Perfect Simulation of Stochastic Automata Networks

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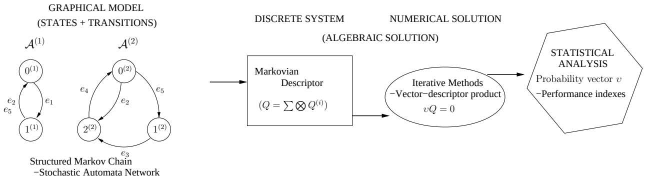
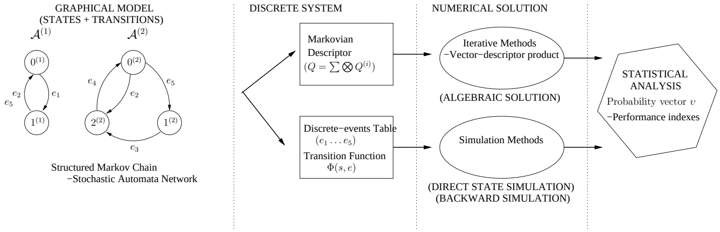
Analysis of complex discrete systems

Markovian modeling

- Structured representations (e.g. SAN, GSPN, PEPAnets)
  - * model complex dynamics (synchronizations, functions)
  - * multidimensional product state space $\mathcal{X}$
- Aim: stationary or transient distribution (for statistical analysis)
- Constraints: deal with state space explosion problem



Analysis of complex discrete systems



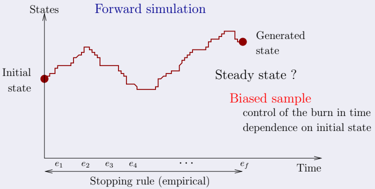
Other Markovian modeling view

- discrete-event simulation to estimate steady-state distribution on long run trajectories
  * establishing transition functions
  * table of events - uniformization

Analysis of complex discrete systems

Classical Simulation Techniques

- Advantage: storage of current/initial state
- Problems:
  - number of iterations needed to steady-state estimation
  - biased samples



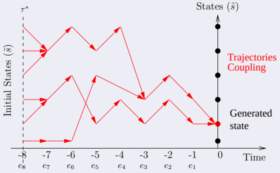
- Complexity: related to the warm-up period

Analysis of complex discrete systems

Backward Simulation Techniques [Propp and Wilson 1996]

- Advantages: samples from the steady-state distribution
  * avoid warm-up period, coupling time τ
- Constraints: $\mathcal{X}$ trajectories in parallel in the worst case

Backward Simulation



Perfect Simulation

- Complexity: mainly related to the cardinality of $\mathcal{X}$ and τ

Adaptation to Structured models

Model Parameters

- set of uniformized events $E = \{e_1, .., e_p\}$
- global states are tuples of local states $\tilde{s} = (s_1, \ldots, s_K)$
- transition function: $\Phi(\tilde{s}, e_i) = \tilde{r}$

* each $\tilde{s} \in \mathcal{X}$ has a set of enabled events and its firing conditions and consequences

Constraints

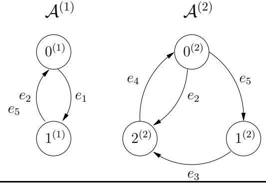
- Well-formed SAN models needed

* exploring the subset $\mathcal{X}^{\mathcal{R}}$ (Reachable state space)

- State space explosion still a problem

Analysis of complex discrete systems

GRAPHICAL MODEL
(STATES + TRANSITIONS)



$\tilde{s} \in \mathcal{X}^{\mathcal{R}}$	$\tilde{r} = \Phi(\tilde{s}, e_p), e_p \in \xi$				
	$\Phi(\tilde{s}, e_1)$	$\Phi(\tilde{s}, e_2)$	$\Phi(\tilde{s}, e_3)$	$\Phi(\tilde{s}, e_4)$	$\Phi(\tilde{s}, e_5)$
{0;0}	**{1;0}**	{0;0}	{0;0}	{0;0}	{0;0}
{0;1}	**{1;1}**	{0;1}	**{0;2}**	{0;1}	{0;1}
{0;2}	**{1;2}**	{0;2}	{0;2}	**{0;0}**	{0;2}
{1;0}	{1;0}	**{0,2}**	{1;0}	{1;0}	**{0;1}**
{1;1}	{1;1}	{1;1}	**{1;2}**	{1;1}	{1;1}
{1;2}	{1;2}	{1;2}	{1;2}	**{1;0}**	{1;2}

