Random generation of Discrete Random variables

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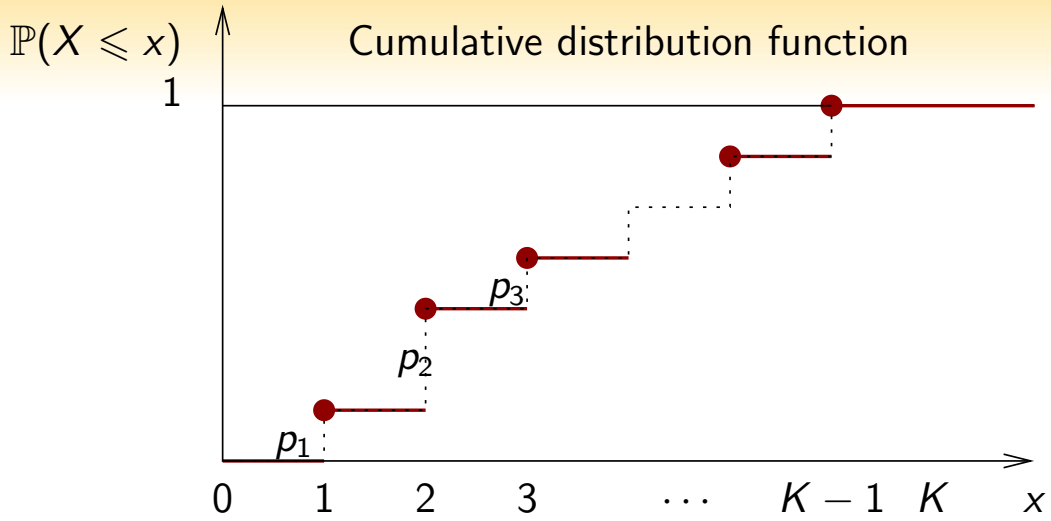
Inverse of PDF



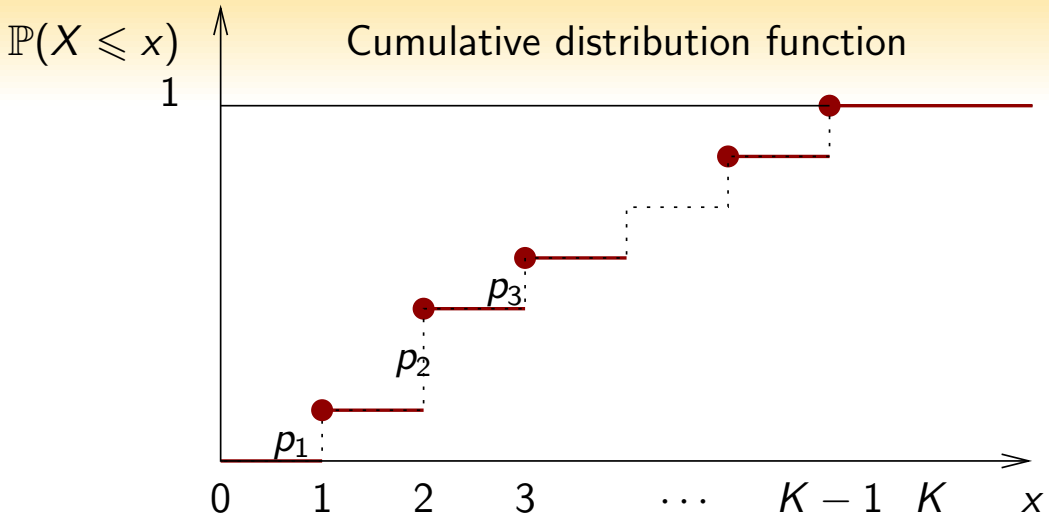
Generation

Divide $[0, 1[$ in intervals with length p_k
Find the interval in which *Random* falls
Returns the index of the interval
Computation cost : $\mathcal{O}(\mathbb{E}X)$ steps
Memory cost : $\mathcal{O}(1)$

Inverse function algorithm

```
s=0; k=0;
u=random()
while u >s do
   k=k+1
   s=s+p_k
end while
return k
```

