Asymptotic properties of object-sharing systems

Nicolas Gast [1]

YEQT VIII, November 2014, Eindhoven

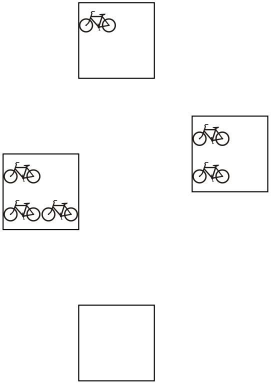
1. joint work with Christine Fricker (Inria) and Vincent Jost (CNRS)

Question : Who has already used a bike-sharing system ? What was your experience ?

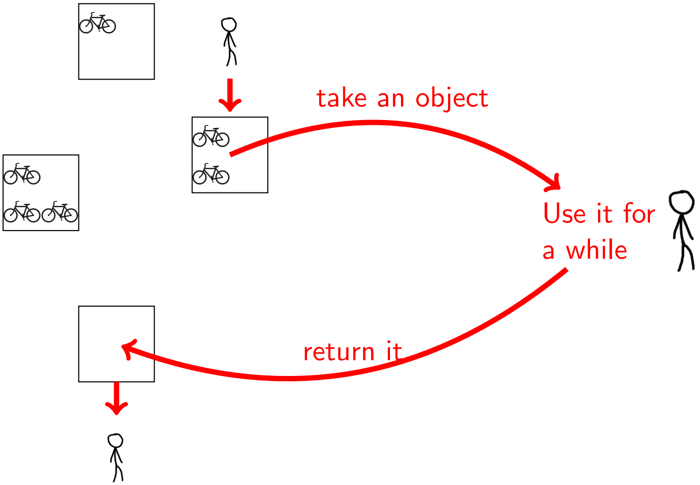
Question : Who has already used a bike-sharing system ? What was your experience ?

- Problems : lack of resources.

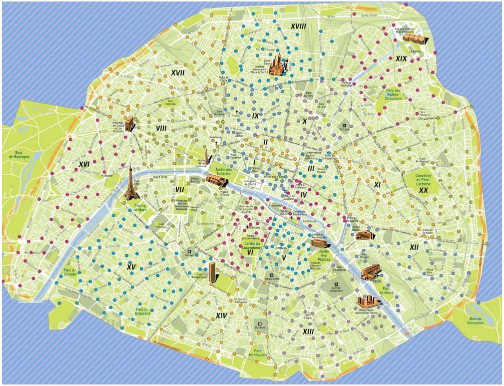
Object-sharing systems



Object-sharing systems



I will focus on large object-sharing systems

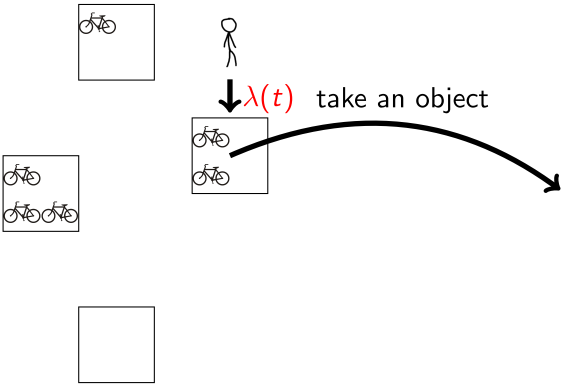


Map of Velib' stations in Paris (France).

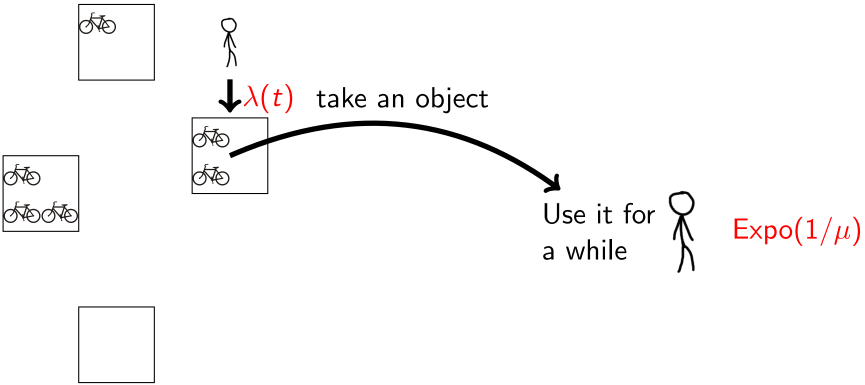
Example of Velib' :

- 20 000 bikes
- 1 200 stations.

