Perfect Sampling of Load Sharing Policies in Large Scale Distributed Systems

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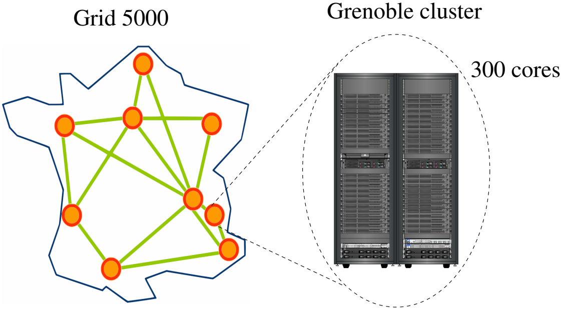
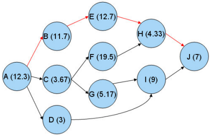
Large scale computing



Computation model



Load sharing middleware

- Distributed control algorithm
- migration of tasks between nodes

Practical needs

Load sharing

A controler (local) checks the utilization of the node and decides to share some work with other nodes.

- When ? $\longrightarrow$ Control triggering
- Who ? $\longrightarrow$ Paradigm Push, Pull
- Decide ? $\longrightarrow$ Local state condition
- How many ? $\longrightarrow$ Amount of work to be transfered
- Where ? $\longrightarrow$ Selection among targets (probing scheme)

Users requirements

- Maximize the utilization of resources (number of active nodes)
- Minimize the network utilization (number of transfers, costly transfers)

$\Rightarrow$ Need of a tool to evaluate the load sharing policies

Performance evaluation of Load sharing systems

Methodology

1. Quantification of the system : **steady-state evaluation**
2. Comparison of systems, paradigms, policies
3. Tuning of system parameters

Numerical approaches

- Markovian modelling and direct numerical solving
- Matrix geometric solution [ELZ86, MTS90]
- Mean field [Mit98, BGY98]
- Simulation [KH02, DKL98]

Key challenge : very large state space (C^K)

Steady-state simulation of Markov models

Generate typical state, i.e. distributed according to the steady-state

forward simulation

Run from an initial state and stop after a sufficiently long period
$\Rightarrow$ Choice of a stopping rule

Perfect simulation [PW96]

Coupling from the past scheme

- Exact stopping criteria
- Unbiased sampling
- **Monotonicity implies simulation efficiency**

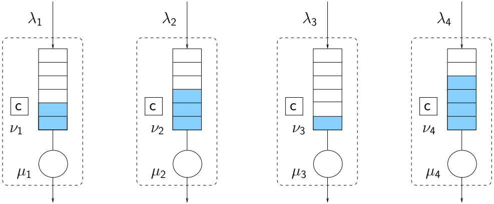
Are the load sharing systems monotone so that we can simulate them efficiently ?

Outline

Outline

Load sharing model

Parallel independent queues



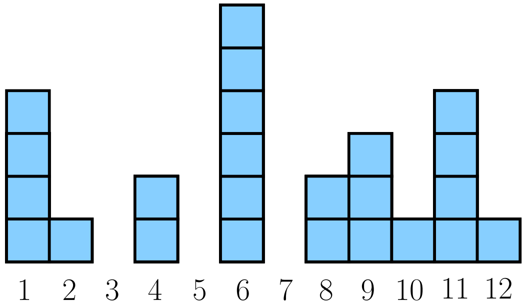
State space : number of tasks in each queue ; $\mathcal{X}_1 \times \cdots \times \mathcal{X}_K$
Dynamics : events driven by Poisson process (Poisson system [Bre99]) :

- Generation of a new task in a queue, with rate λ
- Task completion, with rate μ
- Control, with rate ν

Uniformization $\Rightarrow$ Stochastic Recurence Equation $X_{n+1} = \Phi(X_n, E_{n+1})$

Control event example

Push to the least loaded node among potential targets (Push to the Shortest Queue)



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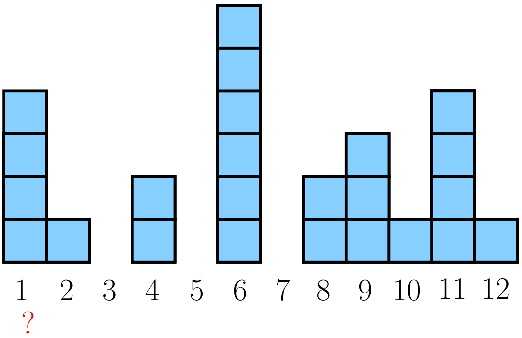

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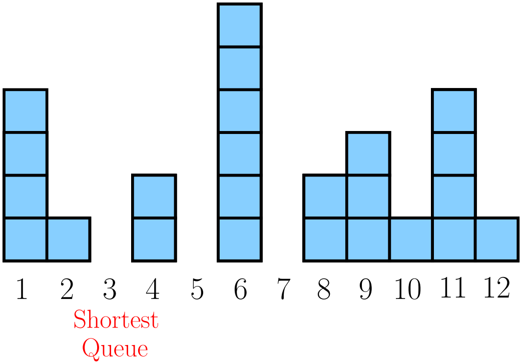

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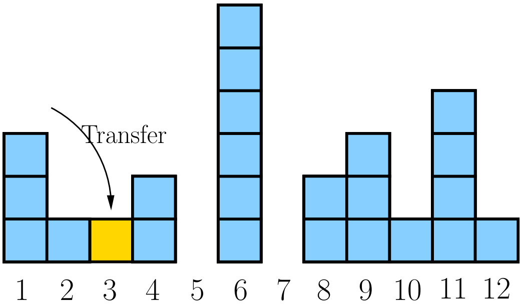
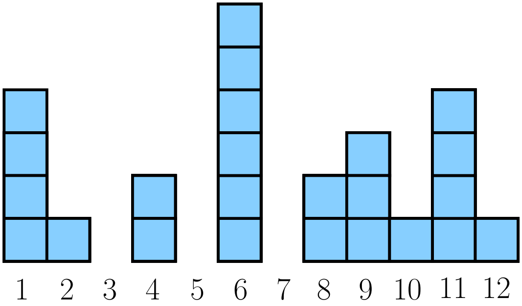


Corresponding index model

Push to the least loaded node among potential targets (Push to the Shortest Queue)



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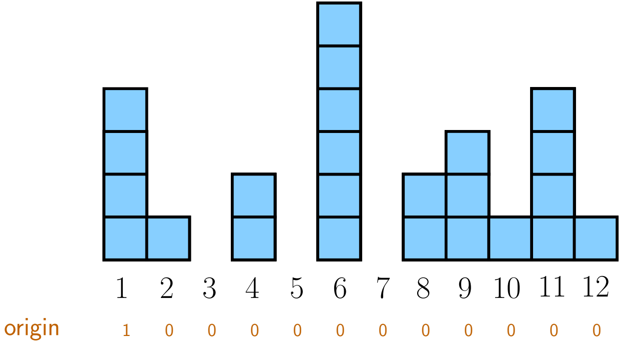

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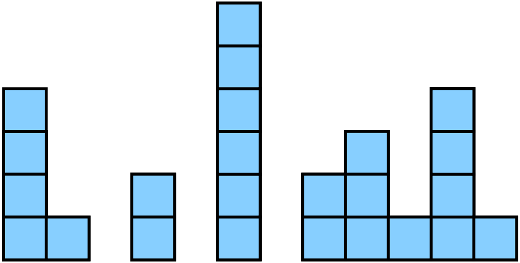
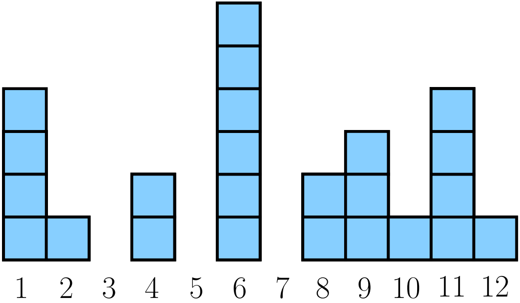