Inverse of PDF



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Searching optimization

Optimization methods

- pre-compute the pdf in a table
- rank objects by decreasing probability
- use a dichotomy algorithm
- use a tree searching algorithm (optimality = Huffmann coding tree)

Comments

- Depends on the usage of the generator (repeated use or not)
- pre-computation usually $\mathcal{O}(K)$ could be huge
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Aliasing technique

Initialization

K objects

list L=∅,U=∅;
for k=1; k⩽ K; k++ **do**
  P[k]=p_k
  if P[k] ⩾ $\frac{1}{K}$ **then**
    U=U+{k};
  else
    L=L+{k};
  end if
end for

Alias and threshold tables

while L ≠ ∅ **do**
  Extract $k \in L$
  Extract $i \in U$
  S[k]=P[k]
  A[k]=i
  P[i] = P[i] - ($\frac{1}{K}$-P[k])
  if P[i] ⩾ $\frac{1}{K}$ **then**
    U=U+{i};
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Combine uniform and alias value when rejection

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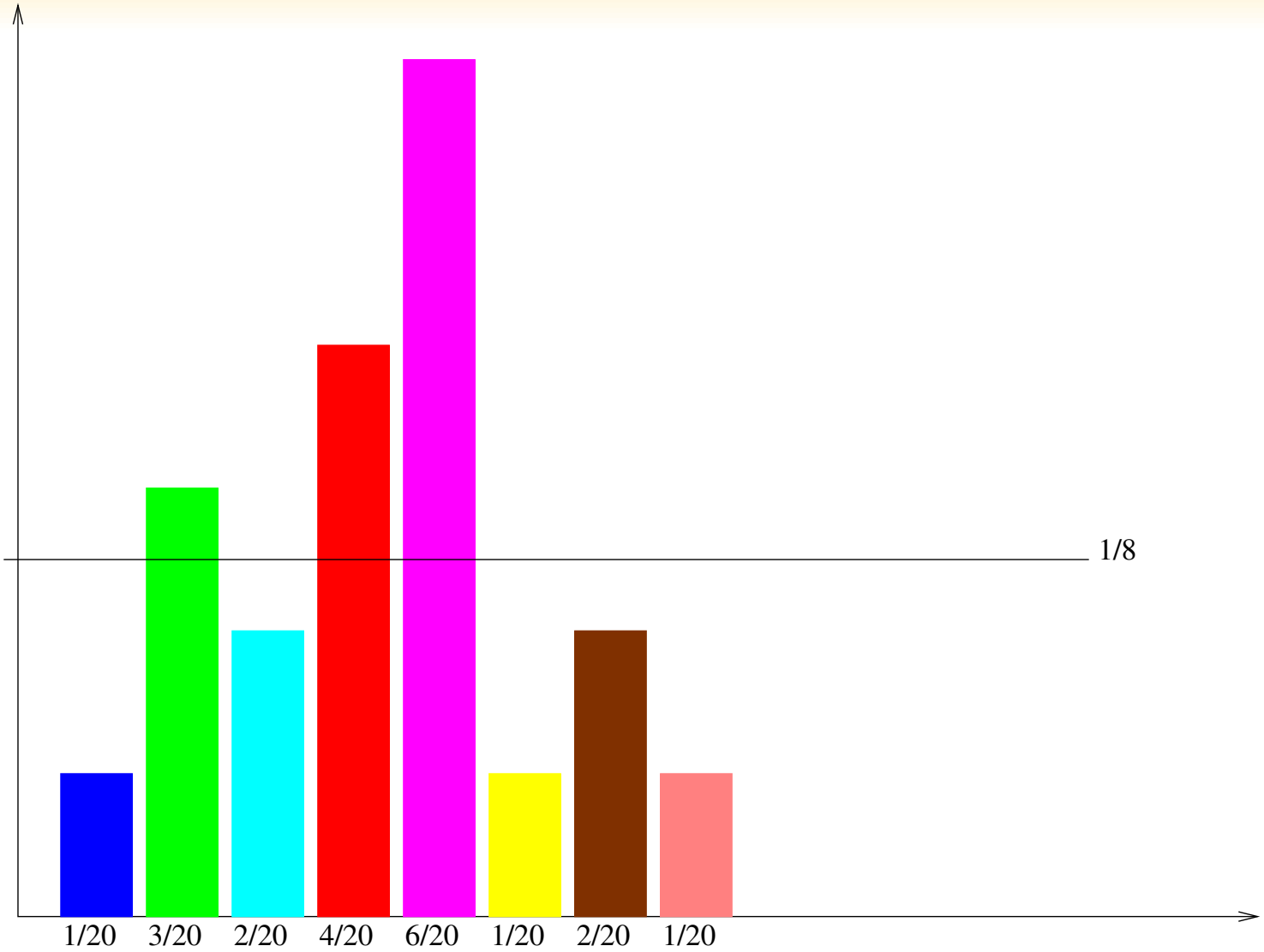