Object-sharing system as closed-queuing networks

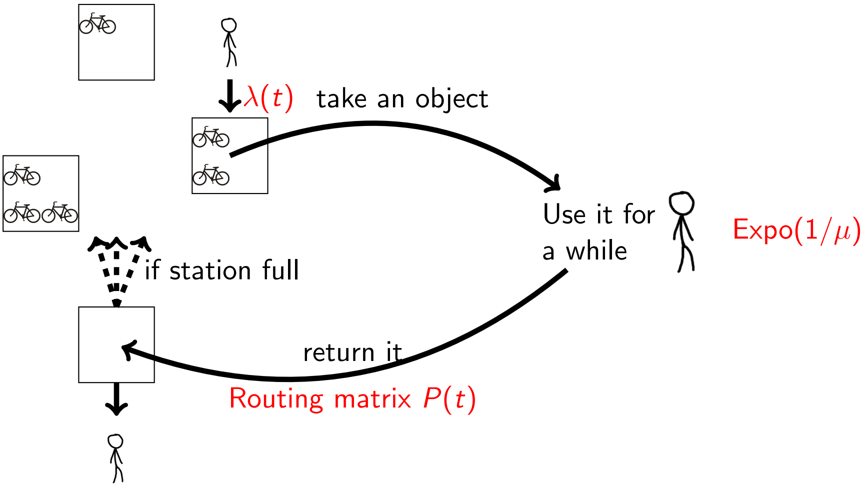


Object-sharing system as closed-queuing networks



Scaling : $N \to \infty$ stations, s objects per station.

Object-sharing system as closed-queuing networks



Scaling : $N \to \infty$ stations, s objects per station.

Main message

Theoretical results : When the system is large :

- if the stations have finite capacities, stations behave as independent M/M/1/K queues.
- if the stations have infinite capacities, there are problems of concentration.

Practical considerations :

- Performance is poor, even for a symmetric city (but simple incentives like a two-choice rule can help a lot).
- Frustrating users can help :
  - It is better to have stations of finite capacities.
  - Frustrating some users can improve the overall usage.
  - We show that the optimal fleet size is not

Outline

The homogeneous model

- All stations are identical.

Motivation :

- Impact of random choices
- Close-form results
- "Best-case analysis"

"Theorem"

Asymptotically, stations are independent and behaves as a $M/M/1/K$.

Distribution of x_i, the fraction of station with i bikes

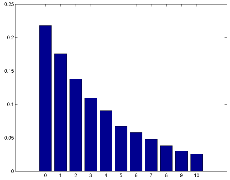
Theorem

There exists ρ, such that in steady state, as N goes to infinity :

$$x_i \propto \rho^i.$$

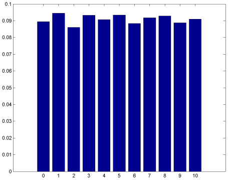
$\rho \leq 1$ *iff* $s \leq \frac{C}{2} + \frac{\lambda}{\mu}$ *where s be the average number of bikes per stations.*

$$s < \frac{C}{2} + \frac{\lambda}{\mu}$$



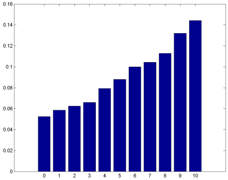
$$\rho < 1$$

$$s = \frac{C}{2} + \frac{\lambda}{\mu}$$



$$\rho = 1$$

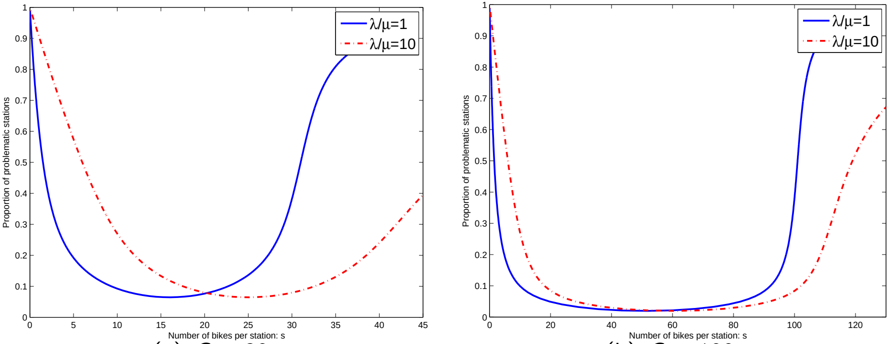
$$s > \frac{C}{2} + \frac{\lambda}{\mu}$$



$$\rho < 1$$

Consequences : optimal performance for $s \approx C/2$

y-axis : Prop. of problematic stations. x-axis : number of bikes/station s.