	1	2	3	4	5	6	7	8	9	10	11	12
origin	1	0	0	0	0	0	0	0	0	0	0	0
target	4	1	0	2	0	6	0	2	3	1	4	1

Corresponding index model

Push to the least loaded node among potential targets (Push to the Shortest Queue)



origin	1	0	0	0	0	0	0	0	0	0	0	0
	Max											
target	4	1	0	2	0	6	0	2	3	1	4	1

Corresponding index model

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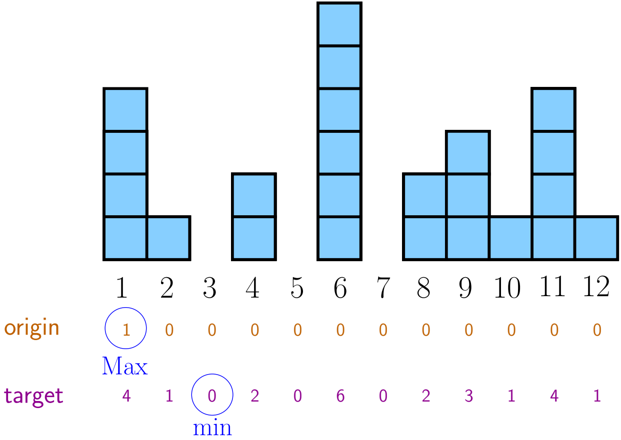

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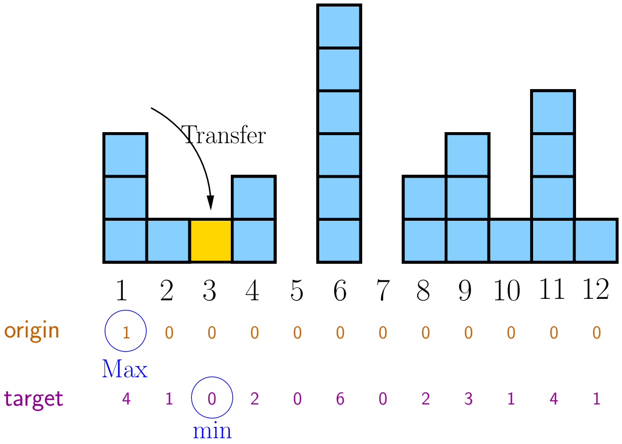
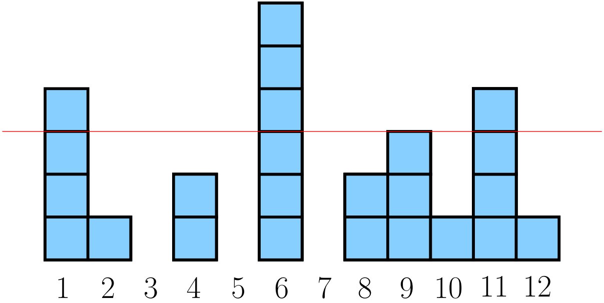


Control event example 2

PSQ Adding a threshold condition (overload condition) on the origin



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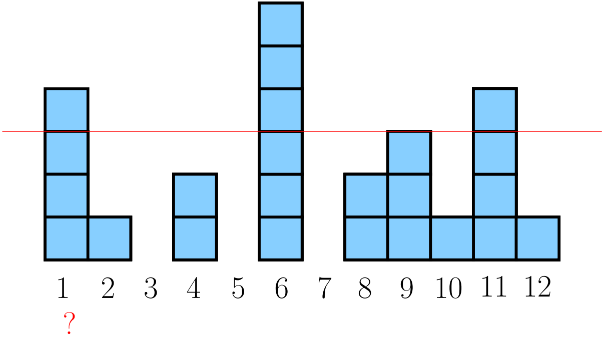

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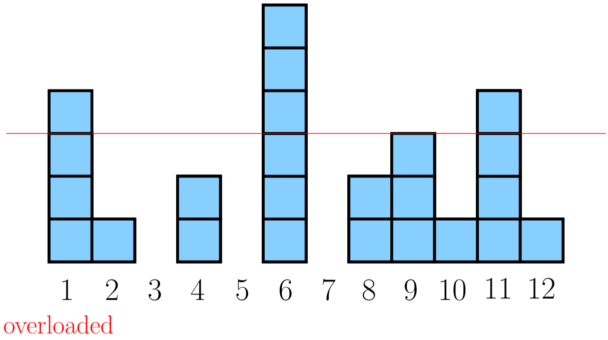

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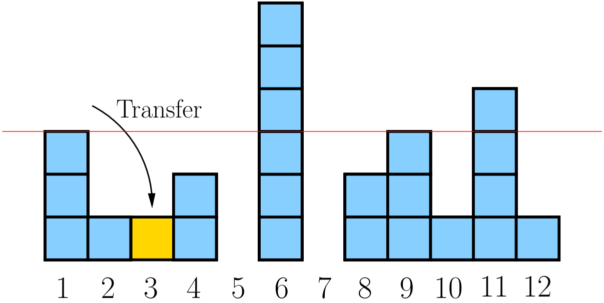

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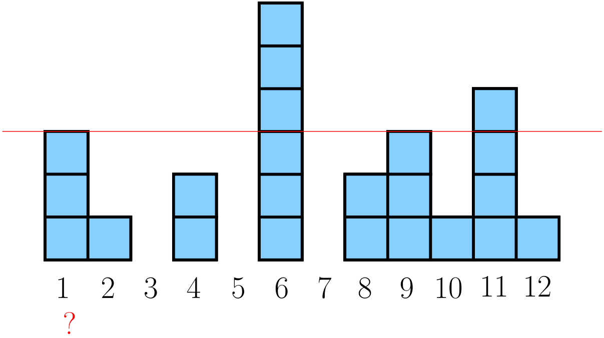

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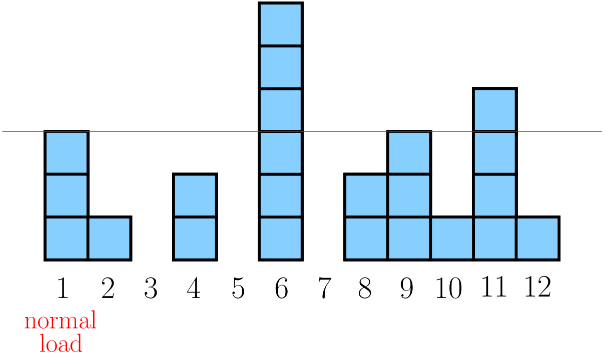

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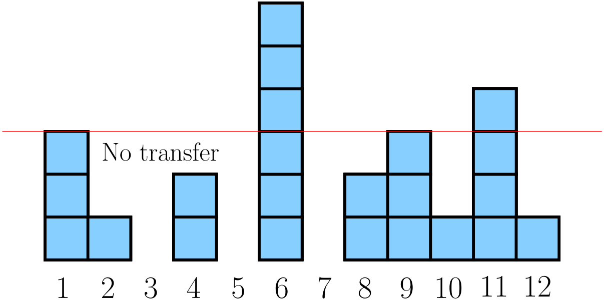
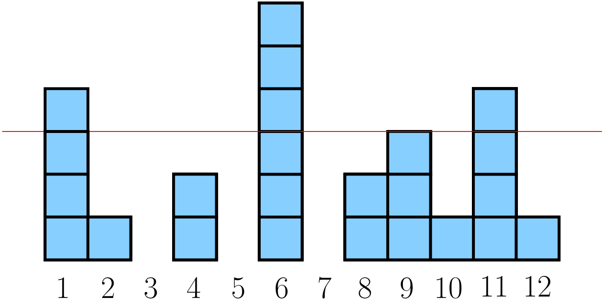


