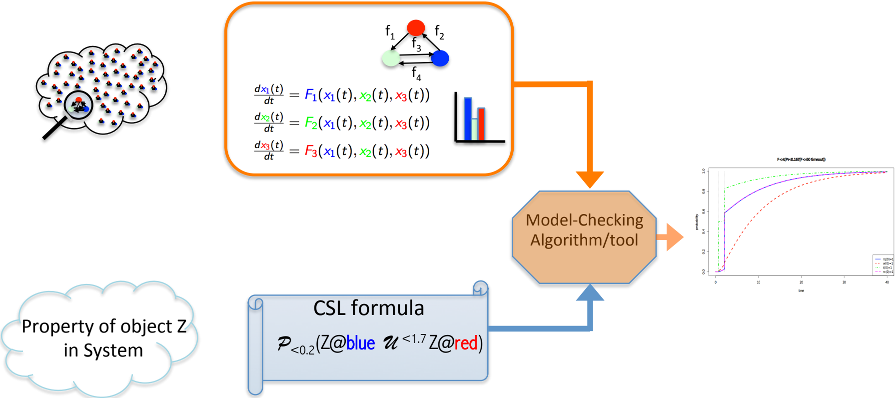
e.g. *agent Z is in the blue state until it enters the red state and this must occur within time 1.7.*

- The agent is considered in the mean field created by the rest (it is represented as a time-inhomogeneous CTMC.)

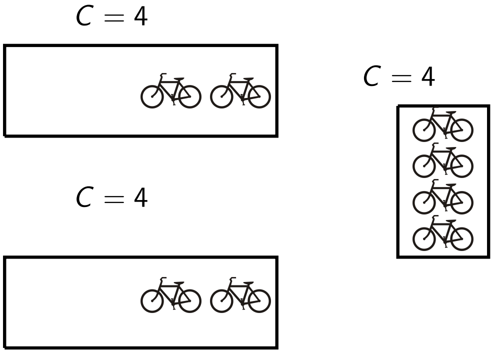
Fluid Model Checking

L.Bortolussi and J.HIllston, Fluid Model Checking, CONCUR 2012

www.quanticol.eu



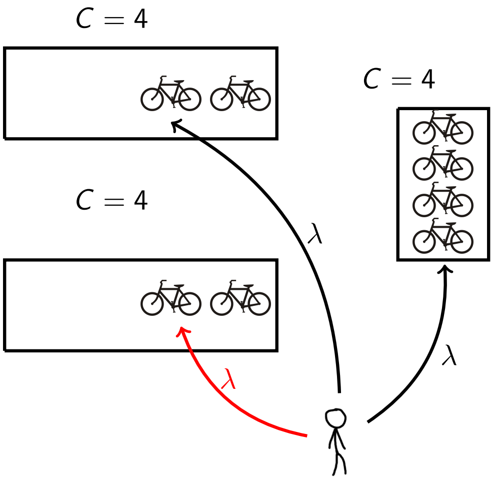
The simplest case: homogeneous system with n stations



For all stations:

- Fixed capacity C

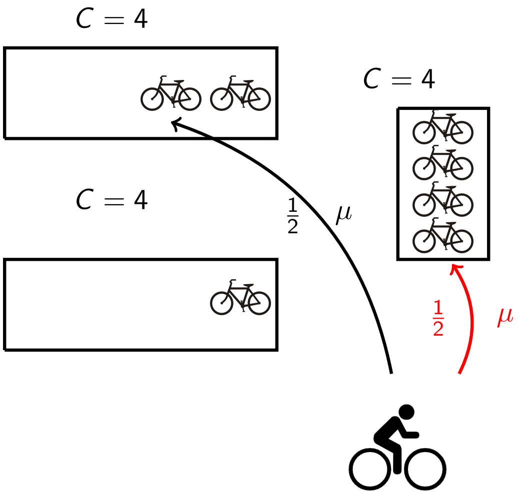
The simplest case: homogeneous system with n stations



For all stations:

- Fixed capacity C
- Arrival rate λ.