(a) $C = 30$. (b) $C = 100$.

Fraction of problematic stations (=empty+full) minimal for $s = \lambda/\mu + C/2$

- Prop. of problematic stations is at least $2/(C+1)$ (6.5% for $C = 30$)

Improvement by dynamic pricing : “two choices” rule

- Users can observe the occupation of stations.
- Users choose the least loaded among 2 stations close to destination to return the bike (ex : force by pricing)

Improvement by dynamic pricing : “two choices” rule

- Users can observe the occupation of stations.
- Users choose the least loaded among 2 stations close to destination to return the bike (ex : force by pricing)

Paradigm known as “*the power of two choices*” :

- Comes from balls and bills [Azar et al. 94]
- Drastic improvement of service time in server farm [Vvedenskaya 96, Mitzenmacher 96]

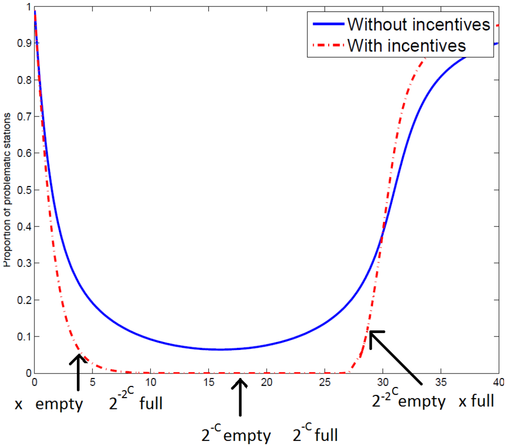
Question : what is the effect on bike-sharing systems ?
Characteristics :

1. Finite capacity of stations.
2. Strong geometry : choice among neighbors.

Two choices – finite capacity but no geometry

With no geometry, we can solve in close-form.

- Proof uses mean field argument.

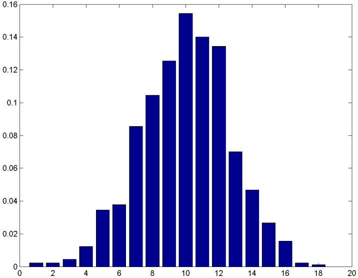


Choosing two stations at random, improves perf. from $1/C$ to $\sqrt{C}2^{-C/2}$

Two choices – taking geometry into account

Mean field do not apply (geometry) :(.

- Existing results for balls and bins (see [Kenthapadi et al. 06])
- Only numerical results exists for server farms (ex : [Mitzenmacher 96])



We rely on simulation

Occupancy of stations
x-axis = occupation of station.
y-axis : proportion of stations.

Recall : with no incentives, the distribution would be uniform.

Empirically :

- with geometry 2D : proportion of problematic stations is $\approx \sqrt{C}2^{-C/2}$. (recall : with no-geometry : 2^{-C}, with no incentive : $1/C$).

Outline

Detailed study of the homogeneous case

Adding some heterogeneity

Improvement by frustrating some demand

Conclusion and future work

We assume that as N goes to infinity, the parameters (λ_i, p_i) of the station have a limiting distribution.

We assume that as N goes to infinity, the parameters (λ_i, p_i) of the station have a limiting distribution.

"Theorem"

When the stations have finite capacities, a station behaves as a M/M/1/K.