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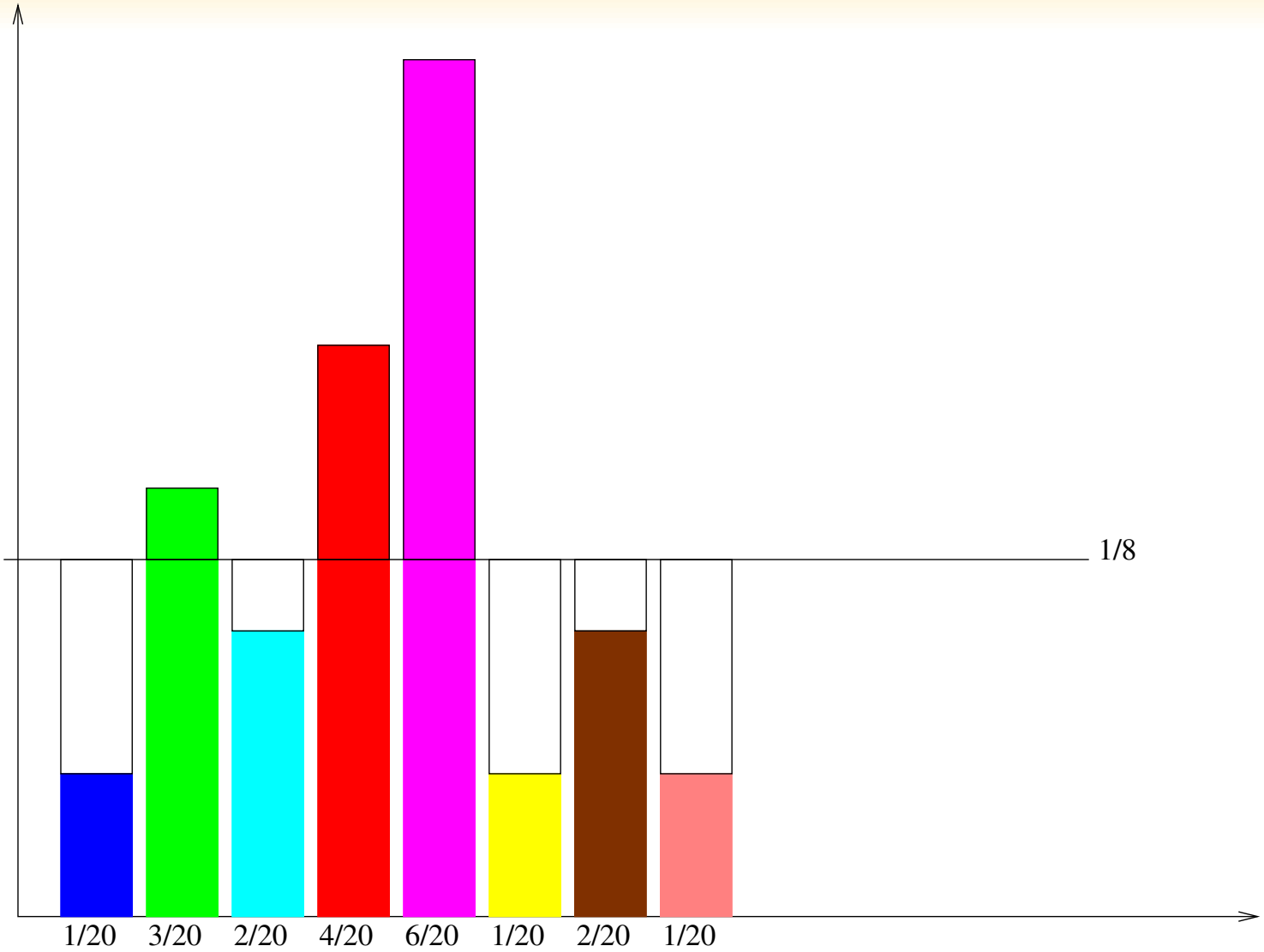