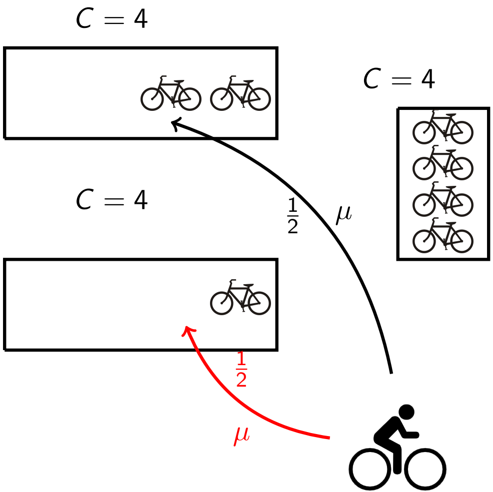
The simplest case: homogeneous system with n stations



For all stations:

- Fixed capacity C
- Arrival rate λ.
- Routing matrix: homogeneous.
- Travel time: exponential of mean $1/\mu$.

The simplest case: homogeneous system with n stations

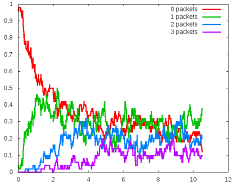


For all stations:

- Fixed capacity C
- Arrival rate λ.
- Routing matrix: homogeneous.
- Travel time: exponential of mean $1/\mu$.
- Other destination chosen if full ($\approx$ local search).

We take the limit as n goes to infinity

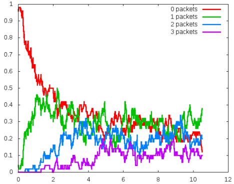
$$x_i = \frac{1}{n}\#\{\text{stations with i bikes}\}$$



For fixed N, X_i is a complicated stochastic process

We take the limit as n goes to infinity

$$x_i = \frac{1}{n}\#\{\text{stations with i bikes}\} \propto \rho^i$$



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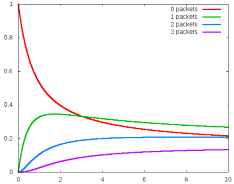
$n \to \infty$



System is described by an ODE

Use mean field approximation [Kurtz 79]

- Study the system when the number of stations N goes to infinity.

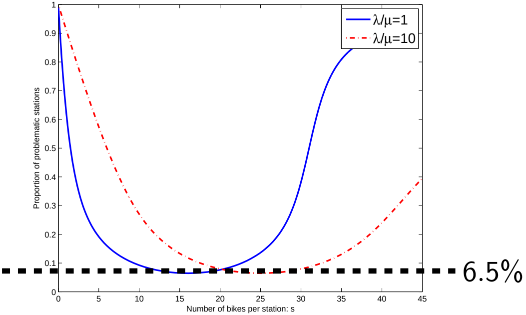
Congestion due to random choices is not negligible

Theorem

- *As n goes to infinity, at least $2n/(C+1)$ stations are problematic.*
- *The optimal fleet size is for $\frac{C}{2}+\frac{\lambda}{\mu}$ bikes per station.*

If the capacity is $C = 30$ bikes and you use the system twice a week, you cannot do a trip once a week.

Proportion of problematic stations



Fleet size (number of bikes per station)

Improvement can be dramatic with simple incentives

Algorithm: we force the users to go to the station that has the least number of bikes among the two closest to his destination.

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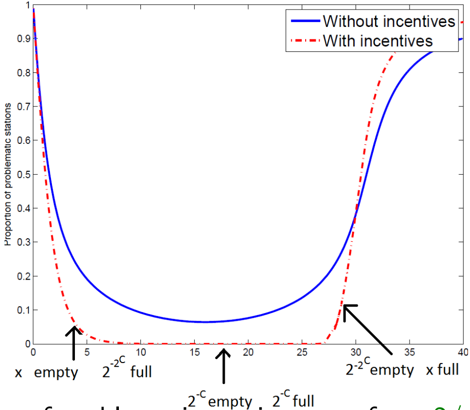

Proportion of problematic station goes from $2/C$ to $\sqrt{C}2^{-C/2}$.