Finite capacities regime

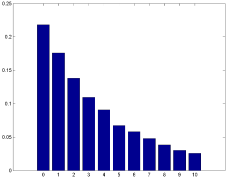
Theorem (Propagation of chaos-like result)

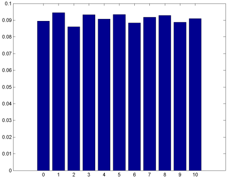
There exists a function $\rho(p)$ such that for all k, if stations $1, \dots k$ have parameter $p_1, \dots p_k$, then, as N goes to infinity :

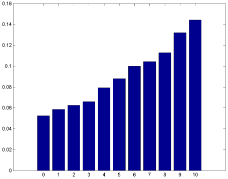
$$P(\#\{\text{bikes in stations j}\} = i_j \text{ for } j = 1..k) \propto \prod_{j=1}^{k} \rho(p_j)^{i_j}$$

Depending on popularity, stations have different behaviors :

Popular start $\rightarrow$ Popular destination

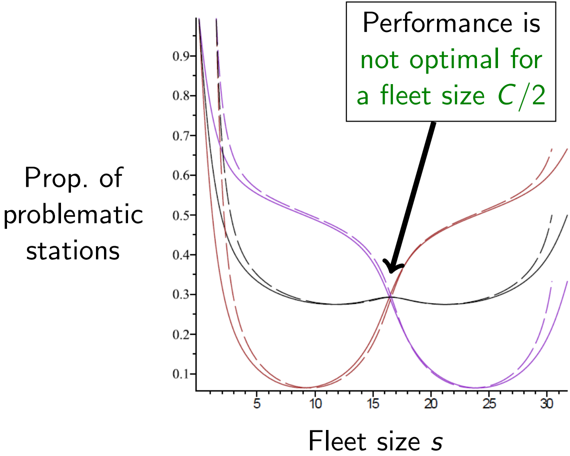






Finite-capacity : numerical example

Two types of stations : popular and non-popular for arrivals : $\lambda_1/\lambda_2 = 2$.



Infinite capacities can worsen the situation

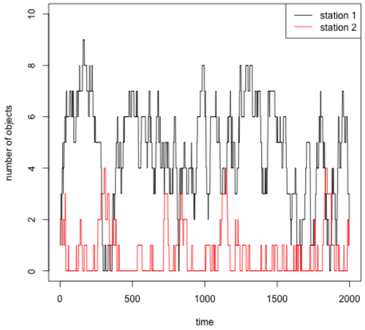
Infinite capacities can worsen the situation

Theorem (Malyshev-Yakovlev 96)

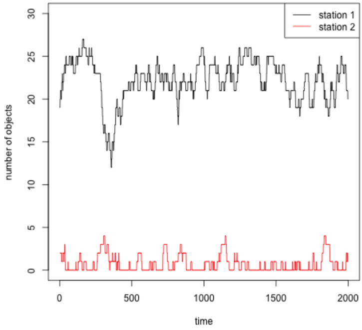
When the stations have infinite capacity, then there exists s_c :

- *if $s < s_c$, a station behaves as a M/M/1/K.*
- *if $s > s_c$, bikes will accumulate in a few stations.*

Example with $\mu = 1$, $p = (2, 1, 1, 1, 1, 1, 1, 1, 1)/10$:



$s = 1 < s_c$



$s = 3 > s_c$

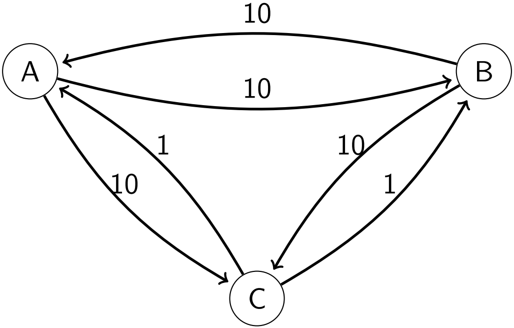
Outline

Having finite capacities prevent saturation of the demand. What if we could frustrate some demand ?

Model : we have a trip demand $\Lambda_{ij}(t)$ and an accepted demand $\lambda_{ij}(t)$.

- Generous policy : $\lambda_{ij}(t) := \Lambda_{ij(t)}$
- Possible control $\lambda_{ij}(t) \leq \Lambda_{ij}(t)$

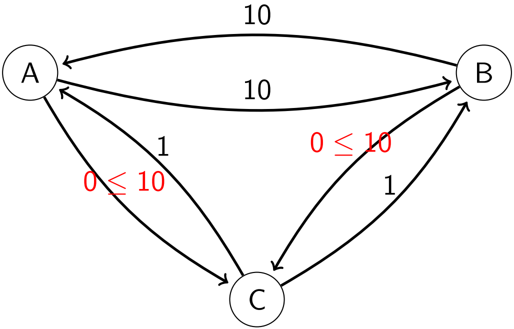
Frustrating demand can improve the balance of objects



Users want to go to C.
Almost nobody wants to go to A or B.