Corresponding index model

PSQ Adding a threshold condition (overload condition) on the origin



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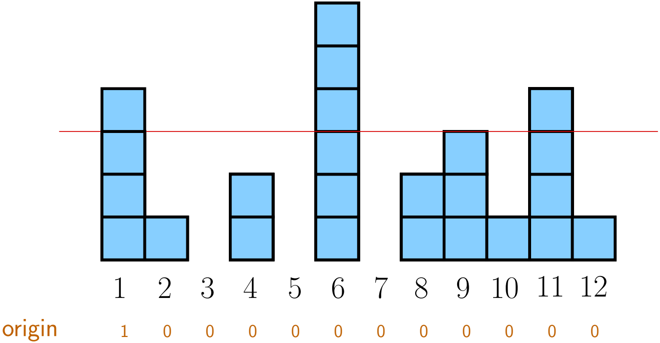

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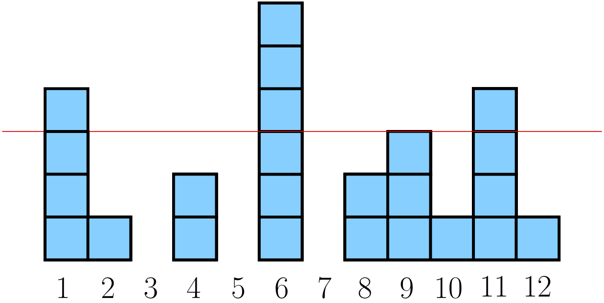

origin	1	0	0	0	0	0	0	0	0	0	0	0
target	4	1	0	2	0	6	0	2	3	1	4	1

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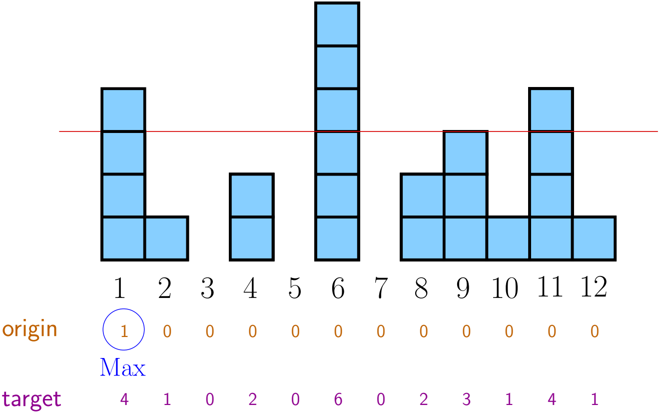

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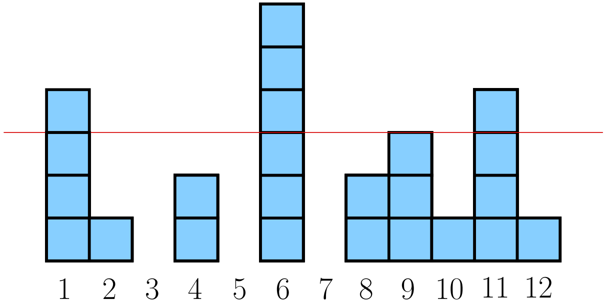
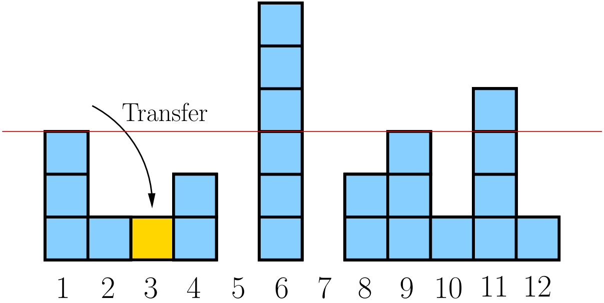


origin	1 (Max)	0	0	0	0	0	0	0	0	0	0	0
target	4	1	0 (min)	2	0	6	0	2	3	1	4	1

Corresponding index model

PSQ Adding a threshold condition (overload condition) on the origin



	1	2	3	4	5	6	7	8	9	10	11	12
origin	1 (Max)	0	0	0	0	0	0	0	0	0	0	0
target	4	1	0 (min)	2	0	6	0	2	3	1	4	1

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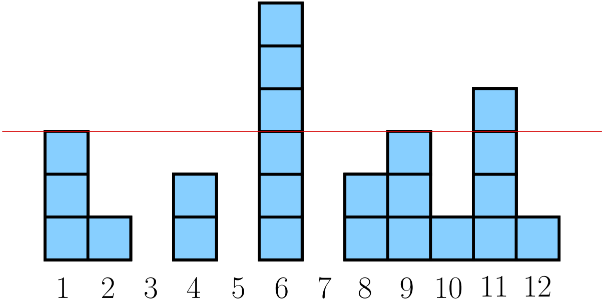

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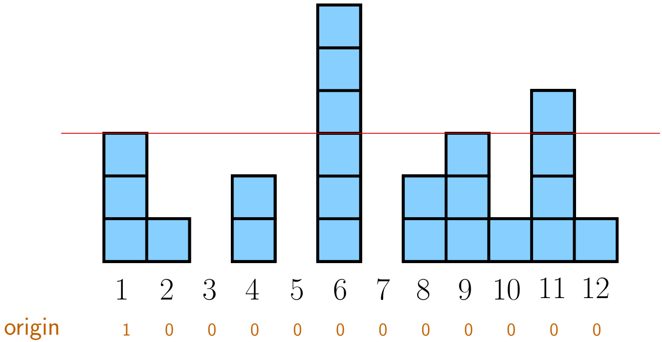

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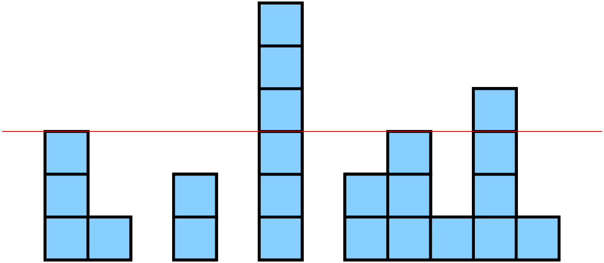

	1	2	3	4	5	6	7	8	9	10	11	12
origin	1	0	0	0	0	0	0	0	0	0	0	0
target	$-\infty$	1	0	2	0	6	0	2	3	1	4	1

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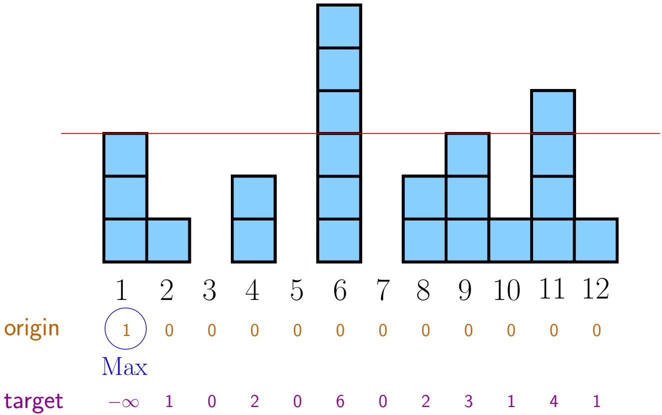