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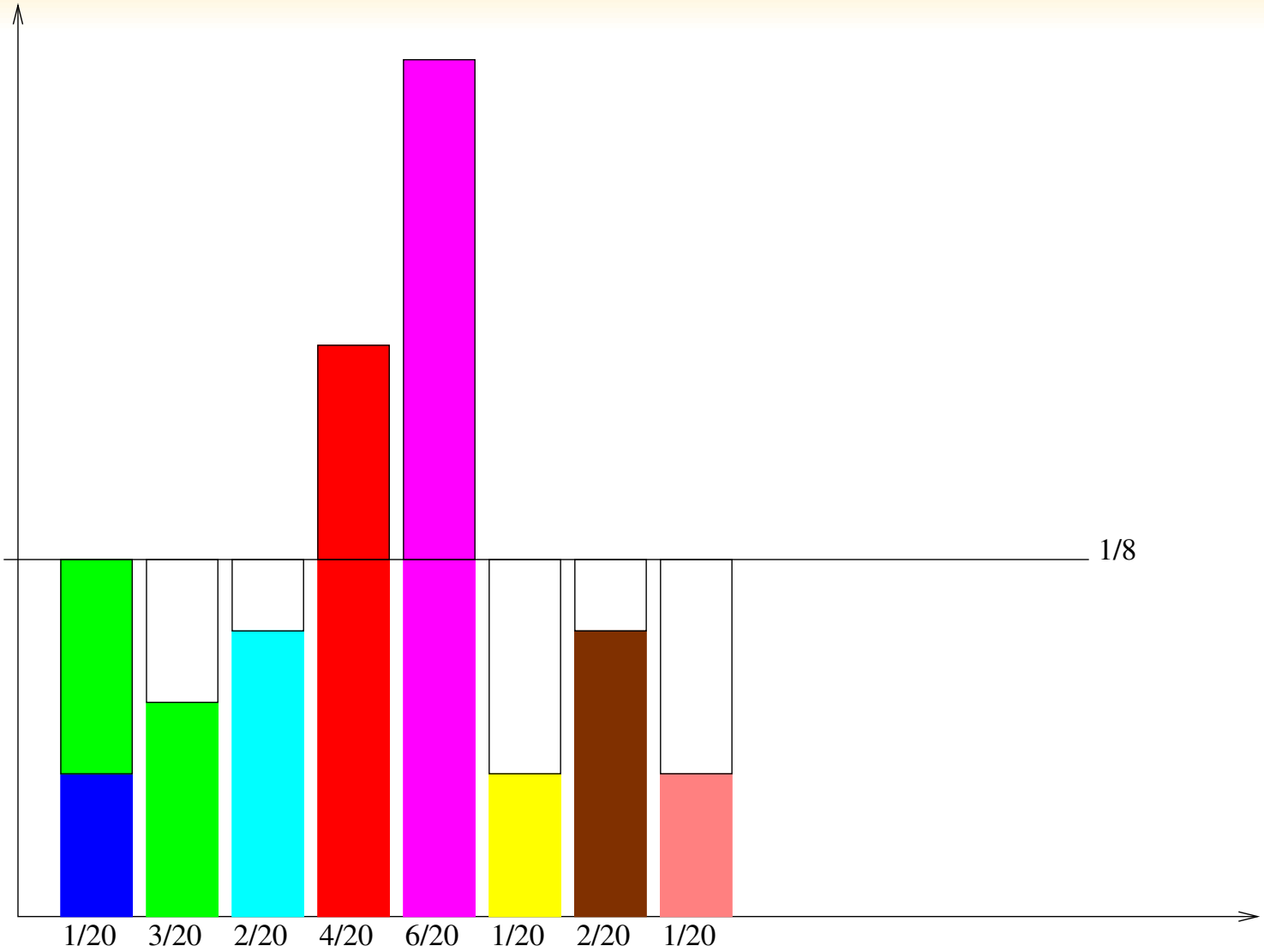

