Optimal Control and Admission in a Queue

Let us consider a system made by one queue with independent exponential service time with rate μ. Arrivals are modeled by Poisson process with rate λ and the queue capacity is C. Whenever a packet arrives and finds the system full, the packet is rejected.

—— Question 1 : Basic Model

Compute analytically the probability p_r of rejection.

Now assume that when the system is full, an incoming packet is not rejected but sent to a secondary queue whose capacity is C_2 and its service rate is μ_2.

—— Question 2 : Simulator

Write a program that simulates the behavior of the system. Validate (partially) your program using the first question.

Let us consider another system made of two queues where packets are sent to queue 1 with probability $1 - p_r$ and to queue 2 with probability p_r, regardless of the current backlog in the buffers.

—— Question 3 : Equivalent Network ?

Are those system different in terms of the average number of packets in both queues ?

In the following we introduce a controller that rejects the packets before full capacity to reduce the cost of the system. The cost is : the average response time of the accepted packets $+ R \times$ "the probability of rejection".

—— Question 4 :

If the controller decides to reject with probability q, compute its average cost and its optimal rejection probability.

Now the controller uses a different strategy of rejection. For each packet, the controller takes a deterministic decision : accept ($a = 1$) or reject ($a = 0$).

—— Question 5 : Transitions

Let us consider the discrete time version of the system. Let $p = \mu/(\mu + \lambda)$ and $q = \lambda/(\mu + \lambda)$. construct the transition matrices for the two actions $a = 1$ and $a = 0$.

—— Question 6 : Cost function

If x is the current number of packets in the system, find the instantaneous cost $g(x, a)$ to minimize the average response time of the packets when they are not rejected while each rejected packet costs R.

—— Question 7 : Finite horizon

Find the finite horizon optimal policy under the following data : $\mu = 3, \lambda = 2, C = 10$ (with an adequate final cost).

—— Question 8 : Infinite horizon

Compute the infinite horizon policy with $\mu = 3, \lambda = 2, C = 10, \alpha = 0.9$.
Can you describe the optimal policy ?

Let us assume the the arrival rate is not well known.
We only know that $\lambda \in [\lambda_1, \lambda_2]$.
—— Question 9 : M

ake a numerical study that tests if this uncertainty degrades the performance of the controller.

—— Question 10 : multiple queues

Consider again the system made of 2 queues with respective rates $/mu_n$. The controller routes the packets.

Adapt the previous approach for optimal control in this case. Can you give properties of the optimal policy in the infinite horizon case ?