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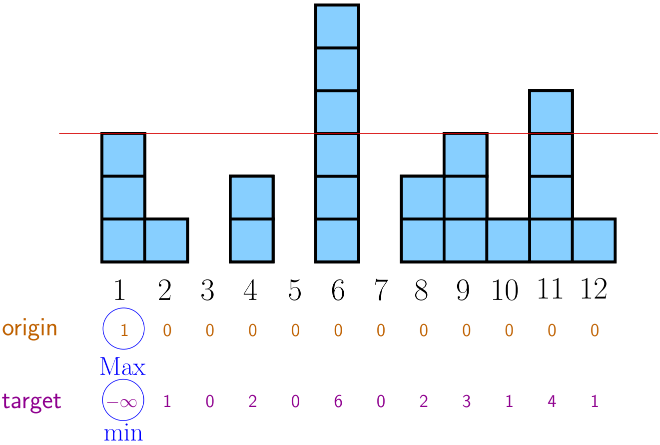

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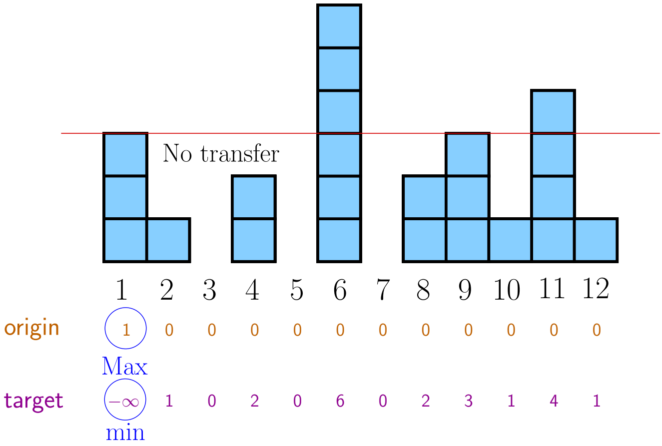
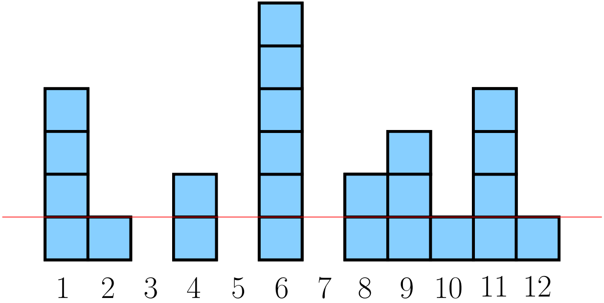


Control event example 3

Pull with probing according to a priority list



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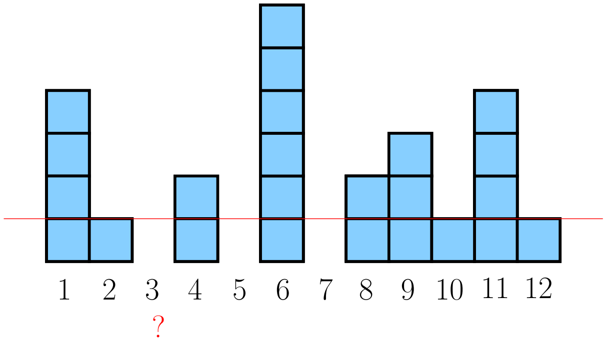

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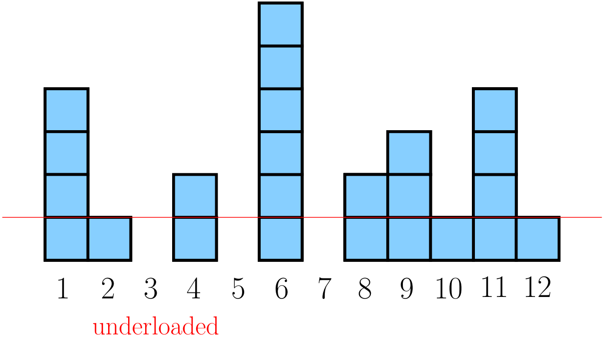

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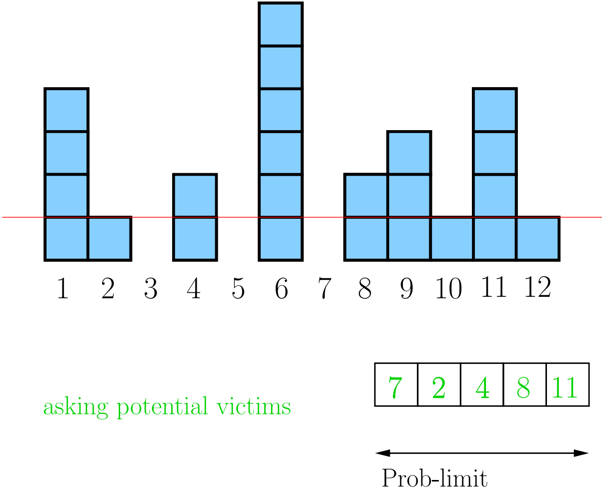

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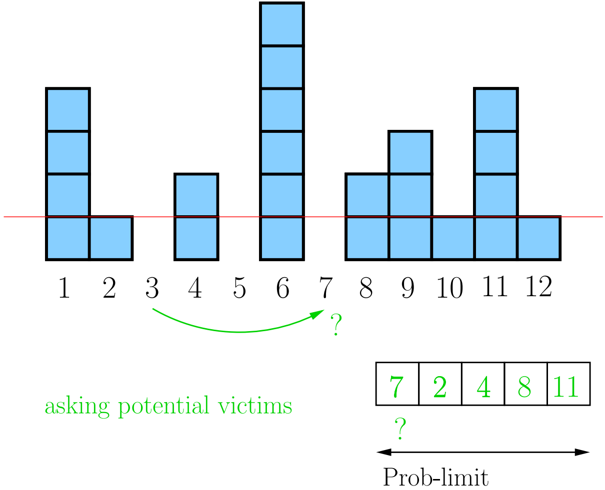

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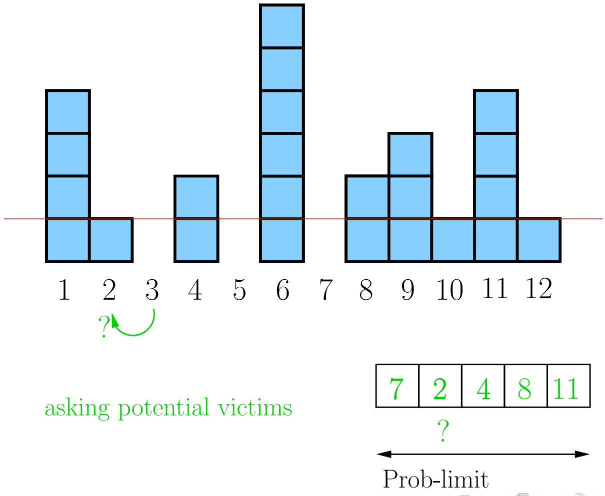

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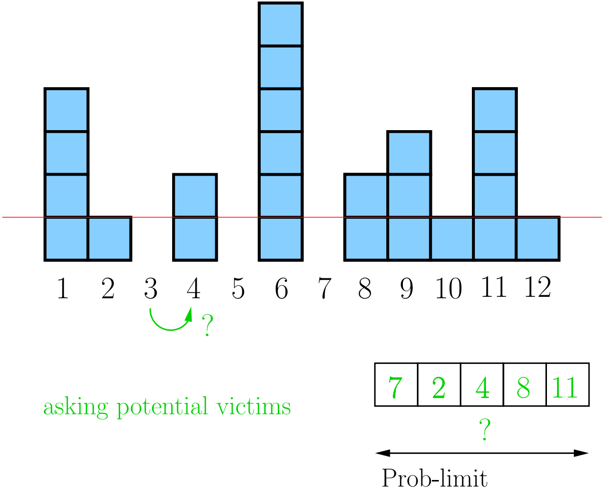