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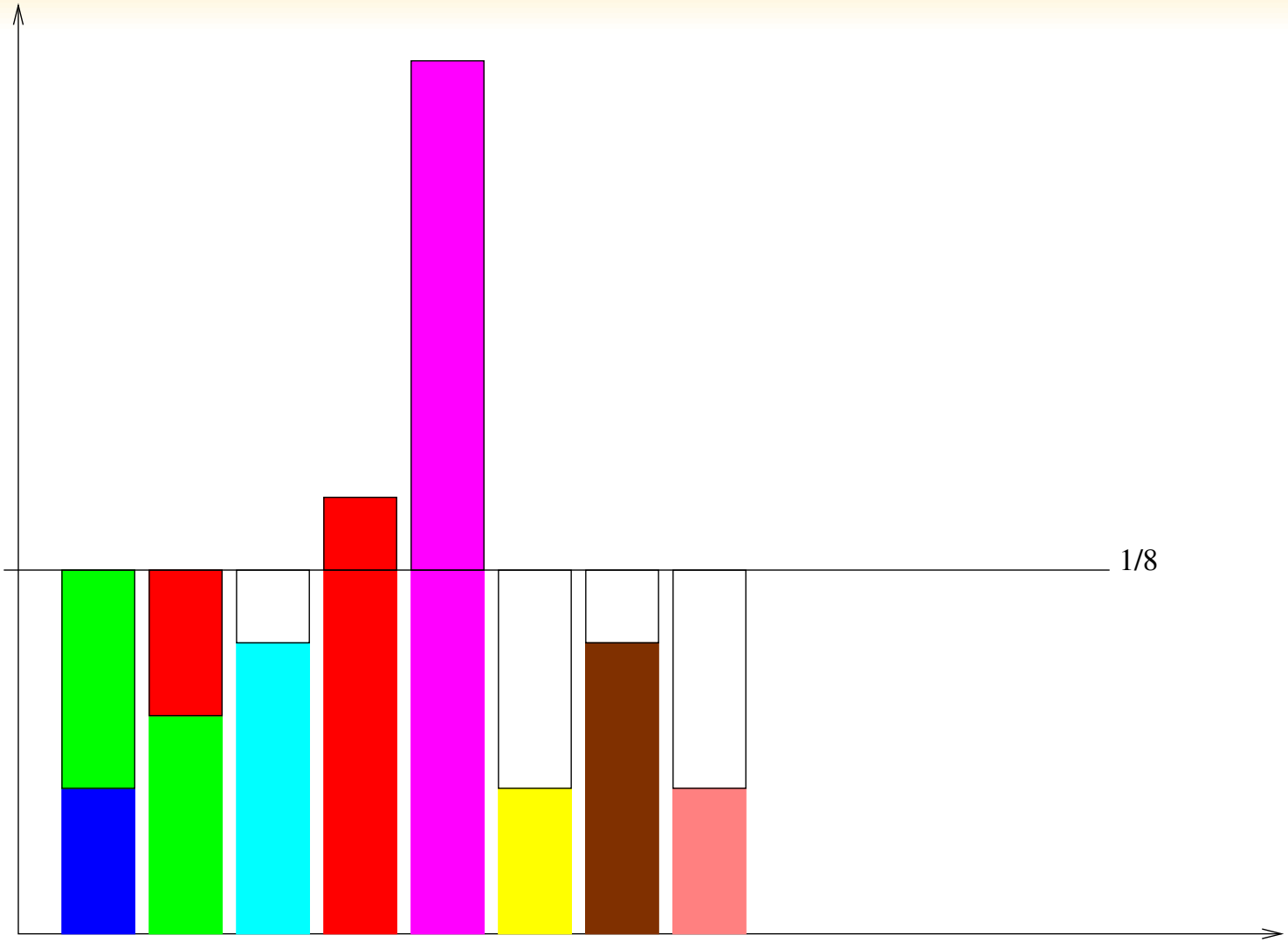