$e_p \in \xi$	Rates	Uniformized Rates
e_1	λ_1	$\lambda_1/(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)$
e_2	λ_2	$\lambda_2/(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)$
e_3	λ_3	$\lambda_3/(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)$
e_4	λ_4	$\lambda_4/(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)$
e_5	λ_5	$\lambda_5/(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)$

SAN Backward coupling simulation

for all $\tilde{s} \in \mathcal{X}^{\mathcal{R}}$ **do**
    $\omega(\tilde{s}) \leftarrow \tilde{s}$ { choice of the initial value of vector ω}
end for
repeat
    e $\leftarrow$ Generate-event( ) { generation of e according the distribution $(\frac{\lambda_1}{\Lambda} \ldots \frac{\lambda_{\mathcal{E}}}{\Lambda})$}
    $\tilde{\omega} \leftarrow \omega$ { copying vector ω to $\tilde{\omega}$}
    for all $\tilde{s} \in \mathcal{X}^{\mathcal{R}}$ **do**
        { computing $\omega(\tilde{s})$ at time 0 of trajectory issued from $\tilde{s}$ at time $-\tau^*$}
        $\omega(\tilde{s}) \leftarrow \tilde{\omega}(\Phi(\tilde{s}, e))$
    end for
until All $\omega(\tilde{s})$ are equal
Return $\omega(\tilde{s})$

Partially Ordered State Spaces

Monotonicity property

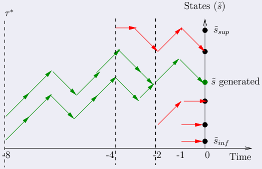
- $e_p \in \mathcal{E}$ is monotone if it preserves the partial order

$$\forall (x, y) \in \mathcal{X} \quad x \leq y \implies \Phi(x, e) \leq \Phi(y, e)$$

Monotone Backward Simulation [Propp and Wilson 1996]

- Advantages: samples from the steady-state distribution
- Complexity: related to τ, two trajectories (instead of $\mathcal{X}$)

* If all events in the model are considered monotone



New solutions for huge SAN models

Monotonicity and Perfect Simulation Idea

- Monotonicity property for SAN related to the analysis of structural conditions
  * component-wise state space formation

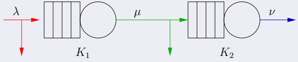
Families of SAN models

- SAN models with a natural partial order (canonical)
  * e.g. derived from Queueing systems models [Vincent 2005]
- SAN models with a given component-wise partial order (non-lattice)

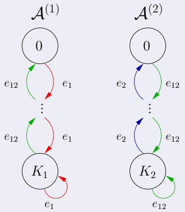
Partially Ordered State Spaces

Canonical component-wise ordering

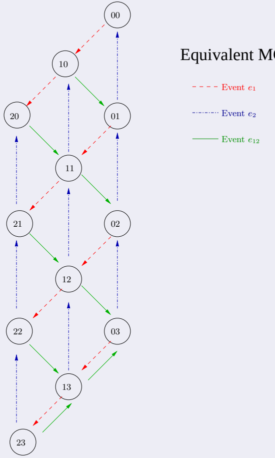
Queueing Network Model



Equivalent SAN Model



Type	Event	Rate
loc	e_1	λ
syn	e_{12}	μ
loc	e_2	ν



Equivalent MC

Partially Ordered State Spaces

Non-lattice component-wise ordering

- Find a partial order of $\mathcal{X}$ demands a high c.c.
- Possible to find extremal global states in the underlying chain
  * $|\mathcal{X}^{\mathcal{M}}|$ states: more than two extremal states
- Complexity: related to τ, but also $|\mathcal{X}^{\mathcal{M}}|$