	Rate of trips (infinite capacities, infinite vehicles)
Generous policy	≈ 6 trips / time unit

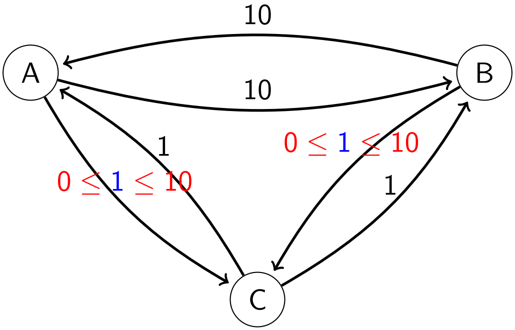
Frustrating demand can improve the balance of objects



Users want to go to C. Almost nobody wants to go to A or B.

	Rate of trips (infinite capacities, infinite vehicles)
Generous policy	≈ 6 trips / time unit
Frustrating policy	20 trips / time unit

Frustrating demand can improve the balance of objects

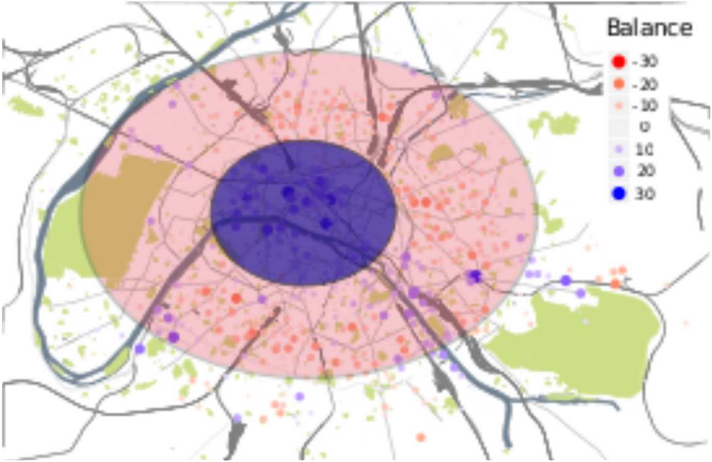


Users want to go to C. Almost nobody wants to go to A or B.

	Rate of trips (infinite capacities, infinite vehicles)
Generous policy	≈ 6 trips / time unit
Frustrating policy	20 trips / time unit
Optimal circulation	24 trips / time unit

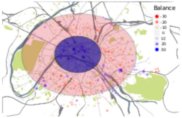
We can explore dynamic scenarios [Waserhole/Jost 2012]