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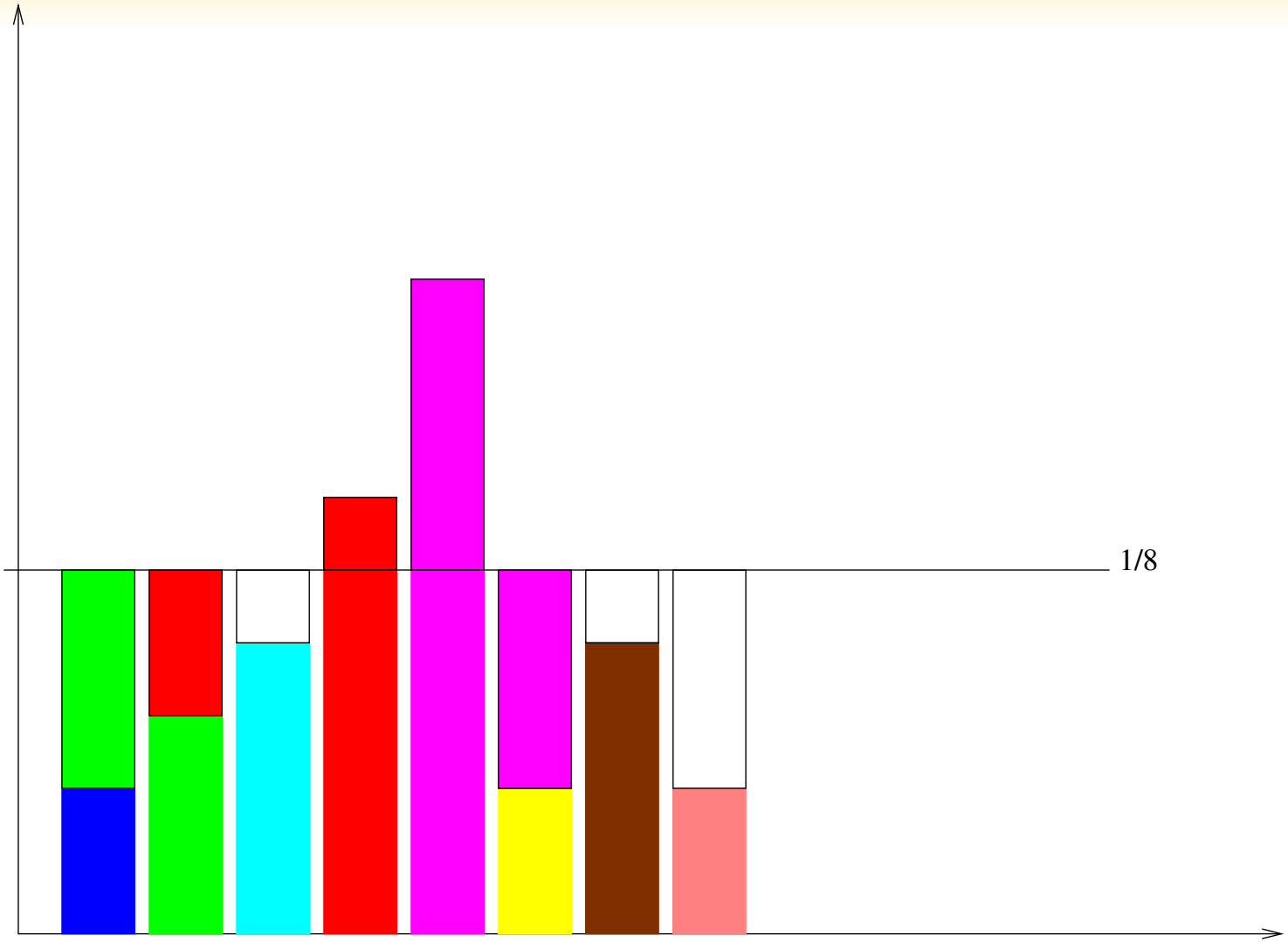