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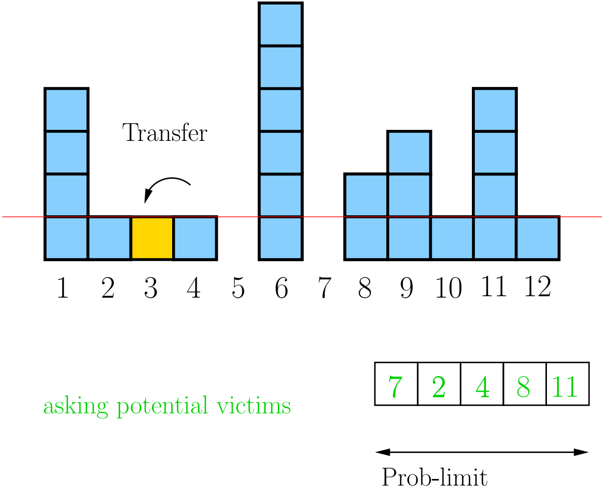
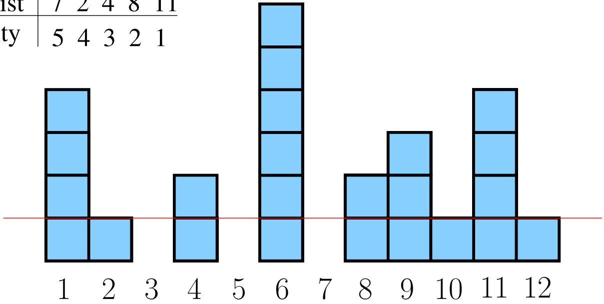


Corresponding index model

Pull with probing according to a priority list

List	7	2	4	8	11
Priority	5	4	3	2	1



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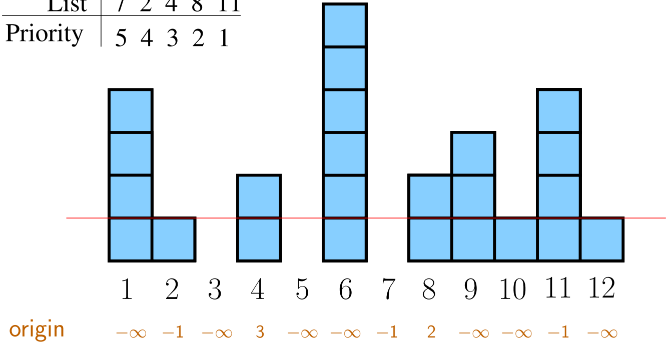

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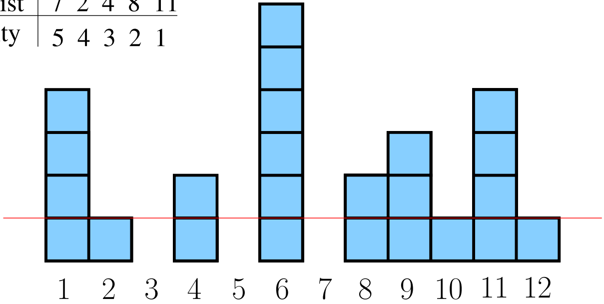
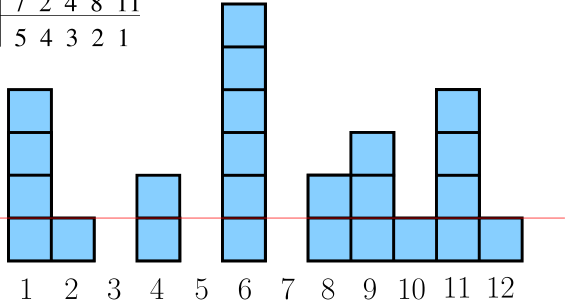


	1	2	3	4	5	6	7	8	9	10	11	12
origin	$-\infty$	-1	$-\infty$	3	$-\infty$	$-\infty$	-1	2	$-\infty$	$-\infty$	-1	$-\infty$
target	0	0	-1	0	0	0	0	0	0	0	0	0

Corresponding index model

Pull with probing according to a priority list

List	7	2	4	8	11
Priority	5	4	3	2	1



	1	2	3	4	5	6	7	8	9	10	11	12
origin	$-\infty$	-1	$-\infty$	3 (Max)	$-\infty$	$-\infty$	-1	2	$-\infty$	$-\infty$	-1	$-\infty$
target	0	0	-1	0	0	0	0	0	0	0	0	0

Corresponding index model

Pull with probing according to a priority list

List	7	2	4	8	11
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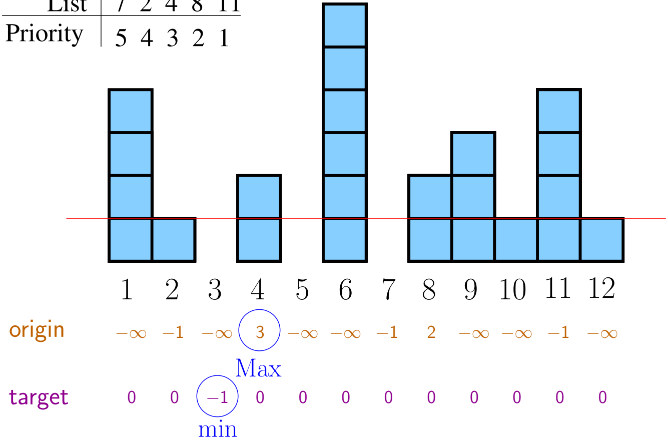

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List	7	2	4	8	11
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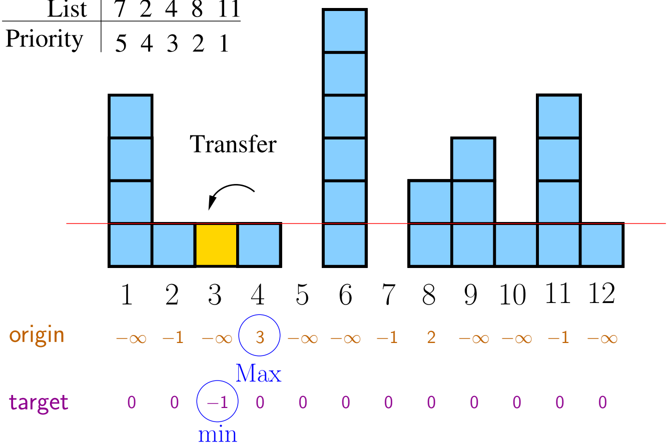
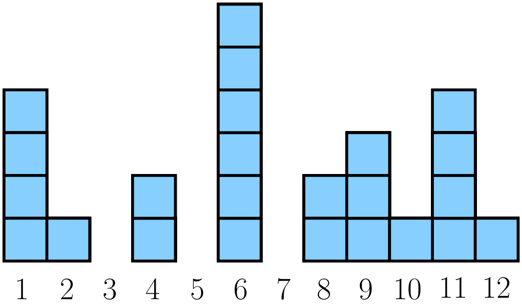


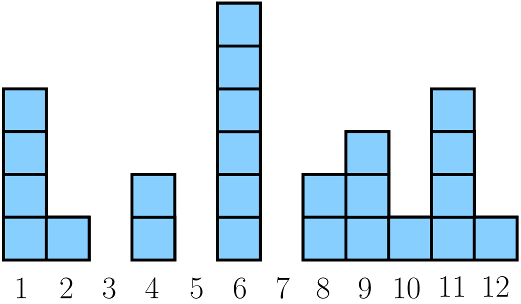
General index model

transfer from an origin (max) to a target (min)