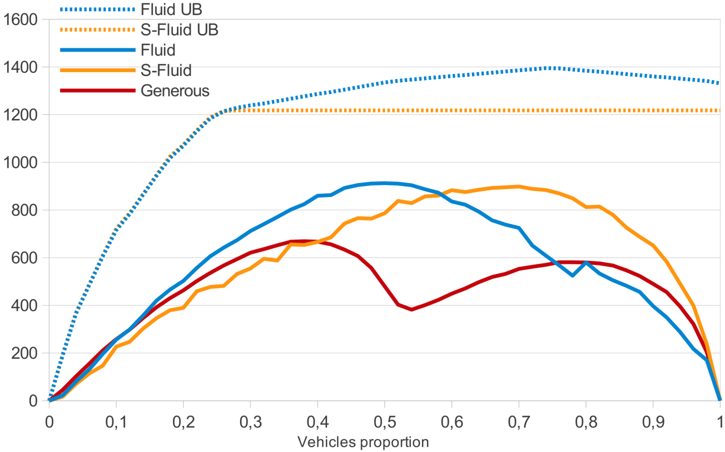
Tides in Paris



Static time-varying frustration of user can improve the situation

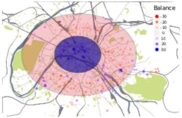
Trips per Second

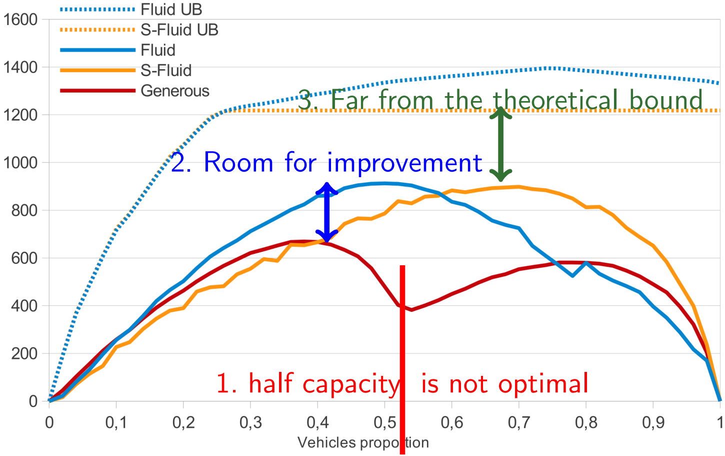




Static time-varying frustration of user can improve the situation

Trips per Second





Outline

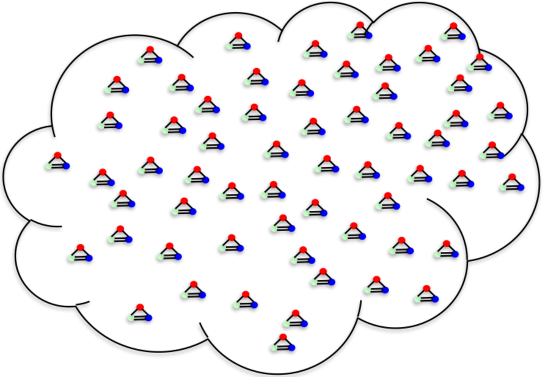
Detailed study of the homogeneous case

Adding some heterogeneity

Improvement by frustrating some demand

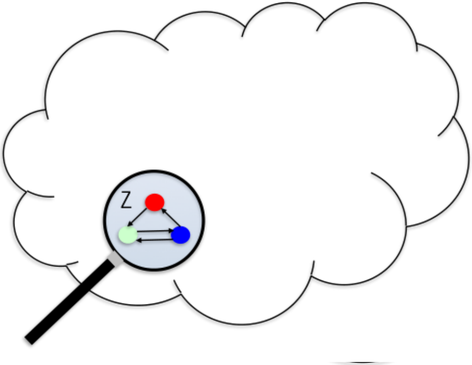
Conclusion and future work

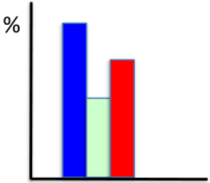
Methodological comments : Asymptotic based on mean-field approximation



- Basic models are reversible.
  - Saddle-points methods can also be used.

Methodological comments : Asymptotic based on mean-field approximation





- Basic models are reversible.
  - Saddle-points methods can also be used.

Take-away message

Asymptotic results for a large class of object-sharing network.

- Performance poor, even for symmetric : $1/C$ problematic stations.
- Simple incentives can help a lot : 2^{-C}.
- Frustrating some users improves overall usage.

Take-away message

Asymptotic results for a large class of object-sharing network.

- Performance poor, even for symmetric : $1/C$ problematic stations.
- Simple incentives can help a lot : 2^{-C}.
- Frustrating some users improves overall usage.

Remarks and future work :

- Metrics are not easy to define.
- Visualization of traces and Influence of geometry ?
- Analyze transient and steady-state behavior.

Take-away message

Asymptotic results for a large class of object-sharing network.

- Performance poor, even for symmetric : $1/C$ problematic stations.
- Simple incentives can help a lot : 2^{-C}.
- Frustrating some users improves overall usage.

Remarks and future work :

- Metrics are not easy to define.
- Visualization of traces and Influence of geometry ?
- Analyze transient and steady-state behavior.

> If an ideal symmetric system works poorly, do not expect perfect service in a real system ;)

References

- C Fricker, N Gast. *Incentives and redistribution in homogeneous bike-sharing systems with stations of finite capacity*. EURO Journal on Transportation and Logistics. 2014.
- C Fricker, N Gast, H Mohamed. *Mean field analysis for inhomogeneous bike sharing systems* DMTCS Proceedings, 2012.
- V.A. Malyshev and A. V. Yakovlev. *Condensation in large closed Jackson networks.* Ann. Appl. Proba. 1996.
- A. Waserhole, V. Jost *Vehicle Sharing System Pricing Regulation : A Fluid Approximation*. 2012