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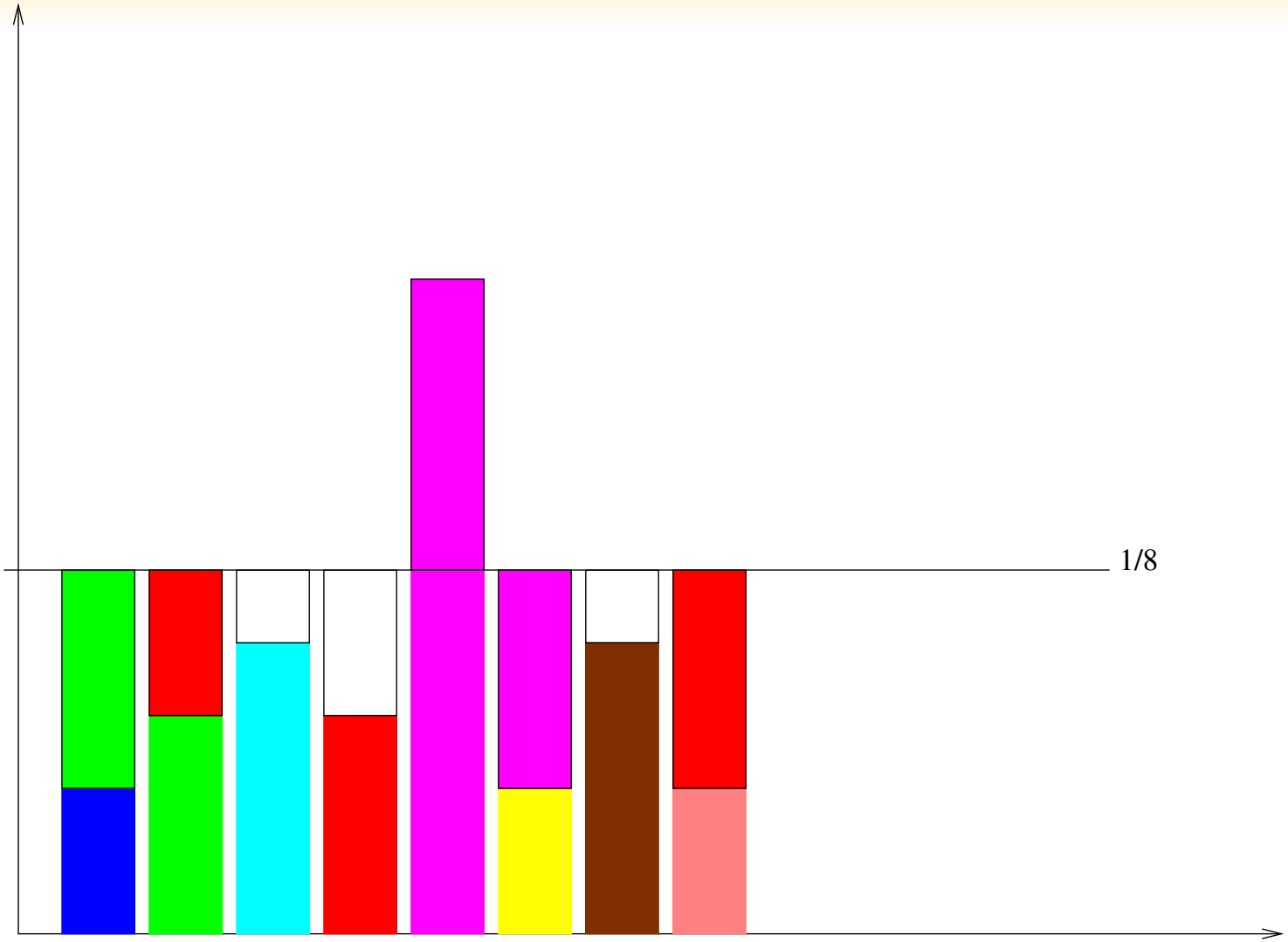