Extremal states

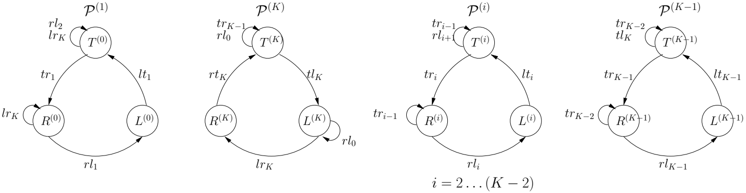
- Component-wise formation has ordered state indexes
  * consider an initial state composing $\mathcal{X}^{\mathcal{M}}$
  * add to $\mathcal{X}^{\mathcal{M}}$ the states without transitions to states with greater indexes

Partially Ordered State Spaces

Non-lattice component-wise ordering

- Resource sharing model with reservation (Dining Philosophers)

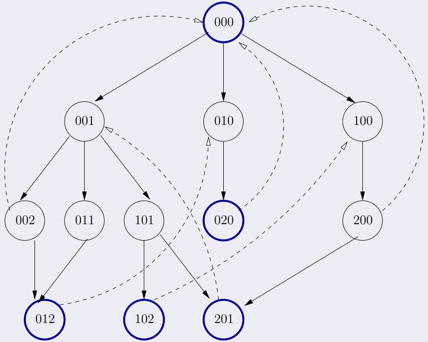
Type	Event	Rate
loc	lt_i	μ
syn	tr_i	λ
syn	rl_i	λ
loc	rt_K	μ
syn	tl_K	λ
syn	lr_K	λ



Partially Ordered State Spaces

Non-lattice component-wise ordering

- e.g. three philosophers with resources reservation, graphical model of the underlying transition chain, extremal states identification



SAN Monotone Backward coupling simulation

$n = 1$
$E[1] \leftarrow$ Generate-event( ) $\{E$ stores the backward sequence of events$\}$
repeat
    $n \leftarrow 2n$ $\{$doubling scheme$\}$
    for each $\tilde{s} \in \mathcal{X}^{\mathcal{M}}$ **do**
        $\omega(\tilde{s}) \leftarrow \tilde{s}$ $\{$initial states at time $-n\}$
    end for
    for $i = n$ downto $(\frac{n}{2} + 1)$ **do**
        $E[i] \leftarrow$ Generate-event( )
    end for
    for $i = n$ downto 1 **do**
        for each $\tilde{s} \in \mathcal{X}^{\mathcal{M}}$ **do**
            $\omega(\tilde{s}) \leftarrow \Phi(\omega(\tilde{s}), E[i])$
        end for
    end for
until All $\omega(\tilde{s})$ are equal
Return $\omega(\tilde{s})$

SAN Perfect Simulation

Resource sharing model with reservation- K Philosophers

K	$\mathcal{X}$	$\mathcal{X}^{\mathcal{R}}$	$\mathcal{X}^{\mathcal{M}}$	*PEPS** (iteration)	*Perfect PEPS* (sample)*
8	6,561	985	43	0.003185 sec.	0.032354 sec.
10	59,049	5,741	111	0.038100 sec.	0.111365 sec.
12	531,441	33,461	289	0.551290 sec.	0.689674 sec.
14	4,782,969	195,025	755	5.712210 sec.	2.686925 sec.
16	43,046,721	1,136,689	1,975	68.704325 sec.	15.793501 sec.
18	**387,420,489**	6,625,109	**5,169**	—-	83.287321 sec.

Numerical results

- 3.2 GHz Intel Xeon processor under Linux, 1 GByte RAM
- times: for one iteration on *PEPS* and for one sample generation on *Perfect PEPS*
- Remarks: $\mathcal{X}$ contraction in $|\mathcal{X}^{\mathcal{M}}|$

* $\mathcal{X}$ limitation 6×10^7 on *PEPS* overcame

Analysis of complex discrete systems

Using model structural information

- model complexity reduction to achieve the numerical solution
- increase of solution bounds overcoming memory constraints
- perfect simulation and monotonicity applied to a structured formalism as SAN

Future Works

- comparative study of convergence control
- deeper understanting of Φ properties
- evaluation or bounds on the coupling time
- strategy adaptation to other structured formalisms

Thank you for your attention!