General index model

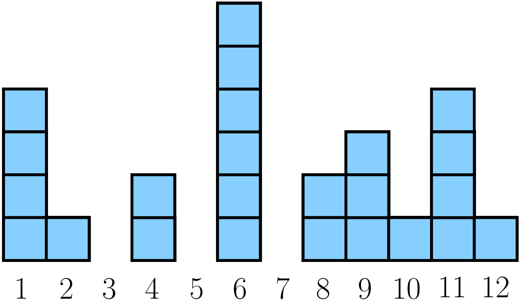
transfer from an origin (max) to a target (min)



origin $I_1(x^1)$ $I_2(x^2)$ $I_3(x^3)$ $I_4(x^4)$ $I_5(x^5)$ $I_6(x^6)$ $I_7(x^7)$ $I_8(x^8)$ $I_9(x^9)$ $I_{10}(x^{10})$ $I_{11}(x^{11})$ $I_{12}(x^{12})$

General index model

transfer from an origin (max) to a target (min)

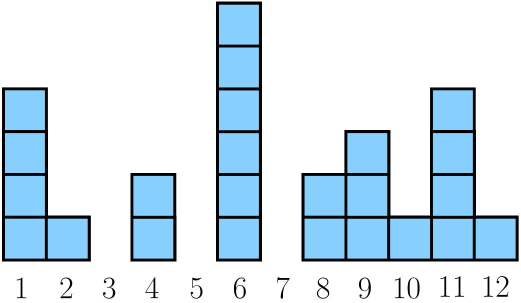


origin $I_1(x^1)$ $I_2(x^2)$ $I_3(x^3)$ $I_4(x^4)$ $I_5(x^5)$ $I_6(x^6)$ $I_7(x^7)$ $I_8(x^8)$ $I_9(x^9)$ $I_{10}(x^{10})$ $I_{11}(x^{11})$ $I_{12}(x^{12})$

target $I_1(x^1)$ $I_2(x^2)$ $I_3(x^3)$ $I_4(x^4)$ $I_5(x^5)$ $I_6(x^6)$ $I_7(x^7)$ $I_8(x^8)$ $I_9(x^9)$ $I_{10}(x^{10})$ $I_{11}(x^{11})$ $I_{12}(x^{12})$

General index model

transfer from an origin (max) to a target (min)



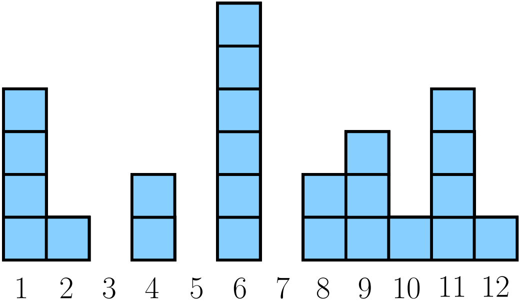
origin $I_1(x^1)$ $I_2(x^2)$ $I_3(x^3)$ $I_4(x^4)$ $I_5(x^5)$ $I_6(x^6)$ $I_7(x^7)$ $I_8(x^8)$ $I_9(x^9)$ $I_{10}(x^{10})$ $I_{11}(x^{11})$ $I_{12}(x^{12})$

Max

target $I_1(x^1)$ $I_2(x^2)$ $I_3(x^3)$ $I_4(x^4)$ $I_5(x^5)$ $I_6(x^6)$ $I_7(x^7)$ $I_8(x^8)$ $I_9(x^9)$ $I_{10}(x^{10})$ $I_{11}(x^{11})$ $I_{12}(x^{12})$

General index model

transfer from an origin (max) to a target (min)



origin $I_1(x^1)$ $I_2(x^2)$ $I_3(x^3)$ $I_4(x^4)$ $I_5(x^5)$ $I_6(x^6)$ $I_7(x^7)$ $I_8(x^8)$ $I_9(x^9)$ $I_{10}(x^{10})$ $I_{11}(x^{11})$ $I_{12}(x^{12})$

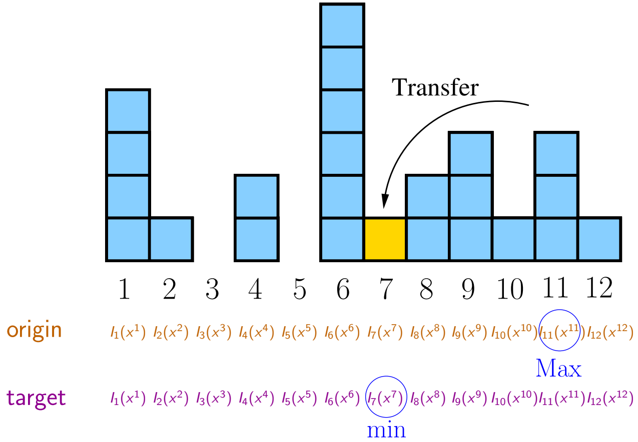
Max

target $I_1(x^1)$ $I_2(x^2)$ $I_3(x^3)$ $I_4(x^4)$ $I_5(x^5)$ $I_6(x^6)$ $I_7(x^7)$ $I_8(x^8)$ $I_9(x^9)$ $I_{10}(x^{10})$ $I_{11}(x^{11})$ $I_{12}(x^{12})$

min

General index model

transfer from an origin (max) to a target (min)



Formalisation

A control event c is defined by :

$$\Phi(x, c) = x - \delta_i + \delta_j$$

i is the **origin**
j is the **target**

Index function

A function $I_k(x^k)$ gives an index, i.e. a cost value to Q_k.

$$i = \mathbf{argmax}_{1 \leqslant k \leqslant K}(I_k^{c,o}(x^k))$$
$$j = \mathbf{argmin}_{1 \leqslant k \leqslant K}(I_k^{c,t}(x^k))$$

Outline

Monotonicity of index load sharing policies

Monotonicity

- $\preceq$ is the natural partial order on the multi-dimensional state space $\mathcal{X} = \mathcal{X}_1 \times \cdots \times \mathcal{X}_K$.

$$x \preceq y \Leftrightarrow x^i \leqslant y^i \quad \forall i$$

- An event e is monotone if it preserves the partial ordering $\preceq$ on $\mathcal{X}$

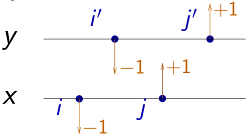
$$\forall (x, y) \in \mathcal{X} \quad x \preceq y \;\Rightarrow\; \Phi(x, e) \preceq \Phi(y, e)$$

Theorem

If all index functions $I_k^{c,o}(x^k)$ and $I_k^{c,t}(x^k)$ are monotone and increasing in function of x^k, then the event c is monotone

Proof

Let $x, y \in \mathcal{X}$ two states with $x \preceq y$,
c a control event, $\Phi(x,c) = x - \delta_i + \delta_j$, $\Phi(y,c) = y - \delta_{i'} + \delta_{j'}$.
Suppose that $i \neq i' \neq j \neq j'$.