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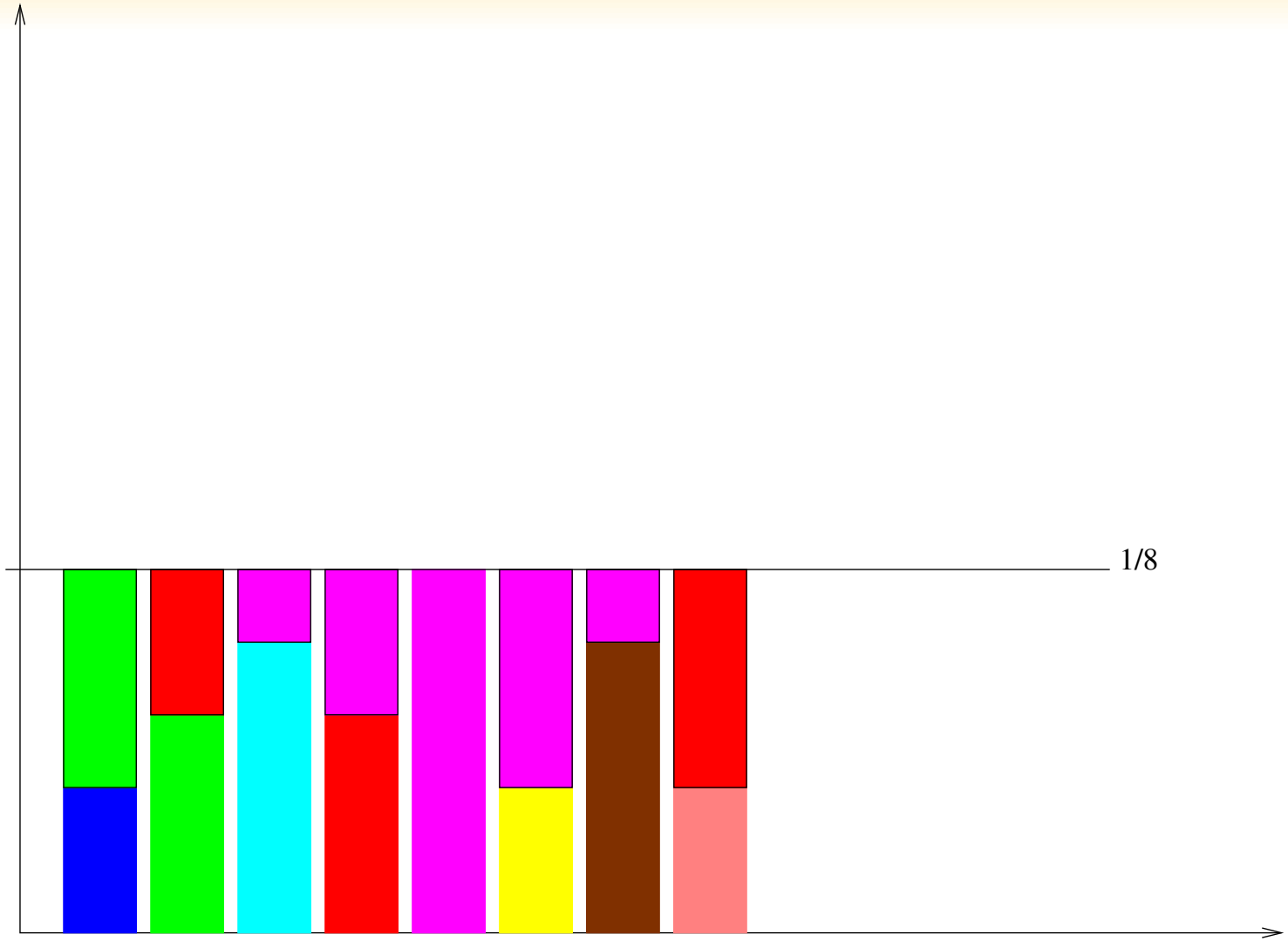

Aliasing technique : generation

Generation

k=alea(K)
if Random . $\frac{1}{K} \leqslant$ S[k] **then**
  return k
else
  return A[k]
end if

Complexity

Computation time :
- $\mathcal{O}(K)$ for pre-computation
- $\mathcal{O}(1)$ for generation

Memory :
- threshold $\mathcal{O}(K)$ (real numbers as probability)
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