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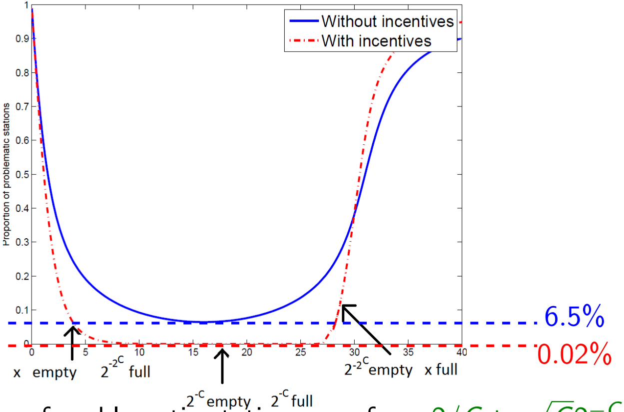


Proportion of problematic station goes from $2/C$ to $\sqrt{C}2^{-C/2}$.

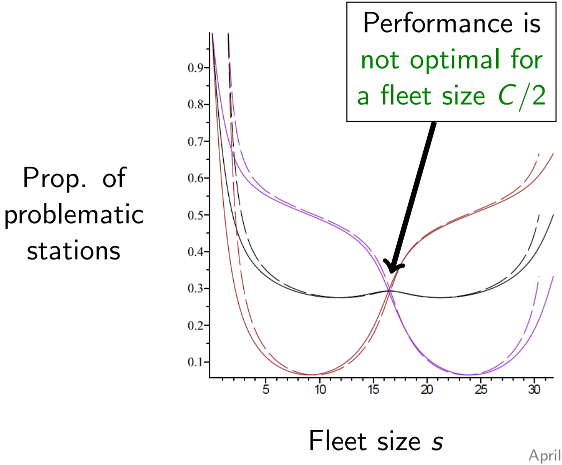
When the stations have different popularities, the previous results do not hold.

Popularity of a station is described by (λ_i, p_i).

- The optimal fleet size can be different than $C/2$.
- Having stations of infinite capacities can worsen the situation.

With two clusters, the optimal fleet size is not $C/2$

Two types of stations: popular and non-popular for arrivals: $\lambda_1/\lambda_2 = 2$.



Infinite capacities can worsen the situation

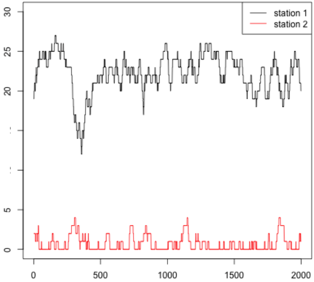
Infinite capacities can worsen the situation

Theorem (Malyshev-Yakovlev 96)

When the stations have infinite capacity, then there exists a critical fleet size s_c such that if $s > s_c$, bikes accumulate in a few stations.

Example: station 1 is a destination twice as popular as stations 2 to 9. There are 27 bikes for 9 stations.

number of bikes in a station



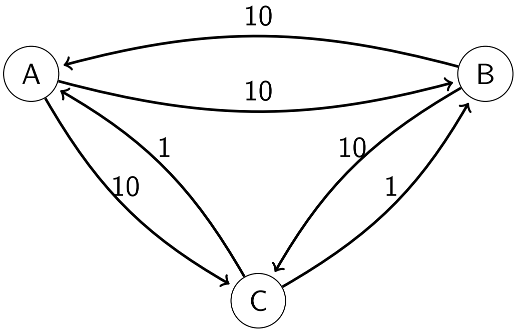
Time

Having finite capacities prevent saturation of the demand. What if we could frustrate some demand?

Model: we have a trip demand $\Lambda_{ij}(t)$ and an accepted demand $\lambda_{ij}(t)$.

- Generous policy: $\lambda_{ij}(t) := \Lambda_{ij(t)}$
- Possible control $\lambda_{ij}(t) \leq \Lambda_{ij}(t)$

Frustrating demand can improve the balance of objects



Users want to go to C. Almost nobody wants to go to A or B.