Then,

$$I_j^{c,t}(x^j) < I_{j'}^{c,t}(x^{j'})$$ j is the argmin for x

$$I_{j'}^{c,t}(x^{j'}) \leqslant I_{j'}^{c,t}(y^{j'})$$ $I_{j'}^{c,t}$ increasing and $x^{j'} \leqslant y^{j'}$

$$I_{j'}^{c,t}(y^{j'}) < I_j^{c,t}(y^j)$$ j' is the argmin for y

$$I_j^{c,t}(x^j) < I_j^{c,t}(y^j)$$ by transitivity

$$x^j < y^j$$ $I_{j'}^{c,t}$ increasing

$\Rightarrow x^j + 1 \leqslant y^j$, and the order is preserved

Synthesis

Impact of the control triggering :

Triggering policy	Independent control	Application dependent
Push	Monotone	Monotone
Pull	Monotone	Non-monotone

⇒ Almost monotone "Pull on completion" can be simulated with envelopes [BGV08]

Prioritization with index function :

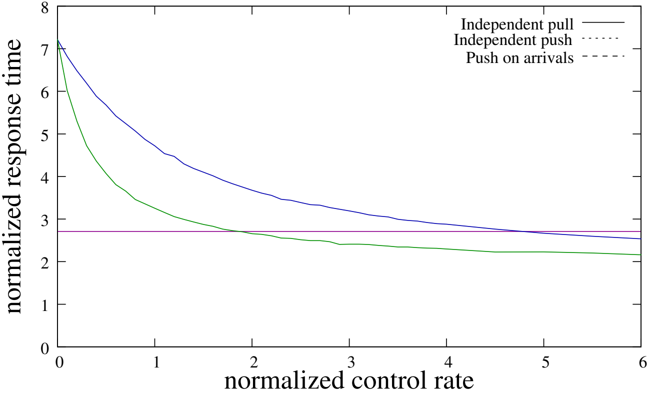
- Using nodes characteristics : CPU speed, capacity ...
- Using system characteristics : network topology
- Threshold criteria
- arbitrary prioritization (priority list)

Random probing : Collection of events with different priority lists

Outline

Estimation of the control rate

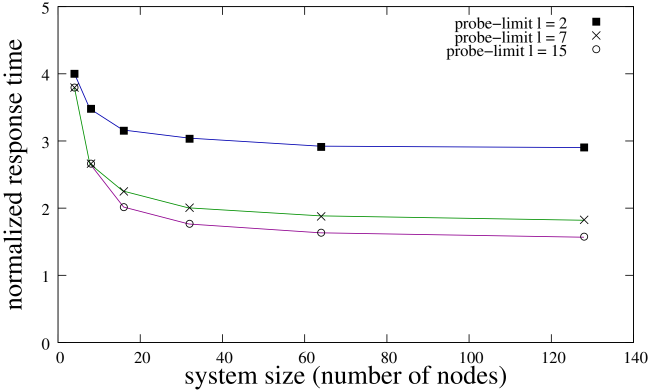
Policy : Controlled Push, Pull and Push on arrivals with random probing of 6 nodes



For a system equipped with a controler, a good operating point is to fix the control rate twice the processor speed.

Estimation of the probe-limit

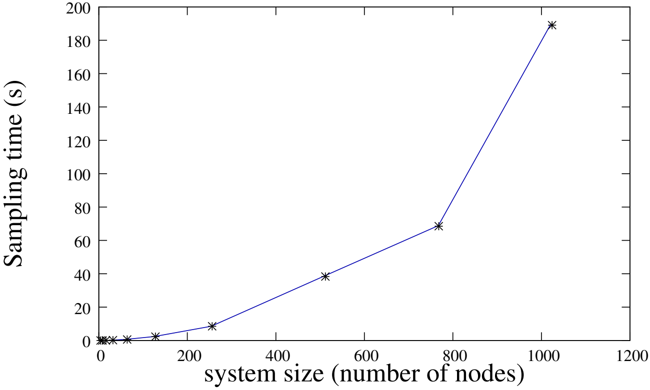
Policy : Controlled Push with random probing of 6 nodes



increasing the Probe-limit further than 7 does not provide a significant performance improvement

Scaling

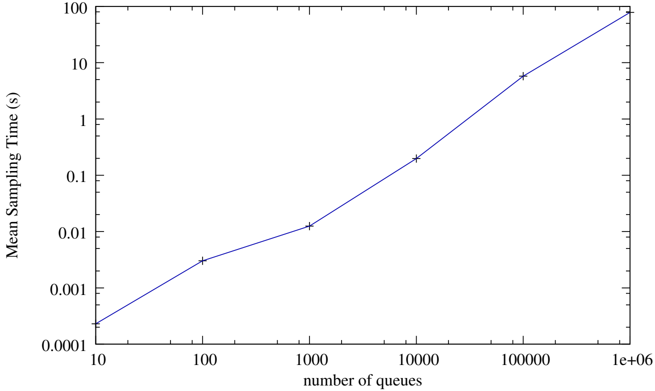
Policy : Controlled Push with random probing of 6 nodes



It is feasible to simulate complex load sharing strategies within a system of 1024 nodes.

Scaling Toward million of nodes

Policy : Threshold Push on Arrival with priority list of 8 nodes



The time to simulate such system is linear with the number of nodes

Outline

Conclusion

Monotonicity result

A framework for modelling monotone control events in load sharing systems

Efficient performance evaluation method and tool

Advantages

- unbiased sampling
- large scale models

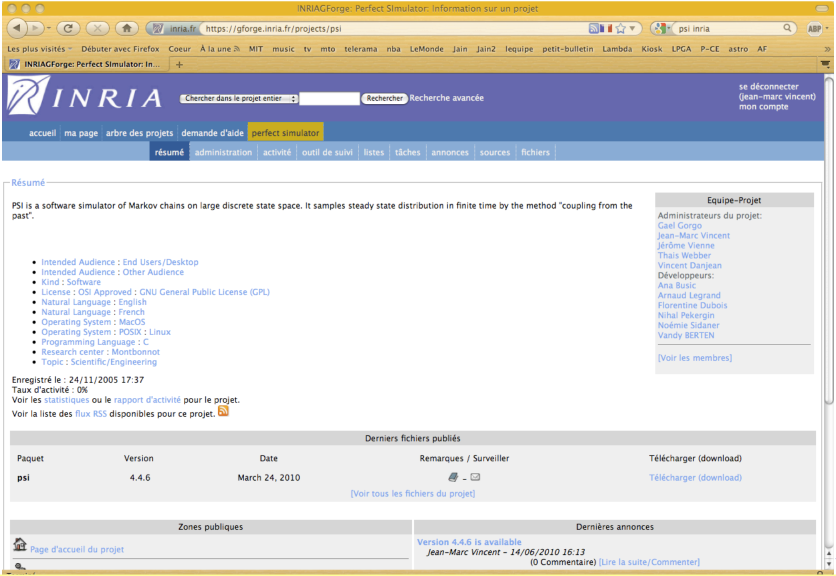
Limitations

- Markovian assumptions everywhere
- No synchronization between jobs
- Migration time neglected

Future work

- Almost monotone events
  $\Rightarrow$ Pull on completion is an almost monotone event and can be simulated with Envelope techniques [BGV08]
- Model refinements
  $\Rightarrow$ Considering transfer costs, more realistic distributions (arrivals, completions)
- More experiments with large scale systems
  $\Rightarrow$ Understanding the behaviour of large scale systems

Download : http ://gforge.inria.fr/projects/psi



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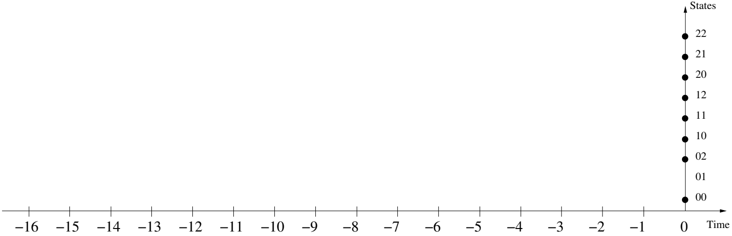
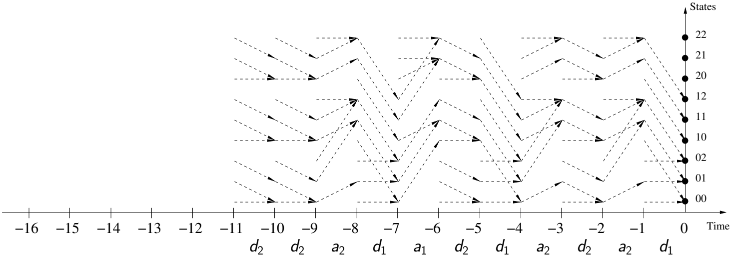
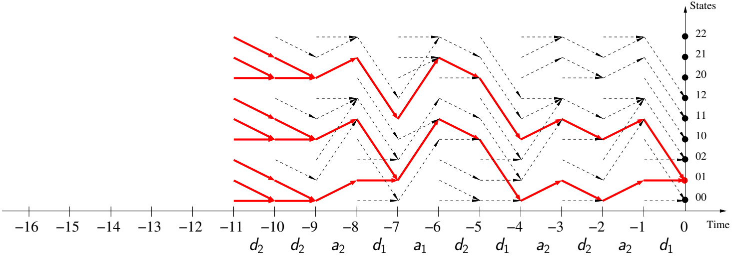
Perfect sampling algorithm



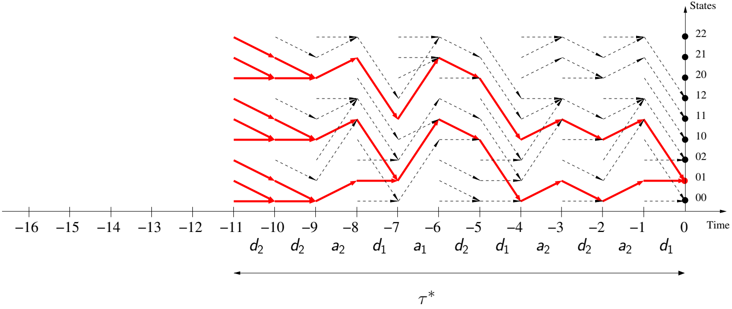
Perfect sampling algorithm



Perfect sampling algorithm



Perfect sampling algorithm

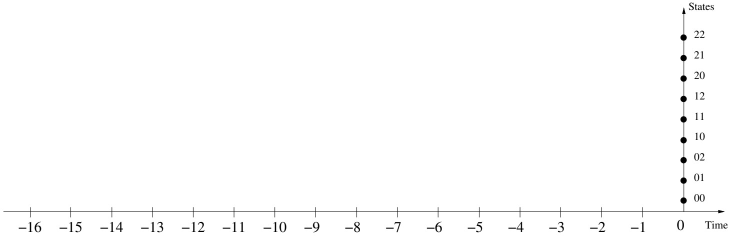


Average time complexity

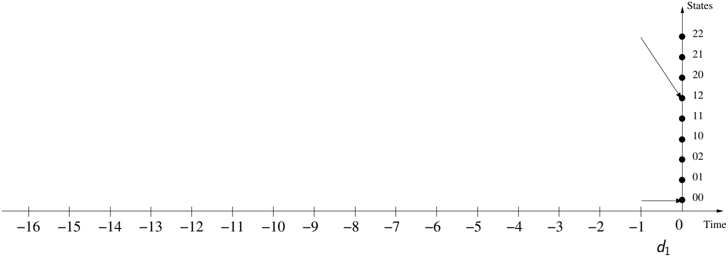
C_Φ mean computation cost of $\Phi(x, e)$ $C = Card(\mathcal{X}).\mathbb{E}(\tau^*).\rfloor_\oplus$

On the example : $C = 9 * 11 = 99$

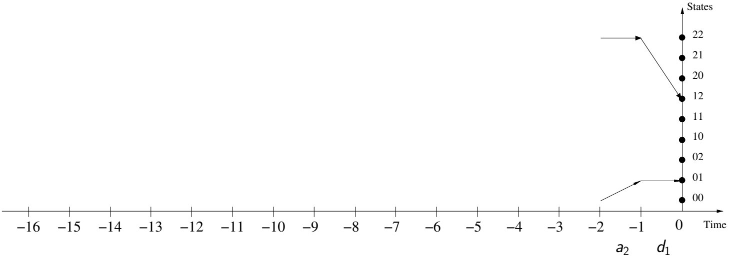
Monotone perfect sampling algorithm



Monotone perfect sampling algorithm



Monotone perfect sampling algorithm



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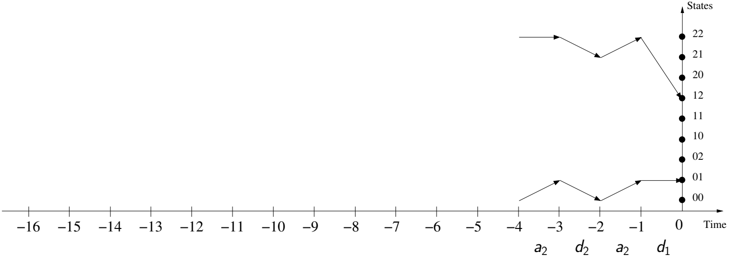

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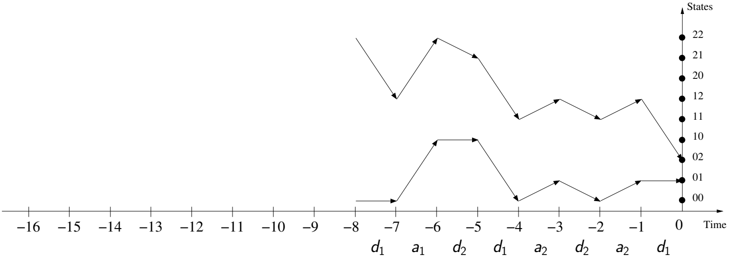
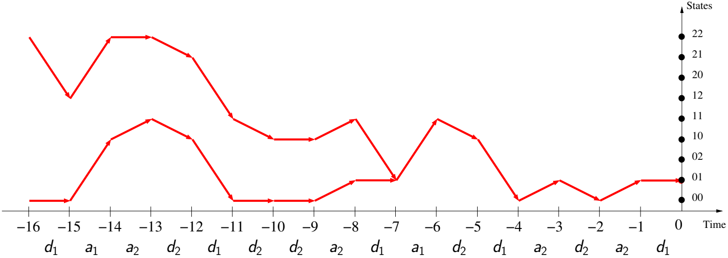
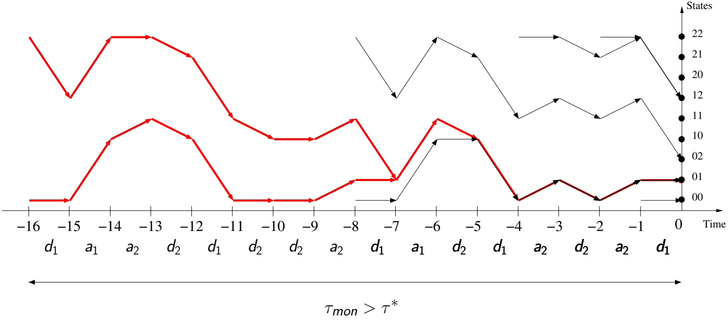


Monotone perfect sampling algorithm



Monotone perfect sampling algorithm



Average time complexity

$C_{mon} \leqslant 2.(2.\mathbb{E}(\tau_{mon}))$

$\Rightarrow$ Reduction factor of $\frac{4}{Card(\mathcal{X})}$

On the example : $C_{mon} = 2 * (1 + 2 + 4 + 8 + 16) = 62$