	Rate of trips (infinite capacities, infinite vehicles)
Generous policy	≈ 6 trips / time unit

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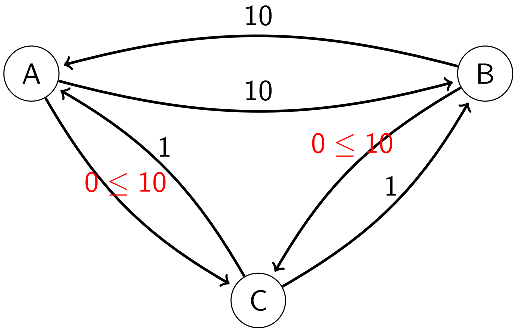
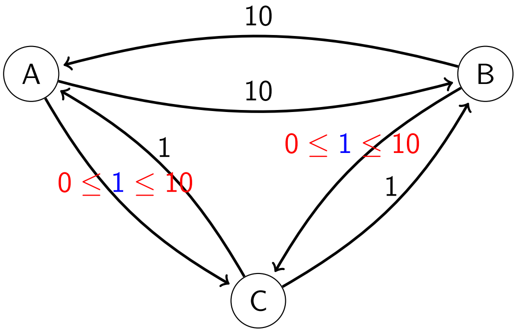


Users want to go to C. Almost nobody wants to go to A or B.

	Rate of trips (infinite capacities, infinite vehicles)
Generous policy	≈ 6 trips / time unit
Frustrating policy	20 trips / time unit

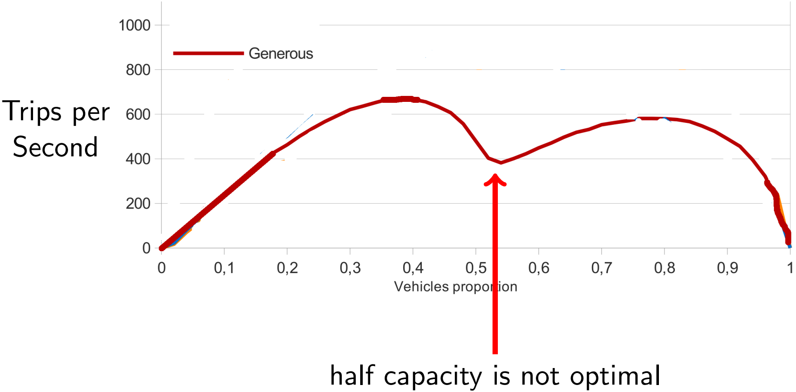
Frustrating demand can improve the balance of objects



Users want to go to *C*. Almost nobody wants to go to A or B.

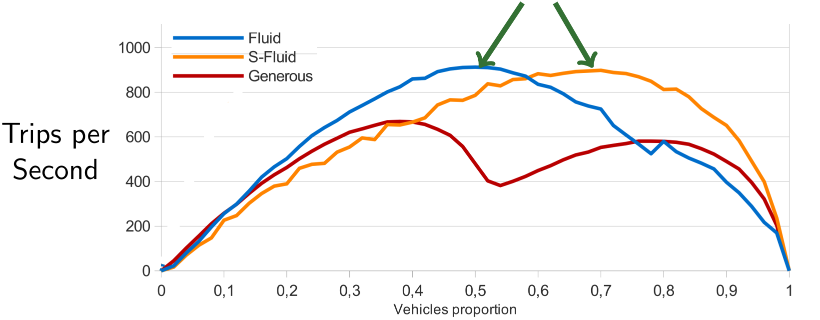
	Rate of trips (infinite capacities, infinite vehicles)
Generous policy	≈ 6 trips / time unit
Frustrating policy	20 trips / time unit
Optimal circulation	24 trips / time unit

Dynamic scenarios have been explored in [Waserhole/Jost 2012]



Dynamic scenarios have been explored in [Waserhole/Jost 2012]

Frustrating policies: +40% of successful trips



Outline

Take-away message

Mean-field approximation makes possible the study of large systems.

Take-away message

Mean-field approximation makes possible the study of large systems.

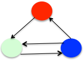
Performance of bike-sharing is poor, even for homogeneous scenarios ($1/C$ of problematic stations). Incentives or frustration can help.

If an ideal symmetric system works poorly, do not expect perfect service in a real system ;)

Limitations of the current approach and future work

This work is part of a bigger project quanticol

Visualization of traces and Influence of geometry.

Language and mathematical foundation.

Distributed control for electric distribution network.

To learn more: `http://mescal.imag.fr/membres/nicolas.gast/`

the slides are online

`www.quanticol.eu` –   

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